Journal of Computer and Mathematical Sciences, *Vol.10(2),287-300 February 2019* *ISSN 0976-5727 (Print)*
(An International Research Journal), *www.compmath-journal.org* *ISSN 2319-8133 (Online)*

Vague β -Soft Continuous and Vague b-soft Continuous Functions

V. Inthumathi[*] and M. Pavithra[1]

[*]Associate Professor,
Department of Mathematics,
Nallamuthu Gounder Mahalingam College,
Pollachi-642 001, Coimbatore, Tamilnadu, INDIA.
[1]Research Scholar,
Department of Mathematics,
Nallamuthu Gounder Mahalingam College,
Pollachi-642 001, Coimbatore, Tamilnadu, INDIA.
[*]email: inthumathi65@gmail.com, [1]email: mk.pavipavithra@gmail.com.

(Received on: February 11, 2019)

ABSTRACT

The purpose of this paper is to study new classes of vague soft open sets namely vague β -soft open sets, vague b-soft open sets in vague soft topological spaces and discuss their relationships among vague α -soft sets, vague semi-soft sets and vague pre-soft sets. The concepts of vague β -soft continuous functions, vague b-soft continuous functions, vague β -soft irresolute and vague b -soft irresolute functions have been introduced and studied.

AMS Subject Classification: 03B52, 54A05, 54A40.

Keywords: Vague β -soft open sets, vague b-soft open sets, vague β -soft continuous functions, vague b-soft continuous functions.

1. INTRODUCTION

Soft set theory was first initiated by the Russian researcher Molodtsov[17] in 1999. He proposed the soft set as a completely generic mathematical tool for modeling uncertainties. There are many Mathematical tools available for modeling complex systems such as probability theory, fuzzy set theory[20], intuitionistic fuzzy set theory[7], interval Mathematics. But there are inherent difficulties associated with each of these techniques. Moreover, all these techniques lack in the parameterization of the tools and hence they could not be applied successfully in tackling problems especially in areas like economic, environmental and social problem domains. Soft set theory is standing in a unique way in the sense that it is free from the above difficulties and has a wider scope for many applications in a multidimensional way.

In[16], Maji *et al*. introduced several operators for soft set theory and made a theoretical study of the soft set theory in more detail. Shabir and Naz[18] introduced the notion of soft topological spaces which are defined over an initial universal set with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. After that Kandil *et al*.[15] have introduced the new notions of semi-open soft sets, pre-open soft sets, α-open soft sets, β-open soft sets and their respective continuous functions in soft topological spaces. Since then, many authors[1,2,3,12] have studied some stronger and weaker forms of soft open sets and their soft continuous functions in soft topological spaces. Recently, many researchers have worked to obtain new decompositions of continuity in soft topological spaces.

In 1993, Gau and Buehrer[11] have introduced the concept of vague sets which allow interval-based membership instead of using single point based membership values as in fuzzy sets. In 2010, vague soft set theory was initiated by Xu *et al*.[19] by combining the notions of vague set theory and soft set theory. They also derived some basic properties and illustrated its potential applications. Vague soft set theory is actually an extension of soft set theory. Its basic concepts and its extensions as well as some interesting applications can be found in[4,5,6,8].

Recently in 2014, C.Wang and Y.Li[10] initiated the study of vague soft topological spaces. They defined vague soft topology on the collection τ of vague soft sets over an initial universe with a fixed set of parameters. Consequently, they defined basic notions of vague soft topological spaces such as vague soft open and closed sets, vague soft interior, vague soft closure, vague soft boundary, vague soft connectedness and vague soft compactness. Later, V. Inthumathi *et al*.[13] defined vague α-soft open sets and obtained its decomposition by using the notions of vague semi-soft open sets and vague pre-soft open sets. In this paper, we introduce new classes of vague soft open sets called vague β-soft open sets, vague b -soft open sets in vague soft topological spaces. We also investigate some of their properties and relations between the existing vague soft open sets. Further, we define vague β-soft continuous functions and vague b-soft continuous functions and obtain some characterizations.

2. PRELIMINARIES

Definition 2.1.[11] A vague set A = $\{(x_i, [t_A(x_i), 1 - f_A(x_i)])\mid x_i \in X\}$ in the universe X = $\{x_1, x_2, \ldots, x_n\}$ is characterized by a truth-membership function $t_A\colon X \to [0,1]$, and a false-membership function $f_A\colon X \to [0,1]$, where $t_A(x_i)$ is a lower bound on the grade of membership of x_i derived from the evidence of x_i, $f_A(x_i)$ is the lower bound on the negation of x_i derived from the evidence against x_i and $0 \leq t_A(x_i) + f_A(x_i) \leq 1$ for any $x_i \in X$. The grade of membership of x_i in the vague set is bounded to a subinterval $[t_A(x_i), 1 - f_A(x_i)]$ of [0,1]. The vague value $[t_A(x_i), 1 - f_A(x_i)]$ indicates that the exact grade of membership $\mu_A(x_i)$ of x_i may be unknown, but it is bounded by $t_A(x_i) \leq \mu_A(x_i) \leq 1 - f_A(x_i)$, where $0 \leq t_A(x_i) + f_A(x_i) \leq 1$.

Definition 2.2.[19] Let X be an initial universe set, V(X) the set of all vague sets on X, E a set of parameters, and A ⊆ E. A pair (F, A) is called a vague soft set over X, where F is a mapping given by

F : A → V (X). The set of all vague soft sets on X is denoted by V$\tilde{S}$(X,E), called vague soft classes.

Definition 2.3.[19] A vague soft set (F,A) over X is said to be a null vague soft set denoted by $\widehat{\emptyset}$, if ∀e ∈ A , $t_{F(e)}(x)$ = 0, $1 - f_{F(e)}(x)$ = 0, x ∈ X.

Definition 2.4.[19] A vague soft set (F,A) over X is said to be an absolute vague soft set denoted by $\hat{X}$, if ∀e ∈ A , $t_{F(e)}(x)$= 1 , $1 - f_{F(e)}(x)$ = 1, x ∈ X.

Definition 2.5.[10] Let X be an initial universe set, E be the nonempty set of parameters and τ be the collection of vague soft sets over X, then τ is said to be a vague soft topology on X if
1. $\widehat{\emptyset}_E$, $\hat{X}_E$ belongs to τ.
2. the union of any number of vague soft sets in τ belongs to τ.
3. the intersection of any two vague soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a vague soft topological space over X.

Definition 2.6.[13] Let (X,τ,E) be a vague soft topological space and (F,E) be a vague soft set over X. Then vague soft interior of (F,E) is defined by, v$\tilde{s}$int (F,E) = ∪{(G,E) / (G,E) ∈ τ and (G,E) ⊆ (F,E)}.

Definition 2.7.[13] Let (X,τ,E) be a vague soft topological space and (F,E) be a vague soft set over X. Then vague soft closure of (F,E) is defined by, v$\tilde{s}$cl (F,E) = ∩{(H, E) / (H,E) ∈ τ^c and (F,E) ⊆ (H,E)}.

