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RESEARCH ARTICLE

Generalized Semi Operators in Soft Multi Topological Spaces

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ABSTRACT

This paper's major goal is to investigate the properties of generalized semi soft multi course and generalized semi soft multi interior with the help of generalized semi closed soft multiset and generalized semi open soft multiset notions and to acquaint the conception of generalized semi soft multi neighbor hoods.

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Keywords and Phrases: Soft multiset, soft multi topology, generalized semi closed soft multiset, generalized semi soft multi closure, generalized semi soft multi interior and generalized semi soft multineighbourhood.

INTRODUCTION

In 2013, Babitha and John [1] was acquainted the conception of soft multisets as a combination of soft sets and multisets. Then Deniz Tokat et al. [2,3] acquainted the concept of the soft multi topology and its basic properties. S. A. El-Sheikh et al. [4,5] acquainted the generalized closed soft multisets and generalization of open soft multisets and mappings in soft multi topological spaces. In 2022, Inthumathi et al. [6] acquainted the concept of generalized semi closed set in soft multi topological space. In this paper, we acquaint the conception of generalized semi soft multi closure, generalized semi soft multi interior and generalized semi soft multi neighbourhood and discuss few important attributes in detail.

PRELIMINARIES

Throughout this paper, SMTS denotes the soft multi topological spaces and smset denotes soft multi set.

Definition 2.1. [3] Let M be an universal mset, L be a set of parameters and $\boldsymbol{I} \subseteq \boldsymbol{L}$. Then, an ordered pair $(\boldsymbol{S},\boldsymbol{I})$ is called a smset where S is a mapping given by $\boldsymbol{S}: \boldsymbol{I} \rightarrow \boldsymbol{P}^*(\boldsymbol{M})$; $\boldsymbol{P}^*(\boldsymbol{M})$is the power set of a mset M. For all l ∈ I, S(l) mset represents by count function $\boldsymbol{C}_{\boldsymbol{S}(\boldsymbol{l})}: \boldsymbol{M}^* \rightarrow \boldsymbol{N}$ where $\boldsymbol{N}$ represents the set of non-negative integers and $\boldsymbol{M}^*$represents the support set of $\boldsymbol{M}$..

Definition 2.2. [3] For two soft msets$(\boldsymbol{S},\boldsymbol{I})$ and $(\boldsymbol{T},\boldsymbol{J})$ over $\boldsymbol{M}$, we say that is a subs mset of if:

Definition 2.2. [3] For two soft m sets(S,I) and (T,J) over M, we say that (S,I) is a subsmsetof(T,J)if:
1.$I \subseteq J$,
2.$C_{S(l)}(v) \leq C_{T(l)}(v), \forall v \in M^*, \forall\ l \in I \cap J$.
We write $(S,I) \,\tilde{\subseteq}\, (T,J)$.

Definition 2.3.[3] The union of two smsets(S,I) and (T,J)over $M$is the smset(H,C),where$C =$ I∪ $J$and$C_{H(l)}(v) = \max\{C_{S(l)}(v), C_{T(l)}(v)\}$, $\forall l \in I \cup J$, $\forall\ v \in M^*$.We write$(S,I) \,\tilde{\cup}\, (T,J)$.

Definition 2.4. [3] The intersection of two soft msets(S,I) and (T,J)over $M$is the smset(H,C),where$C =$ I ∩$J$and$C_{H(l)}(v) = \min\{C_{S(l)}(v), C_{T(l)}(v)\}$, $\forall l \in I \cap J$, $\forall\ v \in M^*$.We write$(S,I) \,\tilde{\cap}\, (T,J)$.

Definition2.5. [3] A soft mset (S,I) over M is said to be a null smset denoted $\tilde{\phi}$ if for all $l \in I, S(l) = \phi$.

Definition2.6. [3] A soft mset(S,I)over M is said to be an absolute smset denoted by $\tilde{M}$ for all $l \in I, S(l) = M$
Definition2.7.[3] The complement of a smset (S,I) is denoted by $(S,I)^c$and is defined by $(S,I)^c = (S^c,I)$ where $S^c: I \rightarrow P^*(M)$ is mapping given by $S^c(l) = M \backslash S\,(l)$for all $l \in I$ where $C_{S^c(l))}(v) = C_{U(l))}(v) - C_{S(l))}(v)$, $\forall\ v \in M^*$.

Definition 2.8.[3] Let Mbe an universal mset and$L$be a set of parameters. Then, the collection of all smsets over M with parameters from$L$is called a soft multi class and is denoted as $SMS(M)_L$.

