RESEARCH ARTICLE

Interval-Valued Intuitionistic Fuzzy Convex Optimization Techniques

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ABSTRACT

Optimization is a procedure of finding and comparing feasible solutions. Convex optimization is one of the fields among several fields of Optimization techniques. This article presents the definition of interval-valued intuitionistic fuzzy local maximum point and interval-valued intuitionistic fuzzy global maximum point and its characterizations.

Keywords: IVIF-convex set, TOPSIS, IVIF-convex objective function, IVIF-convex constraints

INTRODUCTION

In this paper, we introduced the interval-valued intuitive fuzzy local maximum point. (abbreviated $IVIF^{LM}$ point) and the interval-valued intuitionistic fuzzy global maximum point (abbreviated $IVIF^{GM}$ point) of $convex_{IVIF}$ sets and its properties. Throughout this paper U as universal crisp set, I is the collection of all subintervals of the closed interval [0,1] and I^U represents the family of all closed subintervals of [0,1]. with respect to the specified set U

Convex $_{IVIF}$-sets: Characteristics

Definition 2.1.
Let A be an on empty sub set of I^U. An element $x^\rho \in$ *supp* (A)is called an $_{IVIF}{}^{LM}$ point of A if there exists $\epsilon > 0$ such that $[\underline{\mu},\overline{\mu}]_A(x^\rho) \geq [\underline{\mu},\overline{\mu}]_A(x)$ and $[\underline{\nu},\nu]_A(x) \leq [\underline{\nu},\nu]_A(x^\rho)$ for all $x \in B(x,r)$

Definition 2.2.
An element $x^{\rho} \in supp(A)$ is called an $_{IVIF}$ ^{GM} point of an interval-valued intuitionistic fuzzy set A if $[\underline{\mu},\overline{\mu}]_A(x) \le [\underline{\mu},\overline{\mu}]_A(x^{\rho})$ and $[\underline{\nu},\nu]_A(x^{\rho}) \ge [\underline{\nu},\nu]_A(x), \forall x \in U$.

Definition 2.3.
Let A be a *convex IVIF* subset of I^U. An element $x^{\rho} \in supp(A)$ is called a strictly *IVIF* ^{LM} point of A if there exists $\epsilon > 0$ such that $[\underline{\mu},\overline{\mu}]_A(x^{\rho}) > [\underline{\mu},\mu]_A(x)$ and $[\underline{\nu},\nu]_A(x) < [\underline{\nu},\nu]_A(x^{\rho})$ for all $x \in B(x,r)$ and $x \neq x^{\rho}$.

Proposition 2.4. *Let A be a convex $_{IVIF}$ set and $x^{\rho} \in supp(A)$ be a IVIF^{LM} Point of A. Then x^{ρ} be a IVIF ^{GM} of A over supp(A).*
Proof : Given that, A is a convex$_{IVIF}$ set and $x^{\rho} \in supp(A)$ is a IV IF ^{LM} point, implies
$[\underline{\mu},\overline{\mu}]_A(x^{\rho}) \ge [\underline{\mu},\overline{\mu}]_A(x)$ *and* $[\underline{\nu},\nu]_A(x^{\rho}) \le [\underline{\nu},\nu]_A(x)$

We have to prove, $x^{\rho} \in supp(A)$ is a IVIF^{GM} point, that is,
$[\underline{\mu},\overline{\mu}]_A(x) \le [\underline{\mu},\overline{\mu}]_A(x^{\rho})$ *and* $[\underline{\nu},\nu]_A(x) \ge [\underline{\nu},\nu]_A(x^{\rho})$

Suppose that x^{ρ} is not a IV IF ^{GM} point then there exists another point $x^{\rho'} \in supp(A)$ such that, $x^{\rho} \le x^{\rho'}$ implies, $[\underline{\mu},\mu]_A(x) \le [\underline{\mu},\mu]_A(x^{\rho'})$ and $[\underline{\nu},\nu]_A(x) \ge [\underline{\nu},\nu]_A(x^{\rho'})$
Since A is a convex$_{IV\,IF}$ set, then

$$\lambda[\underline{\mu},\mu]_A(x^{\rho'}) + (1-\lambda)[\underline{\mu},\mu]_A(x^{\rho}) \ge min\{[\underline{\mu},\mu]_A(x^{\rho'}), [\underline{\mu},\mu]_A(x^{\rho})\}$$
$$\ge min\{[\underline{\mu},\mu]_A(x_{\rho}), [\underline{\mu},\mu]_A(x_{\rho})\}$$
$$= [\underline{\mu},\mu]_A x^{\rho}$$