Theorem 2.8.[13] Let (X,τ,E) be a vague soft topological space over X, and let (F, E) and (G,E) be two vague soft sets over X. Then the following properties hold:
1. v$\tilde{s}$int($\widehat{\emptyset}_E$) = $\widehat{\emptyset}_E$, v$\tilde{s}$int($\hat{X}_E$) = $\hat{X}_E$, v$\tilde{s}$cl($\widehat{\emptyset}_E$) = $\widehat{\emptyset}_E$ and v$\tilde{s}$cl($\hat{X}_E$) = $\hat{X}_E$.
2. vs$\tilde{s}$int(F,E) ⊆ (F,E) ⊆ vs$\tilde{s}$cl(F,E).
3. (F,E) ∈ τ iff v$\tilde{s}$int(F,E)=(F,E) and (F,E) ∈ τ^c iff v$\tilde{s}$cl(F,E)=(F,E).
4. v$\tilde{s}$int(v$\tilde{s}$int(F,E)) = v$\tilde{s}$int(F,E) and v$\tilde{s}$cl(v$\tilde{s}$cl(F,E)) = v$\tilde{s}$cl(F,E).
5. (F,E) ⊆ (G,E) implies v$\tilde{s}$int(F,E) ⊆ v$\tilde{s}$int(G,E), v$\tilde{s}$cl(F,E) ⊆ v$\tilde{s}$cl(G,E).
6. v$\tilde{s}$int((F,E) ∩ (G,E)) = v$\tilde{s}$int(F,E) ∩ v$\tilde{s}$int(G,E).
7. v$\tilde{s}$int((F,E) ∪ (G,E)) ⊇ v$\tilde{s}$int(F,E) ∪ v$\tilde{s}$int(G,E).
8. (v$\tilde{s}$int(F,E))^c = v$\tilde{s}$cl((F,E)^c) and (v$\tilde{s}$cl(F,E))^c = v$\tilde{s}$int((F,E)^c).
9. v$\tilde{s}$cl((F,E) ∪ (G,E)) = v$\tilde{s}$cl(F,E) ∪ v$\tilde{s}$l(G,E).
10. v$\tilde{s}$cl((F,E) ∩ (G,E)) ⊆ v$\tilde{s}$cl(F,E) ∩ v$\tilde{s}$cl(G,E).

Definition 2.9.[13] A vague soft set (F,A) of a vague soft topological space (X,τ,E) is said to be
1. vague semi-soft open if (F,A) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int (F,A)).
2. vague pre-soft open if (F,A) ⊆ v$\tilde{s}$int(v$\tilde{s}$cl (F,A)).

3. vague α-soft open if (F,A) ⊆ $v\tilde{s}$int($v\tilde{s}$cl($v\tilde{s}$int (F,A))).
4. vague regular-soft open if (F,A) = $v\tilde{s}$int($v\tilde{s}$cl (F,A)).

The complement of vague semi-soft open (resp., vague pre-soft open, vague α -soft open, vague regular-soft open) set is called vague semi-soft closed (resp., vague pre-soft closed, vague α -soft closed, vague regular-soft closed) set. And we denote the family of all vague semi-soft open sets (resp., vague pre-soft open, vague α -soft open sets, vague regular-soft open sets) of a vague soft topological space (X,τ,E) by VS$\tilde{S}$O(X) (resp., VP$\tilde{S}$O(X), Vα$\tilde{S}$O(X), VR$\tilde{S}$O(X)).

Throughout this paper (X,τ,E), (Y,σ,K) are denote the vague soft topological spaces on X , Y respectively.

3. VAGUE β-SOFT OPEN AND VAGUE b-soft Open Sets

Definition 3.1. A vague soft set (F,E) of a vague soft topological space (X,τ,E) is said to be
1. vague β-soft open if (F,E) ⊆ $v\tilde{s}$cl($v\tilde{s}$int($v\tilde{s}$cl(F,E))) .
2. vague b -soft open if (F,E) ⊆ $v\tilde{s}$int($v\tilde{s}$cl(F,E)) ∪ $v\tilde{s}$cl($v\tilde{s}$int(F,E)).

The complement of vague β -soft open (resp., vague b-soft open) set is called vague β-soft closed (resp., vague pre-soft closed) set. We will denote the family of all vague β-soft open (resp., vague β-soft closed, vague b -soft open, vague b- soft closed) sets by Vβ$\tilde{S}$O(X) (resp., Vβ$\tilde{S}$C(X) , Vb$\tilde{S}$O(X) , Vb$\tilde{S}$ (X)).

Theorem 3.2. For a vague soft set (F,E) in a vague soft topological space (X, τ, E),
1. (F,E) ∈ Vβ$\tilde{S}$O(X) iff $(F,E)^c$ ∈ Vβ$\tilde{S}$C(X).
2. (F,E) ∈ Vb$\tilde{S}$O(X) iff $(F,E)^c$ ∈ Vb$\tilde{S}$C(X).

Definition 3.3. Let (X,τ,E) be a vague soft topological space and (F,E) be a vague soft set over X.
Then Vague β-soft interior and Vague b-soft interior of (F,E) are defined as:
vβ$\tilde{s}$int(F,E) = ∪{(G,E) / (G,E) ∈ Vβ$\tilde{S}$O(X) and (G,E) ⊆ (F,E)}.
vb$\tilde{s}$int(F,E) = ∪{(G,E) / (G,E) ∈ Vb$\tilde{S}$O(X) and (G,E) ⊆ (F,E)}.

Theorem 3.4. Let (F,E) be any vague soft set in (X, τ, E). Then
i. vβ$\tilde{s}$cl($(F,E)^c$) = (vβ$\tilde{s}$int$(F,E))^c$ and vβ$\tilde{s}$int($(F,E)^c$) = (vβ$\tilde{s}$cl$(F,E))^c$.
ii. vb$\tilde{s}$cl($(F,E)^c$) = (vb$\tilde{s}$int$(F,E))^c$ and vb$\tilde{s}$int($(F,E)^c$) = (vb$\tilde{s}$cl$(F,E))^c$.
Proof. The proof is obvious from the above Definition 3.3.

Proposition 3.5. Let (F,E) be any vague soft set in (X, τ, E). Then
i. (F,E) ∈ Vβ$\tilde{S}$O(X) iff vβ$\tilde{s}$int(F,E) = (F,E) and (F,E) ∈ Vβ$\tilde{S}$C(X) iff vβ$\tilde{s}$cl(F,E) = (F,E).
ii. (F,E) ∈ Vb$\tilde{S}$O(X) iff vb$\tilde{s}$int(F,E) = (F,E) and (F,E) ∈ Vb$\tilde{S}$C(X) iff vb$\tilde{s}$cl(F,E) = (F,E).

Remark 3.6. Let (F,E) be a vague soft set in (X, τ, E). Then,
i. vβ$\tilde{s}$cl(F,E) = (F,E) ∪ $v\tilde{s}$int($v\tilde{s}$cl($v\tilde{s}$int(F,E))).
ii. vβ$\tilde{s}$int(F,E) = (F,E) ∩ $v\tilde{s}$cl($v\tilde{s}$int($v\tilde{s}$cl(F,E))).

iii. $vb\tilde{s}cl(F,E) = (F,E) \cup [v\tilde{s}int(v\tilde{s}cl(F,E)) \cap v\tilde{s}cl(v\tilde{s}int(F,E))]$.
iv. $vb\tilde{s}int(F,E) = (F,E) \cap [v\tilde{s}int(v\tilde{s}cl(F,E)) \cup v\tilde{s}cl(v\tilde{s}int(F,E))]$.