Definition 2.9. [3] Letτ ⊆ SMS(M)L, then τ is said to be a soft multi topology onMif the following conditions hold:
1. $\tilde{\phi}, \tilde{M}$ belong to τ
2. The union of any number of smsets in τ belongs to τ,
3. The intersection of any two smsets in τ belongs to τ

τ is called a soft multi topology over M and the triple (M, τ, L) is called a soft multi topological space over M. Also, the member of τ are said to be open soft msets in M. A soft mset (S,L) in $SMS(M)_L$ is said to be a closed soft mset in M, if its complement$(S,L)^c$ belongs to τ.

Definition 2.10. Let (M,τ,L) be a SMTS over $M$and (S,L) be a smset over M.Then, the softmulti closure of(S,L), denoted by$cl(S,L)$[$or\ \overline{(S,L)}$] is the intersection of all closed smset containing(S,L).

Definition 2.11. [2] Let (M, τ, L) be a SMTS over M and (S, L)be a smset over M. Then, the soft multi interior of (S, L), denoted by int (S,L) [$or\ (S,L)^o$] is the union of all open soft multiset contained in(S,L).

Definition 2.12. Let (M,τ,L) be a SMTS over M and (G,L) be a smset over$M$and$v \in M$.Then,$(G,L)$is said to be a soft multi neighbourhood of v if there exists a soft multi open set (F,L) suchthat$v \in (F,L) \subseteq (G,L)$.The set of all soft multi neighbourhood of α, denoted by $\widetilde{N}(\alpha)$, is called the family of soft multi neighbourhoods of α,
i.e.$\widetilde{N}(\alpha) = \{(G,L):(G,L) \in \tau, \alpha \in (G,L)\}$.

Definition 2.13. Let(M,τ,L) be a SMTS. A mapping$\gamma : SMS(M)_L \to SMS(M)_L$ is said to be an operation on$OSM(M)_L$, if$N_L \subseteq \gamma(N_L)$for all$N_L \in OSM(M)_L$.The family of all$\gamma$-open soft multi sets is denoted by$OSM(\gamma) = \{N_L : N_L \subseteq \gamma(N_L), N_L \in SMS(M)_L\}$. Also, the complement of γ -open soft multiset is called a γ -closed soft multiset and the set of allγ-closed soft multisets denoted by$CSM(\gamma)$.

Definition 2.14. [5] Let(M, τ, L) be a SMTS. Different cases of -operations on SMS $(M)_L$ are as follows:

i. If γ = int(cl),then γis called a pre-open soft multi operator. The family of all pre-open soft multisets is denoted by POSM $(M)_L$ and the family of all pre-closed soft multisets is denoted by PCSM $(M)_L$.

ii. If γ = int(cl(int)),then γ is called an -open soft multi operator. The family of all - open soft multisets is denoted by αOSM $(M)_L$ and family fall-closed soft multisets is denoted by αCSM $(M)_L$

iii. If γ = cl(int),then is called a semi open soft multi operator. The family of all semi opensoft multisets is denoted by SOSM $(M)_L$ and the family of all semi closed soft multisets is denoted by SCSM (X)L.

iv. Ifγ = cl(int(cl)),then γis called a-open soft multi operator. The family of all -open soft multisets is denoted by βOSM $(M)_L$ and family fall-closed soft multisets is denoted by βCSM $(M)_L$

Definition 2.15. A smset(S,L) in a SMTS(M,τ,L) is said to be a generalized closed (briefly g-closed) smset if$C_{cl(S)(l)}(v) \leq C_{B(l)}(v)$whenever$C_{S(l)}(v) \leq C_{B(l)}(v)$forall$v \in M^*$,$l \in L$ and (B,L) is open smset in M_L.

Definition 2.16. A soft mset S_L in a SMTS $(M,\tau,L)$is said to be a generalized semi closed softmulti set (briefly$gscs$ mset) if $C_{scl(S)(l)}(v) \leq C_{U(l)}(v)$ whenever $C_{(S)(l)}(v) \leq C_{U(l)}(v)$ and $U_L \in OSM(M)_L$.

Definition 2.17.[6] Let f_L be a smset over M_L. f_L is called a smpoint over M, if there exists $l \in L$ and $n/v \in M$, $1 \leq n \leq m$ such that

$$f(\varepsilon) = \begin{Bmatrix} \{n/v\} & if \varepsilon = l, 1 \leq n \leq m \\ \varphi & if\ \varepsilon \in L - \{l\} \end{Bmatrix}$$

We denotef_L by $[(n/v)_l]_L$. The family of all smpoints over$M$is denoted by $P(M,L)$ or *P*.
i.e.$P(M,L) = \{[(n/v_i)l_j]_L : v_i \in M, l_j \in L, 1 \leq n \leq m\}$.

GENERALIZED SEMI SOFT MULTI CLOSURE

Definition 3.1. Let $S_L$be a smset over M_L. Then the generalized semi soft multi closure of S_L,denoted by gssm-cl(S_L),is the intersection of all gsc smsets containing S_L.

