An Innovative Topological approach for Triangular Intuitionistic fuzzy transportation problem by Topologized Graph

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Abstract

In this paper we have introduced and investigated an innovative method to reach the optimal solution for the intuitionistic fuzzy transportation problem using topologized graph. First we transform the intuitionistic fuzzy transportation problem into a crisp transportation problem using prescribed ranking technique then we represent it in a graph using topologized graph. After that, it follows the proposed methodology to reach the optimal cost which starts form the minimum cost amongst all the transportation cost. This approach produces a new relationship of Topological spaces, Intuitionistic Fuzzy transportation problems and Graph theory.

1 Introduction

In this modern era Topology and Graph theory are the two major application areas of Mathematics. Topological spaces and graphs have a considerable role in optimizing techniques, Network analysis, Image processing, Geographic Information Systems (GIS) and Embedding or Printing Circuit in Electronics. In 2005, Antonie vella[1] familiarized the foundations of topological properties on graph theory. In view of that Vimala and Kalpana [20] developed the concept named Topologized Bipartite Graph and applied it in Matching and Coloring.

Presently, there are many multifaceted circumstances in engineering and business, in which experts and decision makers struggle with ambiguity and uncertainty. In actual life situations, collection of crisp data of diverse parameters is challenging due to lack of accurate interactions, inaccuracy in data, market knowledge and customer's satisfaction. The information available is sometimes unclear and inadequate. The real life problems, when defined by the decision maker with uncertainty leads to the notion of fuzzy sets. Due to imprecise information, the exact evaluation of membership values is not possible. Moreover, the evaluation of the non-membership values is always impossible. This leads to an indeterministic environment where hesitation survives. Dealing with inexact information while making decisions, the concept of fuzziness was introduced by Zadeh [21]. K. T, Atanassov [2] introduced the concept of Intuitionistic fuzzy set theory it consists both membership and non-membership values, which is more apt to deal with such problems.

Intuitionistic fuzzy numbers provides lot of applications towards transportation problems. It paves the way to obtain the exact cost of the transportation problems which are having uncertainity and ambiguous in transportation cost.In 2012 Nagoor Gani et al. Solving Intuitionistic Fuzzy Transportation Problem using Zero Suffix Algorithm. Srinivas and Ganesan have introduced Optimal solution for intuitionistic fuzzy transportation problem via revised distribution method. K. Pramila and G.Uthra [13] presented an optimal solution of an intuitionistic fuzzy transportation problem. Paul et al. [8] proposed a new method for solving transportation problem using triangular intuitionistic fuzzy number. Gani et al. [11] proposed the revised distribution method for solving intuitionistic fuzzy transportation problem. Geetharamani et al., [5] introduced an innovative method to solve fuzzy transportation problem via. Robust ranking. Gani et al., [10] suggested a new method for solving intuitionistic fuzzy transportation problem. Hussain et al., [6] discussed algorithmic approach for solving intuitionistic fuzzy transportation problem. In 2014, Pramila and Uthra [13] have discussed the intuitionistic fuzzy transportation problem by using accuracy function to defuzzy the Intuitionistic fuzzy number. Recently in 2017, Bharati[4] introduced the new ranking method of Intuitionistic fuzzy number. Based on this ranking method, in this paper, we discussed the new method to obtain the optimal transportation cost of Triangular Intuitionistic fuzzy transportation problem. Santhi and Kungumaraj [15] have obtained the optimal transportation cost the using ranking technique.

A small account of researchers have evolved various approach to come across optimal solution of a intuitionistic fuzzy transportation problem. But in the topological point of view a few of the researchers analyzed the solution of Transportation problem. Therefore, we have established the latest style of topological approach for intuitionistic fuzzy transportation problem to attain the optimal solution which is developed based on the methods topological solultion of a transportation problem and fuzzy transportation problem using topologized graph by Santhi and Kungumaraj[16] in 2019.