Theorem 3.7. In a vague soft topological space (X, τ, E),
i. an arbitrary union of vague β-soft open sets is a vague β-soft open set.
ii. an arbitrary union of vague b -soft open sets is a vague b -soft open set.
Proof. i. Let $\{(F_\alpha ,E)\}$ be a collection of vague β-soft open sets. Then, for each α
$(F_\alpha,E) \subseteq v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl(F_\alpha,E)))$.
Now, $\cup_\alpha (F_\alpha,E) \subseteq \cup_\alpha [v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl(F_\alpha,E)))]$
$= v\tilde{s}cl[\cup_\alpha v\tilde{s}int(v\tilde{s}cl(F_\alpha,E))]$
$\subseteq v\tilde{s}cl(v\tilde{s}int[\cup_\alpha v\tilde{s}cl(F_\alpha,E)])$
$= v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl[\cup_\alpha (F_\alpha,E)]))$
$\Rightarrow \cup_\alpha (F_\alpha,E) \subseteq v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl[\cup_\alpha (F_\alpha,E)]))$. Hence $\cup_\alpha (F_\alpha,E)$ is a vague β-soft open set.
ii. Let $\{(G_\alpha,E)\}$ be a collection of vague b -soft open sets. Then, for each α
$(G_\alpha,E) \subseteq v\tilde{s}int(v\tilde{s}cl(G_\alpha,E)) \cup v\tilde{s}cl(v\tilde{s}int(G_\alpha,E))$.
Now, $\cup_\alpha (G_\alpha,E) \subseteq \cup_\alpha[v\tilde{s}int(v\tilde{s}cl(G_\alpha,E)) \cup v\tilde{s}cl(v\tilde{s}int(G_\alpha,E))]$
$= \cup_\alpha[v\tilde{s}int(v\tilde{s}cl(G_\alpha,E))] \cup \cup_\alpha[v\tilde{s}cl(v\tilde{s}int(G_\alpha,E))]$
$\subseteq v\tilde{s}int(\cup_\alpha [v\tilde{s}cl(G_\alpha,E)]) \cup v\tilde{s}cl(\cup_\alpha [v\tilde{s}int(G_\alpha,E)])$
$\subseteq v\tilde{s}int(v\tilde{s}cl(\cup_\alpha (G_\alpha,E))) \cup v\tilde{s}cl(v\tilde{s}int(\cup_\alpha (G_\alpha,E)))$
$\Rightarrow \cup_\alpha (G_\alpha,E) \subseteq v\tilde{s}int(v\tilde{s}cl(\cup_\alpha (G_\alpha,E))) \cup v\tilde{s}cl(v\tilde{s}int(\cup_\alpha (G_\alpha,E)))$ is a vague b-soft open set.

Theorem 3.8. If (F,E) and (G,E) are two vague soft sets over (X,τ,E), then the following properties are hold.
i. $v\beta\tilde{s}int(\widehat{\emptyset}_E) = \widehat{\emptyset}_E$ and $v\beta\tilde{s}int(\hat{X}_E) = \hat{X}_E$.
ii. $vb\tilde{s}int(\widehat{\emptyset}_E) = \widehat{\emptyset}_E$ and $vb\tilde{s}int(\hat{X}_E) = \hat{X}_E$.
iii. $v\beta\tilde{s}int(F,E) \subseteq (F,E) \subseteq v\beta\tilde{s}cl(F,E)$.
iv. $vb\tilde{s}int(F,E) \subseteq (F,E) \subseteq vb\tilde{s}cl(F,E)$.
v. $(F,E) \subseteq (G,E) \Rightarrow v\beta\tilde{s}int(F,E) \subseteq v\beta\tilde{s}int(G, E)$ and $v\beta\tilde{s}cl(F,E) \subseteq v\beta\tilde{s}cl(G,E)$.
vi. $(F,E) \subseteq (G,E) \Rightarrow vb\tilde{s}int(F,E) \subseteq vb\tilde{s}int(G,E)$ and $vb\tilde{s}cl(F,E) \subseteq vb\tilde{s}cl(G,E)$.
vii. $v\beta\tilde{s}int((F,E) \cap (G,E)) \subseteq v\beta\tilde{s}int(F,E) \cap v\beta\tilde{s}int(G,E)$.
viii. $v\beta\tilde{s}cl((F,E) \cup (G,E)) \supseteq v\beta\tilde{s}cl(F,E) \cup v\beta\tilde{s}cl(G,E)$.
ix. $vb\tilde{s}int((F,E) \cap (G,E)) \subseteq vb\tilde{s}int(F,E) \cap vb\tilde{s}int(G, E)$.
x. $vb\tilde{s}cl((F,E) \cup (G,E)) \supseteq vb\tilde{s}cl(F,E) \cup vb\tilde{s}cl(G,E)$.

Theorem 3.9. If (F,E) is a vague semi-soft set and (G,E) is a vague pre-soft open set in (X,τ,E) such that $(G,E) \subseteq (F,E) \subseteq v\tilde{s}cl(v\tilde{s}int(G,E))$, then (F,E) is a vague β-soft open set.
Proof. Since (G,E) is a vague pre-soft open set, $(G,E) \subseteq v\tilde{s}int(v\tilde{s}cl(G,E))$. By assumption,
$(F,E) \subseteq v\tilde{s}cl(v\tilde{s}int(G,E))$
$\subseteq v\tilde{s}cl(v\tilde{s}int(v\tilde{s}int(v\tilde{s}cl(G,E))))$
$= v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl(G,E)))$
$\subseteq v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl(F,E)))$.
Hence, (F,E) is a vague β-soft open set.

Theorem 3.10. If (F,E) is a vague β-soft open and vague semi-soft closed set in (X,τ,E), then it is vague semi-soft open.
Proof. Since (F,E) ∈ Vβ$\tilde{S}$O(X) and (F,E) ∈ VS$\tilde{S}$C(X),
v$\tilde{s}$int(v$\tilde{s}$cl(F,E)) ⊆ (F,E) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))).Thus there exist a vague soft open set (H,E) = v$\tilde{s}$int(v$\tilde{s}$cl(F,E)) ∈ τ such that (H,E) ⊆ (F,E) ⊆ v$\tilde{s}$cl(H,E). Hence (F,E) ∈ VS$\tilde{S}$O(X).

Theorem 3.11. If (F,E) is a vague β-soft open and vague α-soft closed set in (X,τ,E), then it is vague soft closed.
Proof. Since (F,E) ∈ Vβ$\tilde{S}$O(X) and (F,E) ∈ Vα$\tilde{S}$C(X),
v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))) ⊆ (F,E) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))).Then (F,E) = v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))) which shows that (F,E) ∈ τ^c.