Proposition 3.2. *For any* $C_{S(l)}(v) \leq C_{M(l)}(v)$,

1. $C_{S(l)}(v) \leq C_{gssm-cl(S)(l)}(v) \leq C_{scl(S)(l)}(v)$
2. $C_{S(l)}(v) \leq C_{gssm-cl(S)(l)}(v) \leq C_{\alpha cl(S)(l)}(v)$
3. $C_{S(l)}(v) \leq C_{gssm-cl(S)(l)}(v) \leq C_{gcl(S)(l)}(v)$

Proof. Since every semi closed (resp.α-closed, g-closed) smsts *gsc* smset.

Proposition 3.3. *For any two sub smsets S_L and T_L of M_L. The following hold,*

1. *If$S_L$isa $gscs$ mset,then* $C_{gssm-cl(S)(l)}(v) = C_{S(l)}(v)$,
2. $C_{gssm-cl(\varphi)(l)}(v) = C_{\varphi(l)}(v)$, *and* $C_{gssm-cl(M)(l)}(v) = C_{M(l)}(v)$,

3. *If* $C_{S(l)}(v) \le C_{T(l)}(v)$, *then* $C_{gssm-cl(S)(l)}(v) \le C_{gssm-cl(T)(l)}(v)$,
4. $C_{gssm-cl(S\cap T)(l)}(v) \le C_{(gssm-cl(S)\cap gssm-cl(T))(l)}(v)$
5. $C_{gssm-cl(S\cup T)(l)}(v) \ge C_{(gssm-cl(S)\cup gssm-cl(T))(l)}(v)$
6. $C_{gssm-cl(gssm-cl(S))(l)}(v) \le C_{gssm-cl(S)(l)}(v)$.

Proof.

1. If S_L is a gscs mset, then the smallest gsc smset containing S_L is itself. Therefore Cgssm-cl(S)lv= $C_{S(l)}$(v).
2. Since $\tilde{\varphi}$ and $\tilde{M}$are *gscs*mset, $C_{gssm-cl(\varphi)(l)}(v) = C_{\varphi(l)}(v)$ *and* $C_{gssm-cl(M)(l)}(v) = C_{M(l)}(v)$ *by (1).*
3. Let$C_{S(l)}(v) \le C_{T(l)}(v)$.Then$C_{S(l)}(v) \le C_{T(l)}(v) \le C_{gssm-cl(T)(l)}(v)$.But $gssm - cl(S_L)$is the smallest *gssm* closure of S_L. Therefore $C_{gssm-cl(S)(l)}(v) \le C_{gssm-cl(T)(l)}(v)$.
4. Since$C_{S\cap T(l)}(v) \le C_{S(l)}(v)andC_{S\cap T(l)}(v) \le C_{T(l)}(v)$,by (3)$C_{gssm-cl(S\cap T)(l)}(v) \le C_{gssm-cl(S)(l)}(v)andC_{gssm-cl(S\cap T)(l)}(v) \le C_{gssm-cl(T)(l)}(v)$.Wherefore$C_{gssm-cl(S\cap T)(l)}(v) \le C_{(gssm-cl(S)\cap gssm-cl(T))(l)}(v)$.
5. Since $C_{S(l)}(v) \le C_{S\cup T(l)}(v)$ and $C_{T(l)}(v) \le C_{S\cup T(l)}(v)$, $C_{gssm-cl(S)(l)}(v) \le C_{gssm-cl(S\cup T)(l)}(v)andC_{gssm-cl(T)(l)}(v) \le C_{gssm-cl(S\cup T)(l)}(v)$.Thus$C_{gssm-cl(S\cup T)(l)}(v) \ge C_{(gssm-cl(S)\cup gssm-cl(T))(l)}(v)$.
6. Let $C_{S(l)}(v) \le C_{F(l)}(v)$ and $F_L$be a *gscs* mset. Then by definition $C_{gssm-cl(S)(l)}(v) \le C_{F(l)}(v)$and $C_{gssm-cl(gssm-cl(S))(l)}(v) \le C_{F(l)}(v)$. Since $C_{gssm-cl(gssm-cl(S))(l)}(v) \le C_{\cap\{T_L \,/\, C_{S(l)}(v)\le C_{T(l)}(v) \text{ and where} T_L \text{be a } gscs \text{ mset}\}}=C_{gssm-cl(S)(l)}(v)$.Ergo$C_{gssm-cl(gssm-cl(S))(l)}(v) =\le C_{gssm-cl(S)(l)}(v)$ by Proposition3.2.

