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On Weakly $\ddot{g}$ Closed Mappings and Weakly $\ddot{g}$ Homomorphism in Intuitionistic Fuzzy Topological Spaces

A. Gnanasoundari*

Assistant Professor,
Department of Mathematics,
Nallamuthu Gounder Mahalingam College, Pollachi – 642 001, Tamil Nadu, INDIA.
email: gnana12483@gmail.com

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ABSTRACT

In this paper, we introduce and study the notions of intuitionistic fuzzy weakly $\ddot{g}$ closed mappings, intuitionistic fuzzy weakly $\ddot{g}$ open mappings, intuitionistic fuzzy weakly $\ddot{g}$ homomorphism and some of its properties.

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1. INTRODUCTION

In 1965, Zadeh[10] introduced fuzzy sets and in 1968, Chang[2] introduced fuzzy topology. The notion of intuitionistic fuzzy sets was introduced by Atanassov[1] as a generalization of fuzzy sets. In 1997, Coker[3] introduced the concept of intuitionistic fuzzy topological spaces. In this paper, we introduce the notions of intuitionistic fuzzy weakly $\ddot{g}$ closed mappings, intuitionistic fuzzy weakly $\ddot{g}$ open mappings, intuitionistic fuzzy weakly $\ddot{g}$ homomorphism and some of its properties.

2. PRELIMINARIES

Throughout this paper, (X, τ) or X denotes the intuitionistic fuzzy topological spaces (IFTS in short). For a subset A of X, the closure, the interior and the complement of A are denoted by cl(A), int(A) and A^c respectively. We recall some basic definitions that are used in the sequel.

Definition 2.1: [1] Let X be a non-empty set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form $A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$ where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: **[1]** Let A and B be IFSs of the form $A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$ and $B = \{<x, \mu_B(x), \nu_B(x)>/ x \in X\}$. Then
(i) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,
(iii) $A^c = \{ <x, \nu_A(x), \mu_A(x)>/ x \in X \}$,
(iv) $A \cap B = \{ <x, \mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x) >/ x \in X \}$,
(v) $A \cup B = \{ <x, \mu_A(x) \leq \mu_B(x), \nu_A(x) \geq \nu_B(x)>/ x \in X \}$.

For the sake of simplicity, we shall use the notation $A = <x, \mu_A, \nu_A>$ instead of $A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$.
The intuitionistic fuzzy sets $0_\sim=\{<x,0,1>: x \in X\}$ and $1_\sim =\{<x,1,0>: x \in X\}$ are respectively the empty and whole set of X.

Definition 2.3: [3] An intuitionistic fuzzy topology(IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:
(i) $0_\sim, 1_\sim \in \tau$,
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$,
(iii) $\cup G_i \in \tau$ for any family $\{G_i / i \in J\} \subseteq \tau$.
In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: **[3]** Let (X, τ) be an IFTS and $A = <x, \mu_A, \nu_A>$ be an IFS in X. Then
1. $int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,
2. $cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$,
3. $cl(A^c)= (int(A))^c$,
4. $int(A^c)=(cl(A))^c$.

Definition 2.5: [4] An IFS $A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$ in an IFTS (X, τ) is said to be an
1. intuitionistic fuzzy semi-open set (IFSOS in short) if $A \subseteq cl(int(A))$,
2. intuitionistic fuzzy α-open set (IFαOS in short) if $A \subseteq int(cl(int(A)))$,
3. intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq int(cl(A))$.

An IFS A is said to be an intuitionistic fuzzy semi-closed set(IFSCS in short), intuitionistic fuzzy α-closed set (IFαCS in short), intuitionistic fuzzy regular closed set (IFRCS in short) and intuitionistic fuzzy pre closed set (IFPCS in short) if the complement of A is an IFSOS, IFαOS, IFROS, IFPOS respectively.

Definition 2.6: [7] An IFS $A = \{<x, \mu_A(x), \nu_A(x)>/ x \in X\}$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy semi-generalized closed set (IFSGCS in short) if scl(A) $\subseteq$U whenever A $\subseteq$U and U is an IFSOS in X.

An IFS A is said to be an intuitionistic fuzzy semi generalized open set (IFSGOS in short) if the complement of A is an IFSGCS.

Definition 2.7:[9]An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy weakly $\ddot{g}$-closed set (IFW$\ddot{g}$CS in short) if cl(int(A)) $\subseteq$U whenever A$\subseteq$ U and U is an IFSGOS in X. The set of all IFW$\ddot{g}$CSs in X is denoted by IFW$\ddot{g}$CS(X).