Theorem 3.12. Every vague semi-soft open (closed) sets is vague β-soft open (closed) set in (X,τ,E).
Proof. Let (F,E) be a vague semi-soft open set. Then, (F,E) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int(F,E)). Since, (F,E) ⊆ v$\tilde{s}$cl(F,E), (F,E) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int(F,E)) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))). Hence, (F,E) ∈ Vβ$\tilde{S}$O(X).

Theorem 3.13. Every vague pre-soft open (closed) sets is vague β-soft open (closed) set in (X,τ,E).
Proof. Let (F,E) be a vague pre-soft open set. Then, (F,E) ⊆ v$\tilde{s}$int(v$\tilde{s}$cl(F,E)). Since, (F,E) ⊆ v$\tilde{s}$cl(F,E), (F,E) ⊆ v$\tilde{s}$int(v$\tilde{s}$cl(F,E)) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))). Hence, (F,E) ∈ Vβ$\tilde{S}$O(X).

Theorem 3.14. For any vague b-soft open set (F,E) in (X,τ,E), v$\tilde{s}$cl(F,E) is vague regular-soft closed set.
Proof. Let (F,E) be vague b-soft open set in X. Then,
(F,E) ⊆ v$\tilde{s}$int(v$\tilde{s}$cl(F,E)) ∪ v$\tilde{s}$cl(v$\tilde{s}$int(F,E))
⇒ v$\tilde{s}$cl(F,E) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E)) ∪ v$\tilde{s}$cl(v$\tilde{s}$int(F,E)))
= v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))) ∪ v$\tilde{s}$cl(v$\tilde{s}$cl(v$\tilde{s}$int(F,E)))
= v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))) ∪ v$\tilde{s}$cl(v$\tilde{s}$int(F,E))
= v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))).
Thus, v$\tilde{s}$cl(F,E) ⊆ v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))). But v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))) ⊆ v$\tilde{s}$cl(F,E). Therefore, v$\tilde{s}$cl(F,E) = v$\tilde{s}$cl(v$\tilde{s}$int(v$\tilde{s}$cl(F,E))). Hence v$\tilde{s}$cl(F,E) is vague regular-soft closed set.

Proposition 3.15. If (F,E) is a vague soft set in (X,τ,E) with v$\tilde{s}$int(F,E) = $\widehat{\emptyset}_E$, then it is vague b-soft closed set.

Proposition 3.16. If (F,E) is a vague soft set in (X,τ,E) with v$\tilde{s}$cl(F,E) = $\hat{X}_E$, then it is vague b -soft open set.

Theorem 3.17. Let (F,E) be a vague soft set in (X,τ,E). Then,
1. vb$\tilde{s}$int(F,E) = vs$\tilde{s}$int(F,E) ∪ vp$\tilde{s}$int(F,E).
2. vb$\tilde{s}$cl(F,E) = vs$\tilde{s}$cl(F,E) ∩ vp$\tilde{s}$cl(F,E).
Proof. 1. vs$\tilde{s}$int(F,E) ∪ vp$\tilde{s}$int(F,E) = [(F,E) ∩ v$\tilde{s}$cl(v$\tilde{s}$int(F,E))] ∪ [(F,E) ∩ v$\tilde{s}$int(v$\tilde{s}$cl(F,E))] = (F,E) ∩ [v$\tilde{s}$cl(v$\tilde{s}$int(F,E)) ∪ v$\tilde{s}$int(v$\tilde{s}$cl(F,E))] = vb$\tilde{s}$int(F,E).

2. $vs\tilde{s}cl(F,E) \cap vp\tilde{s}cl(F,E) = [(F,E) \cup v\tilde{s}int(v\tilde{s}cl(F,E))] \cap [(F,E) \cup v\tilde{s}cl(v\tilde{s}int(F,E))]$
$= (F,E) \cup [v\tilde{s}int(v\tilde{s}cl(F,E)) \cap v\tilde{s}cl(v\tilde{s}int(F,E))] = vb\tilde{s}cl(F,E)$.

Theorem 3.18. Every vague soft set (F,E) is vague b -soft open (closed) in (X,τ,E) iff it is the union (intersection) of vague semi-soft open (closed) set and vague pre-soft open (closed) set.

Theorem 3.19. If (G,E) is a vague soft open set and (F,E) is a vague b -soft open set in (X,τ,E), then $(F,E) \cup (G,E)$ is vague b -soft open set in (X,τ,E).

Proof. $(G,E) \cup (F,E) \subseteq v\tilde{s}int(G,E) \cup [v\tilde{s}cl(v\tilde{s}int(F,E)) \cup v\tilde{s}int(v\tilde{s}cl(F,E))]$
$= [v\tilde{s}int(G,E) \cup v\tilde{s}cl(v\tilde{s}int(F,E))] \cup [v\tilde{s}int(G,E) \cup v\tilde{s}int(v\tilde{s}cl(F,E))]$
$\subseteq [v\tilde{s}cl(v\tilde{s}int(G,E)) \cup v\tilde{s}cl(v\tilde{s}int(F,E))] \cup [v\tilde{s}int(v\tilde{s}cl(G,E)) \cup v\tilde{s}int(v\tilde{s}cl(F,E))]$
$\subseteq [v\tilde{s}cl(v\tilde{s}int((G,E) \cup (F,E)))] \cup [v\tilde{s}int(v\tilde{s}cl((G,E) \cup (F,E)))]$.
Hence, $(F,E) \cup (G,E)$ is vague b -soft open set in (X,τ,E).

Theorem 3.20. If (G,E) is a vague α-soft open set and (F,E) is a vague b-soft open set in (X,τ,E), then $(F,E) \cup (G,E)$ is a vague b -soft open set in (X,τ,E).
Proof. $(G,E) \cup (F,E) \subseteq [v\tilde{s}int(v\tilde{s}cl(v\tilde{s}int(G,E)))] \cup [v\tilde{s}cl(v\tilde{s}int(F,E)) \cup v\tilde{s}int(v\tilde{s}cl(F,E))]$
$= [v\tilde{s}int(v\tilde{s}cl(v\tilde{s}int(G,E))) \cup v\tilde{s}cl(v\tilde{s}int(F,E))] \cup [v\tilde{s}int(v\tilde{s}cl(v\tilde{s}int(G,E))) \cup v\tilde{s}int(v\tilde{s}cl(F,E))]$
$\subseteq [v\tilde{s}cl(v\tilde{s}int(G,E)) \cup v\tilde{s}cl(v\tilde{s}int(F,E))] \cup [v\tilde{s}int(v\tilde{s}cl(G,E)) \cup v\tilde{s}int(v\tilde{s}cl(F,E))]$
$\subseteq [v\tilde{s}cl(v\tilde{s}int((G,E) \cup (F,E)))] \cup [v\tilde{s}int(v\tilde{s}cl((G,E) \cup (F,E)))]$.
Hence, $(F,E) \cup (G,E)$ is vague b-soft open set in (X,τ,E).