Note 3.1. Reverse implication of the above result (1) need not be true. Let (M,τ,L) be a SMTS with mset $M = \{2/v_1, 2/v_2\}$, parameter set $L = \{l_1, l_2\}$ and smtopology $\tau = \{\tilde{\varphi}, \tilde{M}, S_L\}$ where$S(l_1) = \{2/v_1\}, S(l_2) = \{2/v_2\}$ and let $T_L$be a sub smset of $M_L$such that $T(l_1) = \{\varphi\}, T(l_2) = \{1/v_1\}$.Then$C_{gssm-cl(T)(l)}(v) = C_{T(l)}(v)$ but T_L is not a *gscs* mset.

Definitions 3.4. A sub smset$S_L$in$(M,\tau,L)$is called generalized semi open soft multi set (insummary*gsos*mset) if its complement is a *gscs*mset.The set of all *gsos*mset in (M,τ,L) is denoted by$gsosm(M)_L$.

Theorem 3.5. *A subsmset*S_L*of* (M,τ,L) *is gsosmset iff*$C_{F(l)}(v) \le C_{sint(S)(l)}(v)$*whenever* $C_{F(l)}(v) \le C_{S(l)}(v)$ *and* F_L *is a closed smset.*

Proof. Suppose that$C_{F(l)}(v) \le C_{sint(S)(l)}(v)$ whenever$C_{F(l)}(v) \le C_{S(l)}(v)$*and* F_L *is a closed smset.* Let$C_{S^c(l)}(v) \le C_{U(l)}(v)$, where $U_L$is a open smset. Then $C_{U^c(l)}(v) \le C_{S(l)}(v)$, where ${U_L}^c$is a closed smset. By proposal$C_{U^c(l)}(v) \le C_{sint(S)(l)}(v)$,which implies $C_{(sint(S))^c(l)}(v) \le C_{U(l)}(v)$.That is$C_{scl(S^c)(l)}(v) \le C_{U(l)}(v)$. Thus ${S_L}^c$ is a *gscs*mset. Consequently S_L is a *gsos*mset.

Conversely, suppose that$S_L$is a *gsos*mset such that$C_{F(l)}(v) \le C_{S(l)}(v)andF_L$is a closed smset. Then ${F_L}^c$is a open smset and $C_{S^c(l)}(v) \le C_{F^c(l)}(v)$. Ergo$C_{scl(S^c)(l)}(v) \le C_{F^c(l)}(v)$and so$C_{F(l)}(v) \le C_{(scl(S^c))^c(l)}(v)= C_{sint(S)(l)}(v)$. Hence$C_{F(l)}(v) \le C_{sint(S)(l)}(v)$.

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Proposition 3.6.*ForaSMTS* (M,τ,L), *the following hold*

i. *Every open smset is a* gsos *mset.*
ii. *Every-open smset is a* gsos *mset.*
iii. *Every semi open smset is a* gsos *mset.*
iv. *Every* g-*open smset is a* gsos *mset.*

Proof.

(i)Let$S_L$be a open smset and $F_L$be a closed smset such that $C_{F(l)}(v) \leq C_{S(l)}(v)$. Then${F_L}^c$is a open smset and$C_{S^c(l)}(v) \leq C_{F^c(l)}(v)$. Wherefore$C_{scl(S^c)(l)}(v) \leq C_{cl(S^c)(l)}(v)$=$C_{S^c(l)}(v) \leq C_{F^c(l)}(v)$. That is$C_{F(l)}(v) \leq C_{(scl(S^c))^c(l)}(v)$= $C_{sint(S)(l)}(v)$. Hence$C_{F(l)}(v) \leq C_{sint(S)(l)}(v)$. Therefore S_L is a $gsos$ mset.

(ii) Let$S_L$be an α -open smset and $F_L \in CSM\ (M)_L$such that $C_{F(l)}(v) \leq C_{S(l)}(v)$. Then ${F_L}^c$is a open smset and$C_{S^c(l)}(v) \leq C_{F^c(l)}(v)$.Therefore$C_{scl(S^c)(l)}(v) \leq C_{\alpha cl(S^c)(l)}(v)$=$C_{S^c(l)}(v) \leq C_{F^c(l)}(v)$. That is$C_{F(l)}(v) \leq C_{(scl(S^c))^c(l)}(v)$= $C_{sint(S)(l)}(v)$. Hence$C_{F(l)}(v) \leq C_{sint(S)(l)}(v)$. Therefore$S_Lisagsos$mset.

(iii) Let $S_L$be a semi-open smset and $F_L \in CSM\ (M)_L$such that $C_{F(l)}(v) \leq C_{S(l)}(v)$.Then${F_L}^c$is a open smset and$C_{S^c(l)}(v) \leq C_{F^c(l)}(v)$.Therefore $C_{scl(S^c)(l)}(v) = C_{S^c(l)}(v) \leq C_{F^c(l)}(v)$ .Hence$C_{F(l)}(v) \leq C_{sint(S)(l)}(v)$. Ergo S_L is a $gsos$mset.