Definition 2.8: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

1. intuitionistic fuzzy continuous mapping (IF continuous mapping in short)[4] if f^{-1}(B) $\in$ IFO(X) for every B $\in \sigma$,
2. intuitionistic fuzzy α continuous mapping (IF α continuous mapping in short)[4] if f^{-1}(B) $\in$ IF α O(X) for every B $\in \sigma$,
3. intuitionistic fuzzy pre continuous mapping (IFP continuous mapping in short)[4] if f^{-1}(B) $\in$ IFPO(X) for every B $\in \sigma$,
4. intuitionistic fuzzy closed mapping (IF closed mapping in short)[5] if f(A) is an IFCS in Y for each IFCS A in X ,
5. intuitionistic fuzzy α closed mapping (IF α closed mapping in short)[8] if f(A) is an IF αCS in Y for each IFCS A in X ,
6. intuitionistic fuzzy pre closed mapping (IFP closed mapping in short)[8] if f(A) is an IFPCS in Y for each IFCS A in X .

Definition 2.9:[6]Let f be a bijection mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

1. intuitionistic fuzzy homeomorphism (IF homeomorphism in short) if f and f^{-1} are IF continuous mappings,
2. intuitionistic fuzzy α homeomorphism (IF α homeomorphism in short) if f and f^{-1} are IF α continuous mappings.

Definition 2.10: [9] Let **(**X, τ) be an IFTS A be an IFS in X. Then intuitionistic fuzzy weakly $\ddot{g}$ interior and intuitionistic fuzzy weakly $\ddot{g}$ closure of A are defined as

1. $\ddot{g}$int(A) = $\cup$ { G / G is an IFW$\ddot{g}$OS in X and G $\subseteq$ A },
2. $\ddot{g}$cl(A) = $\cap$ { K / K is an IFW$\ddot{g}$CS in X and A $\subseteq$ K }.

3. INTUITIONISTIC FUZZY WEAKLY $\ddot{g}$ CONTINUOUS MAPPING

In this section, we study the notion of intuitionistic fuzzy weakly $\ddot{g}$ continuous mappings and investigate some of their properties.

Definition 3.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy weakly $\ddot{g}$ continuous (IFW $\ddot{g}$ continuous in short) mapping if f^{-1}(V) is an IFW$\ddot{g}$CS in (X, τ) for every IFCS V of (Y, σ).

Theorem 3.2: Every IF continuous mapping is an IF$\ddot{g}$ continuous mapping, but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF continuous mapping and A be an IFCS in Y. Then f^{-1}(A) is an IFCS in X. Since every IFCS is an IFW$\ddot{g}$CS, f^{-1}(A) is an IFW$\ddot{g}$CS in X. Hence f is an IFW$\ddot{g}$continuous mapping.

Example 3.3: Let X={a,b}, Y={u,v} and A=<x,(0.5,0.6),(0.5,0.4)>, B=<x,(0.6,0.6),(0.4,0.4)>. Then τ={$0_{\sim}$,A, $1_{\sim}$} and σ ={$0_{\sim}$,B, $1_{\sim}$} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then IFS S=<x,(0.4,0.4),(0.6,0.6)> is an IFCS in Y and f^{-1}(S) is an IFW$\ddot{g}$CS but not an IFCS in X. Therefore f is an IFW$\ddot{g}$continuous mapping but not an IF continuous mapping.

Theorem 3.4: Every IF pre continuous mapping is an IFW$\ddot{g}$ continuous mapping, but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF pre continuous mapping and A be an IFCS in Y. Then f^{-1}(A) is an IFPCS in X. Since every IFPCS is an IFW$\ddot{g}$CS, f^{-1}(A) is an IFW$\ddot{g}$CS in X. Hence f is an IFW$\ddot{g}$continuous mapping.

Example 3.5: Let X={a,b}, Y={u,v} and A=<x,(0.5,0.3),(0.5,0.7)>, B=<x,(0.1,0.6),(0.9,0.3)>. Then τ={$0_{\sim}$,A, $1_{\sim}$} and σ ={$0_{\sim}$,B, $1_{\sim}$} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then IFS S=<x,(0.9,0.3),(0.1,0.6)> is an IFCS in Y and f^{-1}(S) is an IFW$\ddot{g}$CS but not an IFPCS in X. Therefore f is an IFW$\ddot{g}$continuous mapping but not an IF pre continuous mapping.