Theorem 3.21. In a vague soft topological space (X,τ,E) the followings are hold.

i. Every vague semi-soft open (closed) set is a vague b-soft open (closed) set.
ii. Every vague pre-soft open (closed) set is a vague b-soft open (closed) set.
iii. Every vague b-soft open (closed) set is a vague β-soft open (closed) set.
Proof. i. Let $(F,E) \in VS\tilde{S}O(X)$. Then, $(F,E) \subseteq v\tilde{s}cl(v\tilde{s}int(F,E))$.
But, $(F,E) \subseteq v\tilde{s}cl(v\tilde{s}int(F,E)) \subseteq v\tilde{s}cl(v\tilde{s}int(F,E)) \cup v\tilde{s}int(v\tilde{s}cl(F,E))$. Hence,
$(F,E) \in Vb\tilde{S}O(X)$.
ii. Let $(F,E) \in VP\tilde{S}O(X)$. Then, $(F,E) \subseteq v\tilde{s}int(v\tilde{s}cl(F,E))$.
$\Rightarrow (F,E) \subseteq v\tilde{s}int(v\tilde{s}cl(F,E)) \subseteq v\tilde{s}int(v\tilde{s}cl(F,E)) \cup v\tilde{s}cl(v\tilde{s}int(F,E))$. Hence, $(F,E) \in Vb\tilde{S}O(X)$.
iii. Let $(F,E) \in Vb\tilde{S}O(X)$. Then $(F,E) \subseteq v\tilde{s}int(v\tilde{s}cl(F,E)) \cup v\tilde{s}cl(v\tilde{s}int(F,E))$.
Now, $(F,E) \subseteq v\tilde{s}cl(F,E) \subseteq v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl(F,E)) \cup v\tilde{s}cl(v\tilde{s}int(F,E)))$
$= v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl(F,E))) \cup v\tilde{s}cl(v\tilde{s}int(F,E))$
$= v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl(F,E)))$.
Thus, $(F,E) \subseteq v\tilde{s}cl(v\tilde{s}int(v\tilde{s}cl(F,E)))$. Hence, $(F,E) \in V\beta\tilde{S}O(X)$.

Remark 3.22. The following diagram shows the relationship between some stronger and weaker forms of vague soft-open sets.

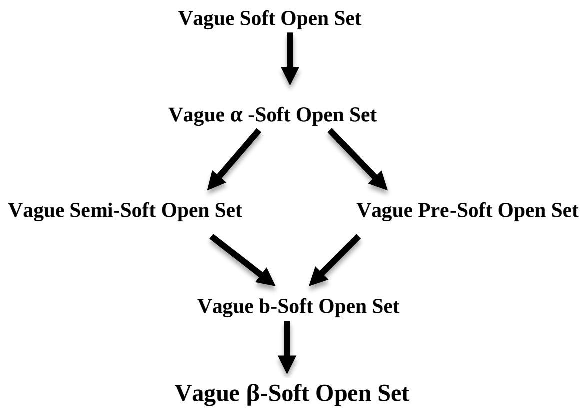

Fig.1

Remark 3.23. In general, the converse of the above implications need not be true as shown by following example.

Example 3.24. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$ and let τ = $\{\widehat{\emptyset}_E$, (F,E), $\hat{X}_E\}$ be a vague soft topological space where (F,E) ={ < e_1, [0.3,0.5]/ x_1, [0.2,0.4]/ x_2>, < e_2, [0.4,0.6]/ x_1, 0.3,0.7]/ x_2 >}. Then
i. the vague soft set (G_1,E) = {< e_1, [0.8,0.9]/ x_1, [0.7,0.9]/ x_2>, < e_2, [0.5,0.7]/ x_1, [0.5,0.8]/ x_2 >} is vague b-soft open set but neither vague semi-soft open set nor vague α-soft open set nor vague soft open set.
ii. the vague soft set (G_2,E) = {<e_1, [0.4,0.6]/ x_1, [0.5,0.8]/ x_2>,<e_2, [0.4,0.6]/ x_1, [0.3,0.7]/ x_2>} is vague b-soft open set but neither vague pre-soft open set nor vague regular-soft open set.
iii. the vague soft set (G_3,E) = {< e_1, [0.4,0.6]/ x_1, [0.2,0.3]/ x_2>,< e_2, [0.2,0.5]/ x_1, [0.3,0.7]/ x_2 >} is vague β-soft open set but not vague b-soft open set.

4. VAGUE β-soft CONTINUOUS AND VAGUE b -soft CONTINUOUS FUNCTIONS

In this section, we introduce vague β-soft continuous and vague b-soft continuous functions and study some of their properties.

Definition 4.1.[9] Let V$\tilde{S}$ (X,E) and V$\tilde{S}$ (Y,K) be two vague soft classes, and let u: X→ Y and p: E→K be mappings. Then a vague soft function g_{pu}=(u,p): V$\tilde{S}$ (X,E)→ V$\tilde{S}$ (Y,K) is defined as: for
(F,A) ∈ V$\tilde{S}$ (X,E), the image of (F,A) under g_{pu} denoted by g_{pu} (F,A) = (u(F), p(A)), is a vague soft set in V$\tilde{S}$ (Y,K) given by

$$t_{u(F)(\beta)}(y) = \begin{cases} \sup\limits_{\alpha\in p^{-1}(\beta)\cap A,\ \ x\in u^{-1}(y)} t_{F(\alpha)}(x) & if\ \ u^{-1}(y) \neq \emptyset, \\ 0 & otherwise. \end{cases}$$

and

$$1 - f_{u(F)(\beta)}(y) = \begin{cases} \sup_{\alpha \in p^{-1}(\beta) \cap A, \; x \in u^{-1}(y)} 1 - f_{F(\alpha)}(x) & if \;\; u^{-1}(y) \neq \emptyset, \\ 0 & otherwise. \end{cases}$$

for all $\beta \in$ p(A) and y∈Y.

Definition 4.2. [9] Let V$\tilde{S}$ (X,E) and V$\tilde{S}$ (Y,K) be two vague soft classes, and let g_{pu} =(u,p): V $\tilde{S}$ (X,E)→ V $\tilde{S}$ (Y,K) be a vague soft function and (G,B) be a vague soft set in V $\tilde{S}$ (Y,K). Then the inverse image of (G,B) under g_{pu} , denoted by g_{pu}^{-1} (G,B) = (u^{-1} (G), p^{-1} (B)) is a vague soft set in V$\tilde{S}$ (X,E) given by $t_{u^{-1}(G)(\alpha)}(x)$ = $t_{G(p(\alpha))}(u(x))$ and $1 - f_{u^{-1}(G)(\alpha)}(x)$ = $1 - f_{G(p(\alpha))}(u(x))$ for all $\alpha \in p^{-1}(B)$ and $x \in$ X.

Theorem 4.3.[9] Let (F ,A), (G,B) be two vague soft sets in V$\tilde{S}$ (X,E)and V$\tilde{S}$ (Y,K) respectively, and g_{pu} =(u,p):V$\tilde{S}$ (X,E)→ V$\tilde{S}$ (Y,K) be a vague soft function. Then

1. (F,A) ⊆ g_{pu}^{-1} (g_{pu} (F,A)) and if g_{pu} is injective, the equality holds.
2. g_{pu} (g_{pu}^{-1} (G,B)) ⊆ (G,B) and if g_{pu} is surjective, the equality holds.