But this is not possible, by the definition of IVIF ^{LM} point. Therefore $x^{\rho} = x^{\rho'}$. Next, for nonmembership degree, we have,

$$\lambda[\underline{\nu},\overline{\nu}]_A(x^{\rho'}) + (1-\lambda)[\underline{\nu},\overline{\nu}]_A(x^{\rho}) \le max\{[\underline{\nu},\overline{\nu}]_A(x^{\rho'}), [\underline{\nu},\overline{\nu}]_A(x^{\rho})\}$$
$$\le max\{[\underline{\nu},\nu]_A(x_{\rho}), [\underline{\nu},\nu]_A(x_{\rho})\}$$
$$= [\underline{\nu},\underline{\nu}]_A(x^{\rho})$$

which is a contradiction to the definition of IVIF LM . Therefore, x ρ is a IV IF GM point.

Proposition 2.5. If A is a strictly convex IVIF set then x^{ρ} is the unique IVIF GM point.

Proof: *Here, we have to prove x^{ρ} is a unique IVIF global maximum point.*
Assume that A is a strictly convex$_{IVIF}$ set, and x^{ρ} is a strictly IV IF^{GM} point, implies, $[\underline{\mu},\mu]_A(x) < [\underline{\mu},\mu]_A(x^{\rho})$ and $[\underline{\nu},\nu]_A(x) > [\underline{\nu},\nu]_A(x^{\rho})$
Suppose that $x^{\rho *}$ is another IVIF^{GM} point, $\Longrightarrow$ $[\underline{\mu},\overline{\mu}]_A(x) < [\underline{\mu},\overline{\mu}]_A(x^{\rho *})$ and $[\underline{\nu},\nu]_A(x) > [\underline{\nu},\nu]_A(x^{\rho *})$
Since A is a strictly convex$_{IVIF}$ set, then

$$\lambda[\underline{\mu}, \mu]_A(x^{\rho*}) + (1-\lambda)[\underline{\mu}, \mu]_A(x^\rho) > min\{[\underline{\mu}, \mu]_A(x^{\rho*}), [\underline{\mu}, \mu]_A(x^\rho)\}$$
$$> min\{[\underline{\mu}, \mu]_A(x^\rho), [\underline{\mu}, \mu]_A(x^\rho)\}$$
$$= [\underline{\mu}, \mu]_A(x^\rho)$$

$[\underline{\mu}, \mu]_A(x^\rho) < [\underline{\mu}, \mu]_A(x^{\rho*})$. *Also we have* $[\underline{\mu}, \mu]_A(x^\rho) > [\underline{\mu}, \mu]_A(x^{\rho*})$. *Thus* $[\underline{\mu}, \mu]_A(x^\rho) = [\underline{\mu}, \mu]_A(x^{\rho*})$.

Similarly for nonmembership interval,

$$\lambda[\underline{\nu}, \nu]_A(x^{\rho*}) + (1-\lambda)[\underline{\nu}, \nu]_A(x^\rho) < max\{[\underline{\nu}, \nu]_A(x^{\rho*}), [\underline{\nu}, \nu]_A(x^\rho)\}$$
$$< max\{[\underline{\nu}, \nu]_A(x^\rho), [\underline{\nu}, \nu]_A(x^\rho)\}$$
$$= [\underline{\nu}, \nu]_A(x^\rho)$$

$[\underline{\nu}, \nu]_A(x^\rho) > [\underline{\nu}, \nu]_A(x^{\rho*})$. *And* $[\underline{\nu}, \nu]_A(x^\rho) < [\underline{\nu}, \nu]_A(x^{\rho*})$. *Thus* $[\underline{\nu}, \nu]_A(x^\rho) = [\underline{\nu}, \nu]_A(x^{\rho*})$

Hence x^ρ *is a unique* $IVIF^{GM}$ *point.*

Proposition 2.6.