2 Preliminaries

Definition 2.1. Fuzzy Set
Let X be a non-empty set. A fuzzy set $\bar{A}$ of X is defined as $\bar{A} = (x, \mu_{\tilde{A}}(x); x \in X)$ where $\mu_{\tilde{A}}(x)$ is called the membership function which maps each element of X to a value between 0 and 1.

Definition 2.2. Fuzzy Number
A fuzzy number $\bar{A}$ is a convex normalized fuzzy set on the real line R such

that:
(i) There exist at least one $x \in R$ *with* $\mu_{\tilde{A}}(x) = 1$
(ii) $\mu_{\tilde{A}}(x)$ *is piecewise continuous.*

Definition 2.3. Triangular Fuzzy Number
A fuzzy number $\bar{A}$ *is denoted by 3- tuples* (a_1, a_2, a_3) *where* a_1, a_2, a_3 *are real numbers and* $a_1 \le a_2 \le a_3$ *with the membership function defined as*

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & if\ a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & if\ a_2 \le x \le a_3 \\ 0, & otherwise \end{cases}$$

Definition 2.4. Intuitionistic Fuzzy set
Let X be a nonempty set. An intuitionistic fuzzy set $\tilde{A}^I$ *of X is defined as* $\tilde{A}^I = (x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x)); x \in X$ *where* $\mu_{\tilde{A}^I}(x)$ *and* $\vartheta_{\tilde{A}^I}(x)$ *are the membership function and non-membership function such that* $\mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x) : X \to [0,1]$ *and* $0 \le \mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$ *for all* $x \in X$

Definition 2.5. Intuitionistic Fuzzy number
An intuitionistic fuzzy subset $\tilde{A}^I = (x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x)); x \in X$ *of the real line R is called an intuitionistic fuzzy number (IFN) if the following conditions holds:*
(i) There exist $x \in R$ *such that* $\mu_{\tilde{A}^I}(x) = 1$ *and* $\vartheta_{\tilde{A}^I}(x) = 0$
(ii) $\mu_{\tilde{A}^I}(x)$ *is continuous function from* $R \to [0,1]$ *such that* $0 \le \mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$ *for all* $x \in X$

Definition 2.6. Triangular intuitionistic fuzzy number
A Triangular intuitionistic fuzzy number(TIFN) $\tilde{A}^I$ *is an Intuitionistic fuzzy set in R with the following membership function* $\mu_{\tilde{A}^I}(x)$ *and non-membership function* $\vartheta_{\tilde{A}^I}(x)$

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & if\ a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & if\ a_2 \le x \le a_3 \\ 0, & otherwise \end{cases} \quad and \quad \vartheta_{\tilde{A}}(x) = \begin{cases} \frac{a_2-x}{a_2-a_1^{'}}, & if\ a_1^{'} \le x \le a_2 \\ \frac{x-a_2}{a_3^{'}-a_2}, & if\ a_2 \le x \le a_3^{'} \\ 1, & otherwise \end{cases}$$

where $a_1^{'} \le a_1 \le a_2 \le a_3 \le a_3^{'}$ *and* $\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \le 1$, *or* $\mu_{\tilde{A}^I}(x) = \vartheta_{\tilde{A}^I}(x)$ *, for all* $x \in R$*. This TIFN is denoted by* $\tilde{A}^I = (a_1, a_2, a_3; a_1{'}, a_2, a_3{'})$.

Definition 2.7. Topologied graph
A topologized graph is a topological space X such that (i)Every singleton is open or closed.
(ii) For every $x \in X$*,* $|\partial(x)| \le 2$*, since* $\partial(x)$ *is denoted by the boundary of a*

point x.
Here the topology is defined on the graph, since the space X is the union of vertices and edges.