Theorem 4.4.[9] Let (X,τ,E) and (Y,σ,K) be two vague soft topological spaces. The vague soft function g_{pu} :V$\tilde{S}$ (X,E)→ V$\tilde{S}$ (Y,K) is called vague soft continuous, if and only if for all (G,K)∈σ , g_{pu}^{-1}(G,K) ∈ τ.

Definition 4.5.[14] Let g_{pu} : (X,τ,E) → (Y,σ,K) be a vague soft function. Then g_{pu} is called;

1. vague semi-soft continuous (VS$\tilde{S}$-continuous in short) if g_{pu}^{-1} (G,K) ∈ VS$\tilde{S}$O(X) for all (G,K) ∈σ.
2. vague pre-soft continuous (VP$\tilde{S}$-continuous in short) if g_{pu}^{-1} (G,K) ∈ V P $\tilde{S}$O(X) for all (G,K) ∈σ.
3. vague α-soft continuous (Vα$\tilde{S}$-continuous in short) if g_{pu}^{-1} (G,K) ∈ Vα$\tilde{S}$O(X) for all (G,K) ∈σ.
4. vague regular-soft continuous (VR$\tilde{S}$-continuous in short) if g_{pu}^{-1} (G,K) ∈ V R$\tilde{S}$O(X) for all (G,K) ∈σ.

Definition 4.6. A vague soft function g_{pu} : (X,τ,E) → (Y,σ,K) is said to be vague β-soft continuous (briefly Vβ$\tilde{S}$-continuous) if the inverse image of each vague soft open set of (Y,σ,K) is a vague β-soft open set in (X,τ,E).

Definition 4.7. A vague soft function g_{pu} : (X,τ,E) → (Y,σ,K) is said to be vague b -soft continuous (briefly Vb$\tilde{S}$-continuous) if the inverse image of each vague soft open set of (Y,σ,K) is a vague b-soft open set in (X,τ,E).

Theorem 4.8. Let g_{pu}: (X,τ,E) → (Y,σ,K) be a vague soft function, then the following statements are equivalent.

1. g_{pu} is Vβ$\tilde{S}$-continuous.
2. The inverse image of each vague soft closed set in (Y,σ,K) is vague β-soft closed in (X,τ,E).

3. v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(g_{pu}^{-1} (S,K)))) ⊆ g_{pu}^{-1} (v$\tilde{s}$cl(G,K)) for each vague soft set (S,K) over (Y,σ,K).
4. g_{pu} (v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(F,E)))) ⊆ v$\tilde{s}$cl(g_{pu} (F,E)) for each (F,E) over (X,τ,E).

Proof. 1 ⇒ 2 : Let (H,K) ∈σ^c , then $(H,K)^c$ ∈σ. Since, g_{pu} is Vβ$\tilde{S}$-continuous, g_{pu}^{-1} ($(H,K)^c$)∈Vβ$\tilde{S}$O(X). But g_{pu}^{-1} ($(H,K)^c$) = $(g_{pu}^{-1}$ (H,K)$)^c$, then we have $(g_{pu}^{-1}$ (H,K)$)^c$ ∈ Vβ$\tilde{S}$O(X).
Thus, g_{pu}^{-1} (H,K) ∈Vβ$\tilde{S}$C(X).
2 ⇒ 3 : Let (S,K) be a vague soft over (Y,σ,K), then g_{pu}^{-1} (v$\tilde{s}$cl(S,K)) ∈ Vβ$\tilde{S}$C(X) .
g_{pu}^{-1} (v$\tilde{s}$cl(S,K)) ⊇ v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(g_{pu}^{-1} (v$\tilde{s}$cl(S,K))))) ⊇ v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(g_{pu}^{-1}(S,K)))).
3 ⇒ 4 : Let (F,E) be a vague soft set over (X,τ,E). Then for the vague soft set g_{pu} (F,E) , we have
v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(g_{pu}^{-1} (g_{pu} (F,E)))) ⊆ g_{pu}^{-1} (v$\tilde{s}$cl(g_{pu} (F,E)) (By part 3)
⇒ v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(F,E))) ⊆ v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(g_{pu}^{-1} (g_{pu} (F,E)))) ⊆ g_{pu}^{-1} (v$\tilde{s}$cl(g_{pu}(F,E)))
⇒ g_{pu} (v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(F,E)))) ⊆ g_{pu} (g_{pu}^{-1} (v$\tilde{s}$cl(g_{pu} (F,E)))) ⊆ v$\tilde{s}$cl(g_{pu}(F,E)).
4 ⇒ 1 : Let (G,K) ∈ σ. Then g_{pu}^{-1} ($(G,K)^c$) is a vague soft set in (X,σ,E) . By 4, we have
g_{pu} (v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(g_{pu}^{-1} ($(G,K)^c$))))) ⊆ v$\tilde{s}$cl(g_{pu} (g_{pu}^{-1} ($(G,K)^c$))) ⊆ v$\tilde{s}$cl($(G,K)^c$)) = $(G,K)^c$.
That is, v$\tilde{s}$int(v$\tilde{s}$cl(v$\tilde{s}$int(g_{pu}^{-1} ($(G,K)^c$)))) ⊆ g_{pu}^{-1} ($(G,K)^c$). Then g_{pu}^{-1} ($(G,K)^c$) ∈ Vβ$\tilde{S}$C(X).
But g_{pu}^{-1} ($(G,K)^c$) =$(g_{pu}^{-1}$ (G,K)$)^c$. Thus, g_{pu}^{-1} (G,K) ∈ Vβ$\tilde{S}$O(X). Hence g_{pu} is Vβ$\tilde{S}$-continuous function.

Theorem 4.9. A vague soft function g_{pu}: (X,τ,E) → (Y,σ,K) is Vb$\tilde{S}$ -continuous if and only if the inverse image of every vague soft closed set in (Y,σ,K) is vague b-soft closed set.
Proof. Let (G,K) be a vague soft closed set in (Y,σ,K), then $(G,K)^c$ ∈ τ. Since, g_{pu} is vague b-soft continuous, g_{pu}^{-1} ($(G,K)^c$)∈ Vb$\tilde{S}$O(X). But g_{pu}^{-1} ($(G,K)^c$) = $(g_{pu}^{-1}$ (G,K)$)^c$,then we have $(g_{pu}^{-1}$(G,K)$)^c$∈Vb$\tilde{S}$O(X). Thus, g_{pu}^{-1} (G,K) ∈ Vb$\tilde{S}$C(X).
Conversely, let (H,K) be a vague soft closed set in (Y,σ,K). Then $(H,K)^c$ is a vague soft open set in (Y,σ,K). By assumption g_{pu}^{-1} ($(H,K)^c$) ∈ Vb$\tilde{S}$C(X). But g_{pu}^{-1} ($(H,K)^c$) = $(g_{pu}^{-1}$ (H,K)$)^c$, so $(g_{pu}^{-1}$ (H,K)$)^c$ ∈Vb$\tilde{S}$C(X). Thus, g_{pu}^{-1} (H,K) ∈ Vb$\tilde{S}$O(X). Hence g_{pu} is Vb$\tilde{S}$-continuous function.