Theorem 3.6: Every IF α continuous mapping is an IFW$\ddot{g}$ continuous mapping, but not conversely.

Proof: Let f: (X, τ) → (Y, σ) be an IF α continuous mapping and A be an IFCS in Y. Then f^{-1}(A) is an IF α CS in X. Since every IF α CS is an IFW$\ddot{g}$CS, f^{-1}(A) is an IFW$\ddot{g}$CS in X. Hence f is an IFW$\ddot{g}$continuous mapping.

Example 3.7: Let X={a,b}, Y={u,v} and A=<x,(0.5,0.3),(0.5,0.7)>, B=<x,(0.6,0.8),(0.4,0.2)>. Then τ={$0_{\sim}$,A, $1_{\sim}$} and σ ={$0_{\sim}$,B, $1_{\sim}$} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then IFS S=<x,(0.4,0.2),(0.6,0.8)> is an IFCS in Y and f^{-1}(S) is an IFW$\ddot{g}$CS but not an IF α CS in X. Therefore f is an IFW$\ddot{g}$continuous mapping but not an IF α continuous mapping.

Theorem 3.8: A mapping f: (X, τ) → (Y, σ) is an IFW$\ddot{g}$continuous mapping if and only if the inverse image of every IFOS in Y is an IFW$\ddot{g}$OS in X.

Proof: Let A be an IFOS in Y. Then A^c is an IFCS in Y. Since f is an IFW$\ddot{g}$continuous mapping, $f^{-1}(A^c)$ is an IFW$\ddot{g}$CS in X. Since $f^{-1}(A^c)$ = $(f^{-1}(A))^c$, $f^{-1}(A)$ is an IFW$\ddot{g}$OS in X.

Theorem 3.9: If f: (X, τ) → (Y, σ) is an IFW$\ddot{g}$ continuous mapping and g: (Y, σ) → (Z,δ) is IF continuous, then gof: (X, τ) → (Z,δ) is IFW$\ddot{g}$ continuous.

Proof: Let f: (X, τ) → (Y, σ) is an IFW$\ddot{g}$continuous and g: (Y, σ) → (Z,δ) is IF continuous. Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y because g is IF continuous. Also $f^{-1}(g^{-1}(A))$ is an IFW$\ddot{g}$CS in X because f is IFW$\ddot{g}$continuous. Therefore $(gof)^{-1}(A)$ = $f^{-1}\big(g^{-1}(A)\big)$ is an IFW$\ddot{g}$CS in X. Hence gof is an IFW$\ddot{g}$continuous mapping.

Theorem 3.10: Let f: (X, τ) → (Y, σ) be an IFW$\ddot{g}$ continuous mapping. Then the following conditions are hold:
1. f(W$\ddot{g}$cl(A))⊆ cl(f(A)), for every IFS A in X,
2. W $\ddot{g}$cl($f^{-1}(B)$) ⊆ $f^{-1}(cl(B))$, for every IFS B in Y.

Proof: 1. Let f: (X, τ) → (Y, σ) be IFW$\ddot{g}$continuous. Let A be an intuitionistic fuzzy set in X. Then cl(f(A)) is an IFCS in Y. Since f is an IFW$\ddot{g}$ continuous, $f^{-1}(cl(f(A)))$ is an IFCS in X. Also A⊆ $f^{-1}(cl(A))$. Thus W$\ddot{g}$cl(A) ⊆W$\ddot{g}$cl($f^{-1}(cl(f(A))))$) = $f^{-1}(cl(f(A)))$ because $f^{-1}(cl(f(A)))$ is intuitionistic fuzzy weakly $\ddot{g}$ closed. Hence f(W$\ddot{g}$cl(A)) ⊆ cl(f(A)) for every IFS A in X.
2. Replacing A by $f^{-1}(B)$ in 1., we have f(W$\ddot{g}$cl($f^{-1}(B)$))⊆ cl(f($f^{-1}(B)$)) ⊆ cl(B). Hence W$\ddot{g}$cl($f^{-1}(B)$) ⊆ $f^{-1}(cl(B))$, for every IFS B in Y.