(iv)Let$S_L$bea *g*-open smset and $F_L \in CSM\ (M\ )_L$suchthat $C_{F(l)}(v) \leq C_{S(l)}(v)$. Then ${F_L}^c$is a open smset and$C_{S^c(l)}(v) \leq C_{F^c(l)}(v)$. Therefore$C_{scl(S^c)(l)}(v) \leq C_{cl(S^c)(l)}(v) \leq C_{F^c(l)}(v)$. Hence$C_{F(l)}(v) \leq C_{sint(S)(l)}(v)$. Ergo S_L is a $gsos$ mset.

Remark 3.7.1.

1. The union of $gsos$ msets need not be a $gsos$mset.

2.The intersection of $gsos$ msets need not be a $gsos$mset.

The Intersection of gsos msets need not be a gsos mset.

Example 3.8. Let (M,τ,L)be a SMTSwith mset $M = \{1/v_1, 2/v_2, 1/v_3\}$, parameter set $L = \{l_1, l_2\}$ and smtopology $\tau = \{\tilde{\varphi}, \widetilde{M}, (S_L)_1, (S_L)_2, (S_L)_3, (S_L)_4\}$ $where S_1(l_1) = \{1/v_1, 1/v_3\}$, $S_1(l_2) = \{1/v_1, 1/v_3\}$, $S_2(l_1) = \{1/v_1\}$, $S_2(l_2) = \{1/v_3\}$, $S_3(l_1) = \{2/v_2\}$, $S_3(l_2) = \{2/v_2\}$,$S_4(l_1) = \{1/v_1, 2/v_2\}$, $S_4(l_2) = \{2/v_2, 1/v_3\}$. Let $S_L$and $T_L$be two subsmsets of $M_L$such that $S(l_1) = \{1/v_3\}$, $S(l_2) = \{2/v_2, 1/v_3\}$and $T(l_1) = \{1/v_2\}$, $S(l_2) = \{1/v_1, 1/v_2, 1/v_3\}$. Then $S_L$and $T_L$are $gsos$msetsbut$S_L \widetilde{\cup} T_L$ is not a $gsos$ mset in (M,τ,L).

Example 3.9. Let (M,τ,L)be a SMTS with mset$M = \{2/v_1, 2/v_2, 2/v_3\}$, parameter set $L = \{l_1, l_2\}$andsmtopology$\tau = \{\tilde{\varphi}, \widetilde{M}, (S_L)_1, (S_L)_2, (S_L)_3\}$$where S_1(l_1) = \{2/v_2\}$, $S_1(l_2) = \{2/v_2\}$,~ $S_2(l_1) = \{2/v_3\}$, $S_2(l_2) = \{2/v_3\}$, $S_3(l_1) = \{2/v_2, 2/v_3\}$, $S_3(l_2) = \{2/v_2, 2/v_3\}$. Let $S_L$and $T_L$betwosubsmsetsof $M_L$suchthat $S(l_1) = \{2/v_1, 2/v_3\}$, $S(l_2) = \{2/v_1, 2/v_3\}$and $T(l_1) = \{2/v_1, 2/v_2\}$, $S(l_2) = \{2/v_1, 2/v_2\}$..Then$S_L$and $T_L$are $gsos$msets but $S_L \widetilde{\cap} T_L$isnota$gsos$msetin(M,τ,L).

Definition 3.10. Let (M, τ, L)be a SMTS over$M_L$and$S_L$be a smset overM_L. Then the generalized semi soft multi Interior of S_L,,denoted by $gssm - int(S_L)$ is the union fall gsosmsets contained in S_L

Proposition 3.11. *Forany* $C_{S(l)}(v) \leq C_{M(l)}(v), C_{int(S)(l)}(v) \leq C_{gssm-int(S)(l)}(v)$.

Proof. Since Every Opens Set a $gsos$mset..

Proposition 3.12. For any *Two subsmsets*$S_L$and T_L *of* (M,τ,L). *The following are hold,*

1. *If* S_L *is a* $gsos$ *mset,then* $C_{gssm-int(S)(l)}(v) = C_{S(l)}(v)$,
2. $C_{gssm-int(\varphi)(l)}(v) = C_{\varphi(l)}(v)$, *and* $C_{gssm-int(M)(l)}(v) = C_{M(l)}(v)$,
3. *If* $C_{S(l)}(v) \leq C_{T(l)}(v)$,*then* $C_{gssm-int(S)(l)}(v) \leq^{\leq} C_{gssm-int(T)(l)}(v)$,
4. $C_{gssm-int(S\cap T)(l)}(v) \geq C_{(gssm-int(S)\cap gssm-int(T))(l)}(v)$,
5. $C_{gssm-int(S\cup T)(l)}(v) \leq C_{(gssm-int(S)\cup gssm-int(T))(l)}(v)$,
6. $C_{gssm-int(gssm-int(S))(l)}(v) =^{\leq} C_{gssm-int(S)(l)}(v)$.