Theorem 4.10. Every VS$\tilde{S}$-continuous function is Vβ$\tilde{S}$-continuous.
Proof. Assume that, g_{pu}: (X,τ,E) → (Y,σ,K) is a vague semi-soft continuous function. Now let (G,K) ∈σ . Then g_{pu}^{-1} (G,K) ∈ VS$\tilde{S}$O(X). From Theorem 3.15, we have g_{pu}^{-1} (G,K) ∈ Vβ$\tilde{S}$O(X). Hence, g_{pu} is Vβ$\tilde{S}$-continuous function.

Theorem 4.11. Every VP$\tilde{S}$-continuous function is Vβ$\tilde{S}$-continuous.
Proof. Let g_{pu}: (X,τ,E) → (Y,σ,K) be a vague pre-soft continuous function and let (G,K) ∈ σ. Then g_{pu}^{-1}(G,K) ∈ VP$\tilde{S}$O(X) . Since every vague pre-soft open set is vague β-soft open as from the

Theorem 3.16, g_{pu}^{-1} (G,K) ∈ Vβ$\tilde{S}$O(X). Hence, g_{pu} is Vβ$\tilde{S}$-continuous function.

Theorem 4.12. Every V$\tilde{S}$-continuous function is Vb$\tilde{S}$ -continuous function.
Proof. Let g_{pu} : (X,τ,E) → (Y,σ,K) be a vague soft continuous function and let (G,K) be a vague soft open set in (Y,σ,K). Then g_{pu}^{-1} (G,K) ∈ τ. And so g_{pu}^{-1} (G,K) ∈ Vb$\tilde{S}$O(X). Hence, g_{pu} is vague b-soft continuous function.

Theorem 4.13. Every VS$\tilde{S}$-continuous function is Vb$\tilde{S}$-continuous.
Proof. Assume that, g_{pu} : (X,τ,E) → (Y,σ,K) is a vague soft function.
1. Suppose that g_{pu} is a VS$\tilde{S}$-continuous function and (G,K) ∈ σ. Then g_{pu}^{-1} (G,K) ∈ VS$\tilde{S}$O(X). Now from Theorem 3.25, we have g_{pu}^{-1}∈ V b $\tilde{S}$O(X). Hence, g_{pu} is Vb$\tilde{S}$ -continuous function.

Theorem 4.14. Every VP$\tilde{S}$-continuous function is Vb$\tilde{S}$-continuous.
Proof. Let g_{pu} : (X,τ,E) → (Y,σ,K) be a VP$\tilde{S}$-continuous function and (G,K) ∈ σ, then g_{pu}^{-1}(G,K) ∈ VP$\tilde{S}$O(X) . Since every vague pre-soft open set is vague b-soft open as from the Theorem 3.25, we have g_{pu}^{-1}∈ Vb$\tilde{S}$O(X). Hence, g_{pu} is Vb$\tilde{S}$ -continuous function.
Theorem 4.15. Every Vb$\tilde{S}$-continuous function is Vβ$\tilde{S}$-continuous.
Proof. Let g_{pu} : (X,τ,E) → (Y,σ,K) be a Vb$\tilde{S}$-continuous function and (G,K) ∈σ , then g_{pu}^{-1} (G,K) ∈ Vb$\tilde{S}$O(X) . Since every vague b-soft open set is vague β-soft open as from the Theorem 3.25, we have g_{pu}^{-1} (G,K) ∈ Vβ$\tilde{S}$O(X). Hence, g_{pu} is Vβ$\tilde{S}$-continuous function.
Remark 4.16. As from the above results we have the following implications.

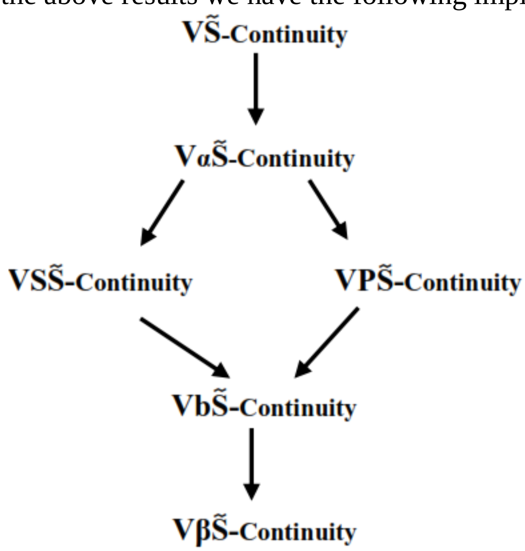

Fig.2

Remark 4.17. The converse of the above implications need not be true as shown in the following examples.

Example 4.18. Let X = $\{x_1, x_2, x_3\}$, Y = $\{y_1, y_2\}$, E = $\{e_1, e_2\}$ and let the vague soft topology τ on X be vague soft indiscrete and the vague soft topology σ on Y be vague soft discrete. If g_{pu} : (X,τ,E) → (Y,σ,K) is a vague soft function where u : X → Y, p : E → E are defined by u(x_1) = y_2, u(x_2) = y_1, u(x_3) = y_1, p(e_1) = e_1, p(e_2) = e_2, then it is Vb$\tilde{S}$-continuous neither VS$\tilde{S}$-continuous function nor V$\tilde{S}$-continuous function.

Example 4.19. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$, Y = $\{y_1, y_2\}$, K = $\{k_1, k_2\}$ and let τ = $\{\widehat{\emptyset}_E$, (F,E), $\hat{X}_E\}$ be a vague soft topological space on X where (F,E) = {<e_1, [0.4,0.6]/ x_1, [0.1,0.7]/ x_2>,<e_2, [0.2,0.5]/ x_1, [0.2,0.4]/ x_2>} and σ = $\{\widehat{\emptyset}_K$, (G,K), $\hat{Y}_K\}$ be a vague soft topological space on Y where (G,K) = {<k_1, [0.4,0.6]/ y_1, [0.2,0.8]/ y_2>,< k_2, [0.4,0.6]/ y_1, [0.3,0.7]/ y_2>}. If g_{pu} : (X,τ,E) → (Y,σ,K) is a vague soft function where u : X → Y , p : E → K are defined by u(x_1) = y_1 , u(x_2) = y_2 , p(e_1) = k_1, p(e_2) = k_2 , then it is Vb$\tilde{S}$-continuous not VP$\tilde{S}$-continuous function.

Example 4.20. Let X = $\{x_1, x_2\}$, E = $\{e_1, e_2\}$, Y = $\{y_1, y_2\}$, K = $\{k_1, k_2\}$ and let τ = $\{\widehat{\emptyset}_E$, (F,E), $\hat{X}_E\}$ be a vague soft topological space on X where (F,E) = {< e_1, [0.3,0.5]/ x_1, [0.2,0.4]/ x_2>,<e_2, [0.4,0.6]/ x_1, [0.3,0.7]/ x_2>} and σ = {$\{\widehat{\emptyset}_K$, (G,K), $\hat{Y}_K\}$ be a vague soft topological space on Y where (G,K) = {< k_1, [0.2,0.3]/ y_1, [0.4,0.6]/ y_2>,< k_2, [0.3,0.7]/ y_1, [0.3,0.6]/ y_2>} . If g_{pu} : (X,τ,E) → (Y,σ,K) is a vague soft function where u : X → Y , p : E → K are defined by u(x_1) = y_2 , u(x_2) = y_1 , p(e_1) = k_1, p(e_2) = k_2, then it is Vβ$\tilde{S}$-continuous not Vb$\tilde{S}$-continuous function.