Let A $\in I^U$ be a strictly convex IVIF set. The set of IVIF points at which A attains its IV IF^{GM} over supp(A) is a convex(crisp) set.

Proof: *Assume that* A *is a* $convex_{IVIF}$ *set. Let* $\{[\underline{\mu}, \mu]_A(x_1), \ldots [\underline{\mu}, \mu]_A(x^\rho), \ldots [\underline{\mu}, \mu]_A(x_n)\}$ *and* $\{[\underline{\nu}, \nu]_A(x_1), \ldots [\underline{\nu}, \nu]_A(x^\rho), \ldots [\underline{\nu}, \nu]_A(x_n)\}$ *be the set of all membership and non-membership intervals of IF points contained in* $supp(A)$. *If* $x^\rho \in supp(A)$ *is an* $IV\ IF^{LM}$ *point of* A *then* x^ρ *is also an* $IV\ IF^{GM}$ *of* A *over* $supp(A)$. *By definition,* $supp(A) = \{x : [\underline{\mu}, \mu]_A(x) > 0, [\underline{\nu}, \nu]_A(x) > 0\}$. *This implies,* $IV\ IF^{GM}$ *point is in convex(crisp) set.*

Proposition 2.7. *Let* $A \in I^U$ *be a strictly* $convex_{IVIF}$ *set, then the following conditions hold:*

1. *If* $x^\rho \in supp(A)$ *is a* $IVIF^{LM}$ *of* A, *then it is a unique* $IVIF^{GM}$ *point.*
2. A *attains its* $IVIF^{GM}$ *point over* $supp(A)$ *at only one point.*

Proof: *Let us assume that* $x^\rho \in supp(A)$ *be a strictly* $IV\ IF^{LM}$ *point. Then there exists* $\epsilon > 0$, *such that* $x \in supp(A)$, $[\underline{\mu}, \mu]_A(x) \neq [\underline{\mu}, \mu]_A(x^\rho)$, $[\underline{\nu}, \nu]_A(x) - [\underline{\nu}, \nu]_A(x^\rho)$

and $\|(x) - x^\rho\| < \epsilon$, $\Longrightarrow$ $[\underline{\mu}, \mu]_A(x^\rho) > [\underline{\mu}, \mu]_A(x)$ *and* $[\underline{\nu}, \nu]_A(x^\rho) < [\underline{\nu}, \nu]_A(x)$

Suppose x^ρ *is not a strictly* $IVIF^{GM}$ *over its* $supp(A)$, *then* $x^{\rho*} \in supp(A)$ *and* $x^{\rho*}$ x^ρ *such that* $[\underline{\mu}, \mu]_{\bar{A}}(x^{\rho*}) > [\underline{\mu}, \mu]_{\bar{A}}(x^\rho)$ *and* $[\underline{\nu}, \nu]_{\bar{A}}(x^{\rho*}) < [\underline{\nu}, \nu]_{\bar{A}}(x^\rho)$. *Since* A *is a strictly* $convex_{IV\ IF}$ *set. For all* $\lambda \in [0, 1]$, *we have*

$$
\begin{aligned}
[\underline{\mu}, \overline{\mu}]_A(x^{\rho *}) &= \lambda[\underline{\mu}, \overline{\mu}]_A(x) + (1-\lambda)[\underline{\mu}, \overline{\mu}]_A(x^{\rho}) \\
&\le \lambda[\underline{\mu}, \overline{\mu}]_A(x) + [\underline{\mu}, \overline{\mu}]_A(x_{\rho}) - \lambda[\underline{\mu}, \overline{\mu}]_A(x_{\rho}) \\
&< \lambda[\underline{\mu}, \overline{\mu}]_A(x^{\rho}) + [\underline{\mu}, \overline{\mu}]_A(x^{\rho}) - \lambda[\underline{\mu}, \overline{\mu}]_A(x^{\rho}) \\
&= [\underline{\mu}, \overline{\mu}]_A(x^{\rho})
\end{aligned}
$$

Similarly, we can prove for nonmembership intervals. Hence x^{ρ} *is a only one strictly* $IVIF^{GM}$ *point.*

Proposition 2.8. Let A be a convex $_{IVIF}$ set with supp (A)φ . If A has a unique IVIF LM point on every closed interval [x, y] in its support then A is a strictly convex $_{IVIF}$ set.