Theorem 3.13. *For a subsmset*S_L *of a SMTS* (M,τ,L), *the following are equivalent*

(i) $cl(S_L) - S_L$ *isa gscsmset,*
(ii) $S_L \cap (cl(S_L))^c$ *isa gsosmset.*

Proof.(*i*)⇒(*ii*) Let $C_{V(l)}(v)$=$C_{(cl(s)-S)(l)}(v)$. Then $C_{V^c(l)}(v) = C_{S\cap(cl(S))^c(l)}(v)$ and $S_L \cap (cl(S_L))^c$ isa *gsos* mset.
(*ii*) ⇒ (*i*) Let $C_{V(l)}(v) = C_{S\cap(cl(S))^c(l)}(v)$. Then $C_{U^c(l)}(v) = C_{(cl(S)-S)(l)}(v)$ and $U_L{}^c$ is a *gscs* mset and so $cl(S_L) - S_L$ *isa gscsmset*.

Proposition 3.14. *If S_L is a gsosmset and* $C_{sint(S)(l)}(v) \leq C_{T(l)}(v) \leq C_{S(l)}(v)$, *then T_L isa gsosmset.*

Proof. Suppose that $C_{sint(S)(l)}(v) \leq C_{T(l)}(v) \leq C_{S(l)}(v)$ and S_L is a *gsos*mset. Then $C_{S^c(l)}(v) \leq C_{T^c(l)}(v) \leq C_{scl(S^c)(l)}(v)$ andsince $S_L{}^c$ isa *gscs* mset, byusing Proposition [6] it follows that $T_L{}^c$ is a gscs mset. Thus T_L is a gsosmset.

Proposition 3.15. *If S_L is a gscsmsetthen $scl(S_L) - S_L$ isa gsosmset*

Proof. Suppose that S_L agscs mset. Let $C_{F(l)}(v) \leq C_{(scl S-S)(l)}(v)$, where FLis closeds mset. It follows that $C_{F(l)}(v)$ =Therefore $C_{F(l)}(v) \leq C_{sint(scl S-S)(l)}(v)$, by Theorem 3.5 and hence, $scl(S_L) - S_L$ isa *gsosmset*

Proposition 3.16. For any $[(n/v)_l]_L \in P(M, L)$, $[(n/v)_l]_L \leq C_{gssm-cl(S)(l)}(v)$ iff $\leq C_{(U\cap S)(l)}(v) \neq C_{\varphi(l)}(v)$ forevery *gsos*mset U_L containing $[(n/v)_l]_L$.

Proof. Let $[(n/v)_l]_L \leq C_{gssm-cl(S)(l)}(v)$ for any $[(n/v)_l]_L \in P(M, L)$.Suppose Subsists a*gsos* mset U_L containing $[(n/v)_l]_L$ such that $C_{U\cap S}v = C_{\varphi(l)}(v)$. Then $C_{S(l)}(v) \leq C_{U^c(l)}(v)$.Since U_L is a*gscs* mset containing S_L, $C_{gssm(S)(l)}(v) \leq C_{U^c(l)}(v)$, which implies that $[(n/v)_l]_L \nleq C_{gssm(S)(l)}(v)$, acontradiction. Thus $C_{(U\cap S)(l)}(v) \neq C_{\varphi(l)}(v)$.
Conversely, Suppose that $[(n/v)_l]_L \nleq$.Thereby Definition, there subsists a *gscs* mset F_L containing S_L such that $[(n/v)_l]_L \nleq C_{F(l)}(v)$.Thus $[(n/v)_l]_L \leq C_{F^c(l)}(v)$ and $F_L{}^c$ is a*gsos* mset.Also $C_{(F^c\cap S)(l)}(v) = C_{\varphi(l)}(v)$ which is a contradiction. There fore $[(n/v)_l]_L \leq C_{gssm-cl(S)(l)}(v)$.

Proposition 3.17. *Let S_L be a sub smset of* (M,τ,L),*then* $C_{(M-gssm-int(S))}v = C_{gssm-cl(M-S)(l)}(v)$.