4. INTUITIONISTIC FUZZY WEAKLY $\ddot{g}$ CLOSED MAPPING AND INTUITIONISTIC FUZZY WEAKLY $\ddot{g}$ OPEN MAPPING

In this section, we study the notion of intuitionistic fuzzy weakly $\ddot{g}$ closed mappings, intuitionistic fuzzy weakly $\ddot{g}$ open mappings and investigate some of their properties.

Definition 4.1: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy weakly $\ddot{g}$ closed (IFW$\ddot{g}$ closed in short) mapping if f(V) is an IFW$\ddot{g}$CS in (Y, σ) for every IFCS V of (X, τ).

Example 4.2: Let X={a,b}, Y={u,v} and G_1=<x,(0.6,0.6),(0.4,0.4)>, G_2=<x,(0.5,0.6),(0.5,0.4)>. Then τ={0~, G_1, 1~} and σ ={0~, G_2, 1~} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IFW$\ddot{g}$ closed mapping.

Definition 4.3: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy weakly $\ddot{g}$ open (IFW$\ddot{g}$ open in short) mapping if f(V) is an IFW$\ddot{g}$OS in (Y, σ) for every IFOS V of (X, τ).

Definition 4.4: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy i weakly $\ddot{g}$ closed (IFiW$\ddot{g}$ closed in short) mapping if f(V) is an IFW$\ddot{g}$CS in (Y, σ) for every IF W$\ddot{g}$CS V of (X, τ).

Example 4.5: Let X={a,b}, Y={u,v} and G_1=<x,(0.5,0.6),(0.5,0.4)>, G_2=<x,(0.6,0.6),(0.4,0.4)>. Then τ={0~, G_1, 1~} and σ ={0~, G_2, 1~} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IFiW$\ddot{g}$ closed mapping.

Definition 4.6: A mapping f: (X, τ) → (Y, σ) is called an intuitionistic fuzzy i weakly $\ddot{g}$ open (IFiW$\ddot{g}$ open in short) mapping if f(V) is an IFW$\ddot{g}$OS in (Y, σ) for every IF W$\ddot{g}$OS V of (X, τ).

Theorem 4.7: Every IF closed mapping is an IFW$\ddot{g}$ closed mapping, but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IF closed mapping. Let A be an IFCS in X. Then f(A) is an IFCS in Y. This implies that f(A) is an IFW$\ddot{g}$CS in Y. Hence f is an IFW$\ddot{g}$ closed mapping.

Example 4.8: In example 4.2, f: (X, τ) → (Y, σ) is an IFW$\ddot{g}$ closed mapping but not an IFCM.

Theorem 4.9: Every IFα closed mapping is an IFW$\ddot{g}$ closed mapping, but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFα closed mapping. Let A be an IFCS in X. Then f(A) is an IFαCS in Y. This implies that f(A) is an IFW$\ddot{g}$CS in Y. Hence f is an IFW$\ddot{g}$ closed mapping.

Example 4.10: Let X={a,b}, Y={u,v} and G_1=<x,(0.6,0.8),(0.4,0.2)>, G_2=<x,(0.5,0.3),(0.5,0.7)>. Then τ={0~, G_1, 1~} and σ ={0~, G_2, 1~} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IFW$\ddot{g}$ closed mapping.

Theorem 4.11: Every IF pre closed mapping is an IFW$\ddot{g}$ closed mapping, but not conversely.

Proof: Assume that f: (X, τ) → (Y, σ) is an IFα closed mapping. Let A be an IFCS in X. Then f(A) is an IFPCS in Y. This implies that f(A) is an IFW$\ddot{g}$CS in Y. Hence f is an IFW$\ddot{g}$ closed mapping.

Example 4.12: Let X={a,b}, Y={u,v} and G_1=<x,(0.1,0.6),(0.9,0.3)>, G_2=<x,(0.5,0.3),(0.5,0.7)>. Then τ={0~, G_1, 1~} and σ ={0~, G_2, 1~} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IFW$\ddot{g}$ closed mapping.

Theorem 4.13: If f: (X, τ) → (Y, σ) is an IFW$\ddot{g}$ closed mapping and A is an IFCS of X, then f|A:A → Y is an IFW$\ddot{g}$ closed mapping.

Proof: Let B⊆ A be an IFCS in A, then B is an IFCS in X, since A is an IFCS in X. f(B) is an IFW$\ddot{g}$ closed set in Y as f is an IF W$\ddot{g}$CM. But f(B)=(f|A)(B). So (f|A)(B) is an IFW$\ddot{g}$ closed set in Y. Therefore f|A is an IF$\ddot{g}$ closed mapping.