Definition 2.8. Topologied Bigraph or Bipartite
Let G be bipartite graph with V vertices and E edges. A topological space is a topologized graph if it satisfies the following conditions : (i)Every singleton is open or closed.
(ii) For every $x \in X$, $|\partial(x)| \leq 2$, since $\partial(x)$ is denoted by the boundary of a point x.
Here the topology is defined on the graph, since the space X is the union of vertices and edges.

Definition 2.9. Defuzzification by Ranking Function
Let $\bar{A}^I = (a_1, a_2, a_3)(b_1, b_2, b_3)$ be triangular intuitionistic fuzzy number. The ranking of a TIFN is given by

$$R(\tilde{A^I}) = \frac{1}{2}\left(\frac{a_3-a_1}{b_3-b_1}\right)(D^S(\bar{A}^I, \bar{O}^I))$$

where $(D^S(\bar{A}^I, \bar{O}^I)) = \frac{(a_1+a_3+4a_2+b_1+b_3)}{8}$

3 Intuitionistic fuzzy Transportation problem using Topological graph

Consider the intuitionistic fuzzy transportation problem with m intuitionistic fuzzy origins (rows) and n intuitionistic fuzzy destinations (columns). Let $\tilde{C_{ij}}$ be the cost of transporting one unit of the product from i^{th} intuitionistic fuzzy origin to j^{th} intuitionistic fuzzy destination. $\tilde{\tilde{a}}_i$ be the quantity of commodity available at intuitionistic fuzzy origin i, $\tilde{b_j}$ be the quantity of commodity needed at fuzzy destination j. x_{ij} is quantity transported from i^{th} intuitionistic fuzzy origin to j^{th} intuitionistic fuzzy destination.

Mathematical model of Intuitionistic Fuzzy Transportation problem is
$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} * x_{ij}$
subject to $\sum_{j=1}^{n} x_{ij} = a_i, i = 1, 2, ..., m$
$\sum_{i=1}^{m} x_{ij} = b_j, j = 1, 2, ..., n$ Here x_{ij} is non-negative triangular Intuitionistic Fuzzy number
n = total number of sources
m = total number of destinations
c_{ij} = the Intuitionistic fuzzy transportation cost for unit quantity of the product from i^{th} source to j^{th} destination.

x_{ij}^{th} = the Intuitionistic fuzzy quantity of the product that should be transported from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost.
$\sum_{i=1}^{m} a_i$ = total Intuitionistic fuzzy availability of the product
$\sum_{j=1}^{n} b_j$ = total Intuitionistic fuzzy demand of the product.

3.1 Topologized Graphical Method

The proposed algorithm of Topologized Graphical Method(TGM) consists of the following steps:
Step 1: Construct the Intuitionistic fuzzy transportation table for the given intuitionistic fuzzy transportation problem, then convert it into balanced one, if it is not.
Step 2: Apply the proposed ranking technique to convert the Intuitionistic fuzzy transportation problem into a crisp transportation problem.
Step 3 Represent the Intuitionistic fuzzy transportation problem in graph by taking supply and demand points as vertices and edges are denoted for unit transportation cost from i^{th} supply point to j^{th} demand point.
Step 4: In each row or column choose the first two minimum transportation cost then modify the graphical representation and generate a topological space whose elements are vertices and edges of the modified graph. Suppose the modified graph satisfies the two condition of the topologized graph go to the next step or else rearrange the graph and make it as a topologized graph.
Step 5: Start the allocation from the vertex which has the minimum transportation cost and allocate the maximum quantity based on the rim requirement. The allocations should be made from both the edges of that vertex. Continue the same process until the demand and supply are exhausted.
Step 6: Suppose any of the quantities are not allocated in any one of the supply points or demand points, select that particular vertex and include one edge and allocate accordingly to satisfy the remaining supply and demand quantities.
Step 7: If the allocation table consists of two or three allocations not more than three in each row and column then it gives the optimal solution for the given intuitionistic fuzzy transportation table. Otherwise change the topologized graph and repeat the steps from 4 to 7.