Proof. Let $[(n/v)_l]_L \leq C_{(M-gssm-int(S)(l)}(v)$.Then $[(n/v)_l]_L \nleq C_{gssm-cl(M-S)(l)}(v)$.That is every *gsos* mset T_L containing $[(n/v)_l]_L$ is that T_L is not contained in S_L. This implies every *gsos*mset T_L containing $[(n/v)_l]_L$ is such that $C_{(T\cap(M-S))(l)}(v) \neq C_{\varphi(l)}(v)$.Then $[(n/v)_l]_L \nleq C_{gssm-cl(M-S)(l)}(v)$. Hence $C_{(M-gssm-int(S)(l)}(v) \leq C_{gssm-cl(M-S)(l)}(v)$. Conversely, Let $[(n/v)_l]_L \leq C_{gssm-cl(M-S)(l)}(v)$. Then by above Proposition Every gsos mset T_L containing $[(n/v)_l]_L$ is such that $C_{(T\cap(M-S))(l)}(v) \neq C_{\varphi(l)}(v)$.That is every *gsos*mset T_L containing $[(n/v)_l]_L$ is such that T_L is not contained in S_L.This implies that $[(n/v)_l]_L \nleq C_{gssm-int(S)(l)}(v)$.Thus $[(n/v)_l]_L \leq C_{(M-gssm-int(S)(l)}(v)$.
Hence $C_{gssm-cl(M-S)(l)}(v) \leq C_{(M-gssm-int(S)(l)}(v)$. Ergo $C_{(M-gssm-int(S)(l)}(v) = C_{gssm-cl(M-S)(l)}(v)$.

Proposition 3.18. *If $gssm - cl(S_L) - S_L$ is a gscsmset, then $C_{S(l)}(v) \leq C_{gssm-int(S\cup gssm-cl(S^c))(l)}(v)$.*

Proof. We know that $C_{S\cup(gssm-cl(S^c))(l)}(v)$=$C_{(gssm-cl(S)-S)^c(l)}(v)$ and by assumption $(gssm - cl(S_L) - S_L)^c$ is a *gsos* mset and so $S_L \cup (gssm - cl(S_L))^c$ is *gsos* mset. Thus $C_{S\cup(gssm-cl(S^c))(l)}(v)$=$C_{gssm-int(S\cup gssm-cl(S^c))(l)}(v)$ and hence $C_{S(l)}(v) \leq C_{gssm-int(S\cup gssm-cl(S^c))(l)}(v)$.

GENERALIZED SEMI SOFT MULTI NEIGHBOURHOOD

Definition 4.1. A sub smset N_L in SMTS(M, τ, L)is said to be generalized semi soft multi neighborhood (in summary *gssm*-nbd) of a smpoint $[(n/v)_l]_L$ in M_L if there subsists an *gsos*mset U_L suchthat $[(n/v)_l]_L \leq C_{U(l)}(v) \leq C_{N(l)}(v)$.

If N_L is a *gsos*mset containing $[(n/v)_l]_L$, then N_L is called generalized semi soft multi open neighborhood (in summary *gssm*-opennbd) of $[(n/v)_l]_L$ and is denoted by $gssmN[(n/v)_l]_L$..

Example 4.2. Let (M,τ,L) be a SMTS with mset $M = \{2/v_1, 2/v_2\}$, parameter set $L = \{l_1, l_2\}$ and smtopology $\tau = \{\tilde{\varphi}, \tilde{M}, (S_L)_1\}$ where $S_1(l_1) = \{2/v_1\}, S_1(l_1) = \{2/v_2\}$. The *gssm*-nbd of $[(2/v_1)_{l_1}]_L$ is $\{\{(l_1,\{2/v_1\}),(l_2,\{\varphi\})\},\{(l_1,\{2/v_1\}),(l_2,\{1/v_1\})\},\{(l_1,\{2/v_1\}),(l_2,\{2/v_1\})\},\{(l_1,\{2/v_1\}),(l_2,\{1/v_2\})\},\{(l_1,\{2/v_1\}),(l_2,\{2/v_2\})\},\{(l_1,\{2/v_1\}),(l_2,\{1/v_1,1/v_2\})\},\{(l_1,\{2/v_1\}),(l_2,\{1/v_1,2/v_2\})\},\{(l_1,\{2/v_1\}),(l_2\{2/v_1,1/v_2\})\},\{(l_1,\{2/v_1\}),(l_2,\{M\})\},\{(l_1,\{2/v_1,1/v_2\}),(l_2,\{\varphi\})\},\{(l_1,\{2/v_1,1/v_2\}),(l_2,\{1/v_1\})\},\{(l_1,\{2/v_1,1/v_2\}),(l_2,\{2/v_1\})\},\{(l_1,\{2/v_1,1/v_2),(l_2,\{1/v_2\})\},\{(l_1,\{2/v_1,1/v_2\}),(l_2,\{2/v_2\})\},\{(l_1,\{2/v_1,1/v_2\}),(l_2,\{1/v_1,1/v_2\})\},\{(l_1,\{2/v_1,1/v_2\}),(l_2,\{1/v_1,2/v_2\})\},\{(l_1,\{2/v_1,1/v_2\}),(l_2,\{2/v_1,1/v_2\})\},\{(l_1,\{2/v_1,1/v_2\}),(l_2,\{M\})\},\{(l_1,\{M\}),(l_2,\{\varphi\})\},\{(l_1,\{M\}),(l_2,\{1/v_1\})\},\{(l_1,\{M\}),(l_2,\{2/v_1\})\},\{(l_1,\{M\}),(l_2,\{1/v_2\})\},\{(l_1,\{M\}),(l_2,\{2/v_2\})\},\{(l_1,\{M\}),(l_2,\{1/v_1,1/v_2\})\},\{(l_1,\{M\}),(l_2,\{1/v_1,2/v_2\})\},\{(l_1,\{M\}),(l_2,\{2/v_1,1/v_2\})\},\tilde{M}\}$.