Theorem 4.14: If f: (X, τ) → (Y, σ) is an IF closed mapping and g: (Y, σ) → (Z,δ) is an IFW$\ddot{g}$ closed mapping, then gof: (X, τ) → (Z,δ) is an IFW$\ddot{g}$ closed mapping.

Proof: Let H be an IFCS in X. Then f(H) is an IFCS. But (g o f)(H) = g(f(H)) is an IFW$\ddot{g}$CS as g is an IFW$\ddot{g}$ closed mapping. Thus g o f is an IFW$\ddot{g}$ closed mapping.

Theorem 4.15: If f: (X, τ) → (Y, σ) is a bijective mapping, then the following statements are equivalent

1. f is an IFW$\ddot{g}$OM
2. f is an IFW$\ddot{g}$CM
3. f^{-1}: (Y, σ) → (X, τ) is an IFW$\ddot{g}$ continuous.

Proof: (1)⇒(2): Let U be an IFCS in X and f be an IFW$\ddot{g}$OM. Then X – U is an IFOS in X. By assumption, we get f(X-U) is an IFW$\ddot{g}$OS in Y. That is Y-f(X-U) = f (U) is IFW$\ddot{g}$CS in Y.
(2)⇒(3): Let U be an IFCS in X. By assumption, f(U) is an IFW$\ddot{g}$CS in Y. As f(U) = $(f^{-1})^{-1}$(U), f^{-1} is an IFW$\ddot{g}$ continuous.
(3)⇒(1): Let U be an IFOS in X. By assumption f (U) = $(f^{-1})^{-1}$(U). That is, f(U) is an IFW$\ddot{g}$OS in Y. Hence f is an IFW$\ddot{g}$OM.

Theorem 4.16: If f: (X, τ) → (Y, σ) and g: (Y, σ) → (Z,δ) are IFiW$\ddot{g}$ closed mappings, then gof: (X, τ) → (Z,δ) is an IFiW$\ddot{g}$CM.

Proof: Let V be an IFW$\ddot{g}$CS in X. Since f is an IFiW$\ddot{g}$closed mapping, f(V) is an IFW$\ddot{g}$CS in Y. Then g(f(V)) is an IFW$\ddot{g}$CS in Z. Hence gof is an IFiW$\ddot{g}$CM.

Theorem 4.17: If f: (X, τ) → (Y, σ) is an IF W$\ddot{g}$closed mapping and g: (Y, σ) → (Z,δ) is an IFiW$\ddot{g}$ closed mapping, then gof: (X, τ) → (Z,δ) is an IFW$\ddot{g}$ closed mapping.

Proof: Let V be an IFCS in X. Since f is an IFW$\ddot{g}$closed mapping, f(V) is an IFW$\ddot{g}$CS in Y. Then g(f(V)) is an IFW$\ddot{g}$CS in Z. Hence gof is an IFW$\ddot{g}$CM.

5. INTUITIONISTIC FUZZY WEAKLY $\ddot{g}$ HOMEOMORPHISM

In this section, we study the notion of intuitionistic fuzzy weakly $\ddot{g}$ homeomorphism and investigate some of their properties.

Definition 5.1: A bijection mapping f: (X, τ) → (Y, σ) is called intuitionistic fuzzy weakly $\ddot{g}$ homeomorphism (IFW $\ddot{g}$ homeomorphism in short) if f is both an IFW $\ddot{g}$ continuous mapping and IFW $\ddot{g}$ closed mapping.

Theorem 5.2: Every IF homeomorphism is an IFW $\ddot{g}$ homeomorphism, but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an IF homeomorphism. Then f is IF continuous and IF closed. Since every IF continuous function is IF W $\ddot{g}$continuous and every IF closed mapping is IFW $\ddot{g}$ closed, f is IF W $\ddot{g}$continuous and IFW $\ddot{g}$closed. Hence f is an IFW $\ddot{g}$homeomorphism.

Example 5.3: Let X={a,b}, Y={u,v} and A=<x,(0.5,0.6),(0.5,0.4)>, B=<x,(0.4,0.4), (0.6,0.6)>. Then τ={$0_\sim$, A, $1_\sim$} and σ ={$0_\sim$, B, $1_\sim$} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IFW$\ddot{g}$ closed homeomorphism but not an IF homeomorphism.

Theorem 5.4: Every IF α homeomorphism is an IFW $\ddot{g}$ homeomorphism, but not conversely.