Definition 2.9. Let $f_A : U^n \rightarrow U$ be a convex function and $f_A : U \rightarrow D[0, 1]$ be a nonincreasing interval-valued IF set. The composition is denoted by $f_A \circ f_A = A$, then the membership composite interval function, $[\underline{\mu}, \overline{\mu}]_A : U^n \rightarrow D[0, 1]$ defined by

$$
[\underline{\mu}, \overline{\mu}]_A(x) = \begin{cases} f_A(f_A(x)) & \text{if } f_A \in supp(f_A) \\ \square & \text{if } f_A \notin supp(f_A) \end{cases}
$$

and nonmembership composite interval function $[\nu^L, \nu^U]_A : U^n \rightarrow D[0, 1]$ defined by

$$
[\underline{\nu}, \overline{\nu}]A(x) = \begin{cases} f_A(f_A(x)) & \text{if } f_A \in supp(f_A) \\ \varphi & \text{if } f_A \notin supp(f_A) \end{cases}
$$

Definition 2.10. Let $A : I^U \rightarrow D[0, 1]$ be a $convex_{IVIF}$ set and $B : I^U \rightarrow D[0, 1]$ be an another $convex_{IVIF}$ set. Then the membership composite interval sets of $(A \circ B)(x) = A(B(x))$ defined by $([\underline{\mu}, \overline{\mu}]_A \circ [\underline{\mu}, \overline{\mu}]_B)(x) = \vee\{\wedge([\underline{\mu}, \overline{\mu}]_A, [\underline{\mu}, \overline{\mu}]_B)(x)\}$, and the nonmembership composite interval sets of $(A \circ B)(x) = A(B(x))$ defined by $([\underline{\nu}, \overline{\nu}]_A \circ [\underline{\nu}, \overline{\nu}]_B)(x) = \wedge\{\vee([\underline{\nu}, \overline{\nu}]_A, [\underline{\nu}, \overline{\nu}]_B)(x)\}$

Multitask $convex_{IVIF}$ -Optimization Problems

In this part we introduce the general approach of decision making problems under $convex_{IVIF}$ sets. A decision maker wants to evaluate n-number of goals as convex functions $G_1, G_2, \ldots G_n$ and m-number of IVIF-constraints $C_1, C_2, \ldots C_m$ defined the solution space $U \subseteq U^n$ are assumed to be given. An IVIF-decision D in U is defined by,

$$
[\underline{\mu}, \overline{\mu}]^D(x) = max\{[\underline{\mu}, \overline{\mu}]_{G_1} * [\underline{\mu}, \overline{\mu}]_{G_2} * \ldots * [\underline{\mu}, \overline{\mu}]_{G_n} * [\underline{\mu}, \overline{\mu}]_{C_1} * [\underline{\mu}, \overline{\mu}]_{C_2} * \ldots * [\underline{\mu}, \overline{\mu}]_{C_n}\}(x)
$$

$$
[\underline{\nu}, \overline{\nu}]^D(x) = min\{[\underline{\nu}, \overline{\nu}]_{G} * [\underline{\nu}, \overline{\nu}]_{G} * \ldots * [\underline{\nu}, \overline{\nu}]_{G} * [\underline{\nu}, \overline{\nu}]_{C} * [\underline{\nu}, \overline{\nu}]_{C} * \ldots * [\underline{\nu}, \overline{\nu}]_{C}\}(x)
$$

where $x \in$ U and * denotes an aggregation operator. Many different aggregation operators have been proposed. Here we desired to use min operator to aggregate the convex combination of goals and constraints. Due to computational simplicity, D might be expressed as a convex combination of the goals and constraints with weighting co-efficient reflecting the relative importance of the various terms. If thereexists a subset M ⊆ U for which A(x) reaches its maximum, then M is called the set of maximizing decisions.

CONCLUSION

In further we can work towards the problem of assigning location centers in convex sets with interval-valued intuitionistic fuzzy sets. That is, a convex set is characterized by an interval-valued intuitionistic fuzzy convex-objective function and interval-valued intuitionistic fuzzy convex constraint functions over a convex set which is the set of the decision variables. Also, assign a new location with the given convex sets using a TOPSIS-based computational procedure.

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