3.2 Numerical Example

Consider an Intuitionistic fuzzy transportation problem whose cost, supply and demand values are triangular intuitionistic fuzzy number. Find the optimal solution of the triangular intuitionistic fuzzy transportation prob-

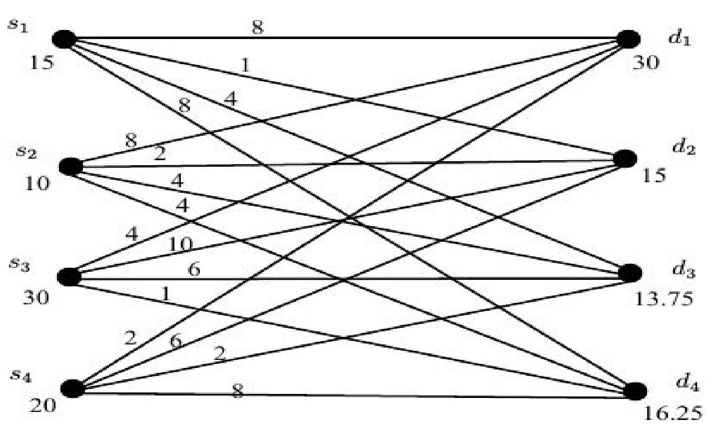
	d_1	d_2	d_3	d_4	IF Capacity
s_1	(14,16,18;12,16,20)	(0,1,2;0,1,3)	(7,8,9; 6,8,10)	(11,13,15;10,13,16)	(4,8,12;2,8,14)
s_2	(8,11,14;7,11,15)	(3,4,5;2,4,6)	(5,7,9;4,7,10)	(8,10,12;6,10,14)	(10,12,14;8,12,16)
s_3	(6,8,10;5,8,11)	(13,15,17;12,15,18)	(7,9,11;6,9,12)	(1,2,3;0,2,4)	(14,16,18;12,16,20)
s_4	(5,6,7;4,6,8)	(11,12,13;10,12,14)	(3,5,7;1,5,9)	(12,14,16;11,14,17)	(16,20,24;12,20,28)
IF Demand	(3,4,5;2,4,6)	(3,5,7;1,5,9)	(10,12,14;8,12,16)	(6,7,8;5,7,9)	

lem (TIFTP).

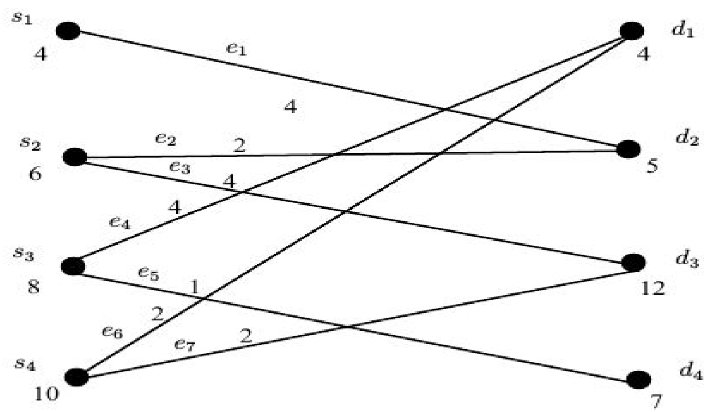
As per the proposed algorithm we proceed towards the optimal solution of the above TIFTP. By using the prescribed Ranking technique we convert the above intuitionistic fuzzy transportation problem into a crisp transportation problem. It is given below

	d_1	d_2	d_3	d_4	IF Supply
s_1	8	1	4	8	4
s_2	8	2	4	4	6
s_3	4	10	6	1	8
s_4	2	6	2	8	10
IF Demand	4	15	12	7	

The graphical representation of the above intuitionistic fuzzy transportation problem is given below.



The modified representation of the above intuitionistic fuzzy transportation problem shown here.