Proof: Let f : (X, τ) → (Y, σ) be an IF α homeomorphism. Then f is IF α continuous and IF α closed. Since every IFα continuous function is IF W $\ddot{g}$continuous and every IF α closed mapping is IFW $\ddot{g}$ closed, f is IF W $\ddot{g}$continuous and IFW $\ddot{g}$closed. Hence f is an IFW $\ddot{g}$ homeomorphism.

Example 5.5: Let X={a,b}, Y={u,v} and A=<x,(0.5,0.3),(0.5,0.7)>, B=<x,(0.4,0.2),(0.6,0.8)>. Then τ={$0_{\sim}$, A, $1_{\sim}$} and σ ={$0_{\sim}$, B, $1_{\sim}$} are IFTs on X and Y respectively. Define f: (X, τ) → (Y, σ) by f(a)=u and f(b)=v. Then f is an IFW$\ddot{g}$ closed homeomorphism but not an IF homeomorphism.

Theorem 5.6: For any bijection mapping f : (X, τ) → (Y, σ) the following statements are equivalent

(i) The inverse map f^{-1}: (Y, σ) → (X, τ) is IFW $\ddot{g}$ continuous,
(ii) f is an IFW $\ddot{g}$ open mapping,
(iii) f is an IFW $\ddot{g}$ closed mapping.

Proof: (i) ⇒(ii). Let V be an IFOS in X. Since f^{-1} is IFW $\ddot{g}$continuous, the inverse image of V under f^{-1} , namely f(V) is an IFW $\ddot{g}$ OS in Y and so f is an IFW $\ddot{g}$ open mapping.
(ii) ⇒ (iii). Let V be any IFCS in X. Then V^c is an IFOS in X. Since f is IFW $\ddot{g}$open, f(V^c) is an IFW $\ddot{g}$ OS in Y . But f(V^c) = Y − f(V) and so f(V) is an IFW $\ddot{g}$CS in Y . Therefore f is an IFW $\ddot{g}$ closed mapping.
(iii) ⇒ (i). Let V be any IFCS in X. Then the inverse image of V under f^{-1}, namely f(V) is IFW $\ddot{g}$ CS in Y, since f is an IFW $\ddot{g}$ closed mapping. Therefore f^{-1} is an IFW $\ddot{g}$ continuous mapping.

Theorem 5.7: Let f : (X, τ) → (Y, σ) be bijective mapping and IFW $\ddot{g}$ continuous. Then the following statements are equivalent

(i) f is an IFW $\ddot{g}$ open mapping,
(ii) f is an IFW $\ddot{g}$ homeomorphism,
(iii) f is an IFW $\ddot{g}$ closed mapping.

Proof: (i) ⇒(ii). Given f : (X, τ) → (Y, σ) is bijective, IFW $\ddot{g}$ continuous and IFW $\ddot{g}$ open. Then by definition, f is and IFW $\ddot{g}$ homeomorphism.
(ii) ⇒ (iii). Given f : (X, τ) → (Y, σ) is bijective and an IFW $\ddot{g}$ homeomorphism. By above theorem f ia an IFW $\ddot{g}$ closed mapping.
(iii) ⇒ (i). Given f : (X, τ) → (Y, σ) is bijective and an IFW $\ddot{g}$ closed. By above theorem, f is an IFW $\ddot{g}$ open mapping.

Theorem 5.8: Let f : (X, τ) → (Y, σ) be an IFW $\ddot{g}$ homeomorphism, then f is an IF homeomorphism if X and Y are IFW $\ddot{g}$ $T_{1/2}$ space.

Proof: Let B be an IFCS in Y. Then f^{-1}(B) is an IFW $\ddot{g}$CS in X, by hypothesis. Since X is an IFW$\ddot{g}T_{1/2}$space, f^{-1}(B) is an IFCS in X. Hence f is an IF continuous mapping. By hypothesis f^{-1}: (Y, σ) $\rightarrow$ (X, τ) is IFW $\ddot{g}$ continuous mapping. Let A be an IFCS in X. Then $(f^{-1})^{-1}$(A) =f(A) is an IFW $\ddot{g}$CS in Y, by hypothesis. Since Y is an IFW$\ddot{g}T_{1/2}$space, f(A) is an IFCS in Y. Therefore f^{-1} is an IF continuous mapping. Hence the mapping f is an IF homeomorphism.

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