Definition 4.3. The family of all *gssm*-nbd of a point $[(n/v)_l]_L \leq C_{M(l)}(v)$ is called the *gssm*- neighbourhood system of $[(n/v)_l]_L$..

Proposition 4.4. *Every sm nbd of*[(n/v)l]L*is a gssm-nbd of* $[(n/v)_l]_L$

Proof. Let N_L be a smnbd of $[(n/v)_l]_L \leq C_{M(l)}(v)$. Then there subsists an open smset U_L such that $[(n/v)_l]_L \leq C_{U(l)}(v) \leq C_{N(l)}(v)$..Since every open smset is a $gscs$mset, U_L

Remark 4.5. *The converse of the above proposition need not be true as seen from the following example.*

Example 4.6. In Example4.2,The smset $\{(l_1,\{2/v_1\}),(l_2,\{2/v_1,2/v_2\})\}$ is *gssm*-nbd of $[(2/v_1)_{l_1}]_L$.but not a smnbd of $[(2/v_1)_{l_1}]_L$.

Proposition 4.7. *Every gsosmset is a gssm-nbd of each of its points.*

Proof. Let N_L be a *gsos* mset and $[(n/v)_l]_L \leq C_{N(l)}(v)$. Then $[(n/v)_l]_L \leq C_{N(l)}(v) \leq C_{N(l)}(v)$. Since $[(n/v)_l]_L$ is an arbitrary point of N_L it follows thatNLisgssm-nbdof each of its points.

Proposition 4.8. For a SMTS (M,τ,L), the following holds:

a. Every $[(n/v)_l]_L \in \tilde{M}$ has atleast onegssm–nbd.
b. Every gssm-nbd of $[(n/v)_l]_L \in \tilde{M}$ and [(n/v)l]L
c. Every super smset of a gssm-nbd of $[(n/v)_l]_L \in \tilde{M}$ and [(n/v)l]L
d. If N_L is a $gssm$-nbd of $[(n/v)_l]_L \in \tilde{M}$ then there subsists a $gssm$-nbd Z_L of $[(n/v)_l]_L$ such that $C_{Z(l)}(v) \leq C_{N(l)}(v)$ and Z_L is a gssm-nbd of each of its sm points.

Proof.(a)M being gsos-mset it is a gsm-nbd of each of its sm points. So each [(n/v)l] LM has at least one gssm-nbdnamely M.

(b) Let N_L be a $gssm$-nbd of $[(n/v)_l]_L$.Then there subsists a *gsos*-mset G_L such that $[(n/v)_l]_L \leq C_{G(l)}(v) \leq C_{N(l)}(v)$. Clearly $[(n/v)_l]_L \leq C_{N(l)}(v)$.So each $gssm$-nbd of $[(n/v)_l]_L$ contains $[(n/v)_l]_L$.

(c) Let N_L be a $gssm$-nbd of $[(n/v)_l]_L$ and let $K_L$be a super sm set of N_L. Then by Definition ofgssm-nbd of as m point, there subsists gsos-mset G_L such that$[(n/v)l]_L C_{G(l)}(v) \leq C_{N(l)}(v) \leq C_{K(l)}(v)$.This shows that KL is also a gssm-nbdof[(n/v)l] L. Let NL be a gssm-nbd of[(n/v)l]L. Then there subsists a gsos-mset ZL such that $[(n/v)l]_L C_{Z(l)}(v) \leq C_{N(l)}(v)$.Now ZL being gsos-mset, it is a gssm-nbd of each of its sm points.

RESULTS AND DISCUSSION

In this work we have acquainted with the conception of generalized semi soft multi closure, generalized semi soft multi interior and generalized semi soft multi neighborhood and also some basic characteristics. In future we extend the conception of generalized semi soft multi continuous mappings, their respective open mappings and homeomorphisms.

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