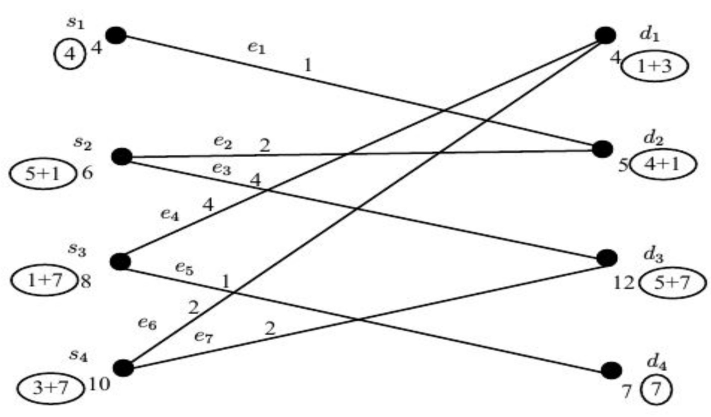


Here s_1, s_2, s_3, s_4, d_1, d_2, d_3, d_4 are vertices of the graph which are denoted for the supply and demand points and $e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8$ are the edges between the supply points and the demand points. Now we should verify the above graph is a topologized graph.

Let $X = \{s_1, s_2, s_3, s_4, d_1, d_2, d_3, d_4\}$ be a topological space with the topology
$\tau = \{\{\phi\}, X, \{s_1\}, \{s_3\}, \{s_1, s_3\}, \{s_1, e_1, d_2, e_2, s_2, e_3, d_3, e_7, s_4\}, \{e_6, d_1, e_4, s_3, e_5, d_4\}, \{s_3, s_1, e_1, d_2, e_2, s_2, e_3, d_3, e_7, s_4\}, \{s_1, e_6, d_1, e_4, s_3, e_5, d_4\}\}$.

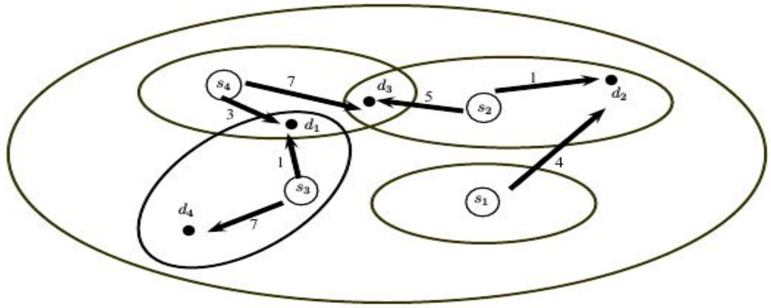
Since every singleton sets are either closed or open in X and every vertex having the boundary as 2 thus the above graph is a topologized graph.

Hence we start our allocation from s_3 which possess the minimum unit transportation cost to d_4. Similarly allocate all the quantities as per our procedure. The allocation between supply points and demand points is as follows



The consequent diagram gives the topological allocation of given intuitionistic fuzzy transportation problem based on the above stated methodology

Topological Allocation of the TIFTP



The same allocation also given in the traditional transportation allocation table;

	d_1	d_2	d_3	d_4	IF Supply
s_1	8	1(4)	4	8	4
s_2	8	2(1)	4(5)	4	6
s_3	4(1)	10	6	1(7)	8
s_4	2(3)	6	2(7)	8	10
IF Demand	4	15	12	7	

The total transportation cost of the quantities as per the TGM $= 4*1+2*1+4*5+4*1+1*7+2*3+2*7 = 57$

4 Conclusion

This paper recommended an innovative approach to reach the optimal solution for Intuitionistic Fuzzy Transportation Problem through topologized graph. It procures significant association amidst Topological space, Fuzzy Transportation Problem and graph theory. Subsequently it puts forward to conquer minimum intuitionistic fuzzy transportation cost comparing any other existing methods. Another advantage of operating this methodology is, it involves less amount of time and minimum number of steps for calculation.

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