Definition 4.21. A vague soft function g_{pu} : (X,τ,E) → (Y,σ,K) is said to be vague b -soft irresolute (briefly, Vb$\tilde{S}$-irresolute) if g_{pu}^{-1} (G,K) ∈ Vb$\tilde{S}$O(X) for every (G,K) ∈ Vb$\tilde{S}$O(X).

Theorem 4.22. A vague soft function g_{pu} : (X,τ,E) → (Y,σ,K) is said to be vague b -soft irresolute (briefly, Vb$\tilde{S}$-irresolute) if the inverse image of every vague b -soft open set in (Y,σ,K) is vague b -soft open set in (X,τ,E).

Definition 4.23. A vague soft function g_{pu} : (X,τ,E) → (Y,σ,K) is said to be vague b -soft open (vague b-soft closed) function if the image of every vague soft open (vague soft closed) set in (X,τ,E) is vague b-soft open (vague b-soft closed) set in (Y,σ,K).

Theorem 4.24. Every Vb$\tilde{S}$-irresolute function is Vb$\tilde{S}$-continuous.
Proof. Let g_{pu} : (X,τ,E) → (Y,σ,K) be a Vb$\tilde{S}$-irresolute function. Let (G,K) be a vague soft open set in (Y,σ,K). Since every vague soft open set is vague b -soft open, (G,K) ∈ Vb$\tilde{S}$O(X). Thus,
g_{pu}^{-1} (G,K) ∈ Vb$\tilde{S}$O(X) . Hence g_{pu} is a Vb$\tilde{S}$-continuous function.

Theorem 4.25. Let g_{pu} : (X,τ,E) → (Y,σ,K), h_{pu} : (Y,σ,K) → (Z,υ,R) be two vague soft functions.
i. If g_{pu} is Vb$\tilde{S}$- continuous and h_{pu} is vague soft continuous function then
$h_{pu} \circ g_{pu}$: (X,τ,E) → (Z,υ,R) is a Vb$\tilde{S}$-continuous function.

ii. If g_{pu} and h_{pu} are Vb$\tilde{S}$- irresolute functions then $h_{pu} \circ g_{pu}$: (X,τ,E) → (Z,υ,R) is a Vb$\tilde{S}$-irresolute function.

iii. If g_{pu} is V b $\tilde{S}$- irresolute and h_{pu} is Vb$\tilde{S}$-continuous function then $h_{pu} \circ g_{pu}$: (X,τ,E) → (Z,υ,R) is a Vb$\tilde{S}$- continuous function.

iv. If g_{pu} is vague soft open function and h_{pu} is Vb$\tilde{S}$-open function then $h_{pu} \circ g_{pu}$: (X,τ,E) → (Z,υ,R) is a Vb$\tilde{S}$-open function.

Proof. i. Let (H,R) ∈ υ. Then h_{pu}^{-1} (H,R) ∈ σ, since h_{pu} is vague soft continuous function. Now since g_{pu} is Vb$\tilde{S}$-continuous function, $(h_{pu} \circ g_{pu})^{-1}$ (H,R) = g_{pu}^{-1} (h_{pu}^{-1} (H,R)) ∈ Vb$\tilde{S}$O(X). Hence, $h_{pu} \circ g_{pu}$ is a Vb$\tilde{S}$-continuous function.

The proof of the results ii, iii and iv as similar.

Definition 4.26. A vague soft function g_{pu} : (X,τ,E) → (Y,σ,K) is said to be vague β-soft irresolute (briefly, Vβ$\tilde{S}$-irresolute) if g_{pu}^{-1} (G,K) ∈ Vβ$\tilde{S}$O(X) for every (G,K) ∈ Vβ$\tilde{S}$O(X).

Theorem 4.27. A vague soft function g_{pu} : (X,τ,E) → (Y,σ,K) is said to be vague β-soft irresolute (briefly, Vβ$\tilde{S}$-irresolute) if the inverse image of every vague β-soft open set in (Y,σ,K) is vague β-soft open set in (X,τ,E).

Definition 4.28. A vague soft function g_{pu} : (X,τ,E) → (Y,σ,K) is said to be vague β-soft open (vague β-soft closed) function if the image of every vague soft open (vague soft closed) set in (X,τ,E) is vague β-soft open (vague β-soft closed) set in (Y,σ,K).

Theorem 4.29. Every Vβ$\tilde{S}$-irresolute function is Vβ$\tilde{S}$-continuous.

Proof. Let g_{pu} : (X,τ,E) → (Y,σ,K) be a Vβ$\tilde{S}$-irresolute function. Let (G,K) be a vague soft open set in (Y,σ,K). Since every vague soft open set is vague β-soft open, (G,K) ∈ Vβ$\tilde{S}$O(X). Thus,

g_{pu}^{-1} (G,K) ∈ Vβ$\tilde{S}$O(X). Hence, g_{pu} is a Vβ$\tilde{S}$-continuous function.

Theorem 4.30. Let g_{pu} : (X,τ,E) → (Y,σ,K), h_{pu} : (Y,σ,K) → (Z,υ,R) be two vague soft functions.

i. If g_{pu} is Vβ$\tilde{S}$-continuous and h_{pu} is vague soft continuous function then $h_{pu} \circ g_{pu}$: (X,τ,E) → (Z,υ,R) is a Vβ$\tilde{S}$-continuous function.

ii. If g_{pu} and h_{pu} are Vβ$\tilde{S}$ - irresolute functions then $h_{pu} \circ g_{pu}$: (X,τ,E) → (Z,υ,R) is a Vβ$\tilde{S}$-irresolute function.

iii. If g_{pu} is Vβ$\tilde{S}$ - irresolute and h_{pu} is Vβ$\tilde{S}$-continuous function then $h_{pu} \circ g_{pu}$: (X,τ,E) → (Z,υ,R) is Vβ$\tilde{S}$-continuous function.

iv. If g_{pu} is vague soft open function and h_{pu} is Vβ$\tilde{S}$-open function then $h_{pu} \circ g_{pu}$: (X,τ,E) → (Z,υ,R) is a Vβ$\tilde{S}$-open function.

REFERENCES

1. M. Akdag, A. Ozkan, Soft α- open sets and soft α-continuous functions, *Abstract and Applied Analysis*, Vol, Article ID 891341, 7 pages (2014).

2. M. Akdag, A. Ozkan, Soft β- open sets and soft β-continuous functions, *The Scientific World Journal*, Vol, Article ID 843456, 6 pages (2014).
3. M. Akdag, A. Ozkan, Soft b - open sets and soft b -continuous functions, *Math Sci*., 8:124, DOI 10,1007/s40096-014-0124-7 (2014).
4. K. Alhazaymeh, N. Hassan, Generalized Vague soft set relations and its applications, *International Journal of Pure and Applied Mathematics*, 77(3), 391-401, (2012).
5. K. Alhazaymeh, N. Hassan, Possibility Vague soft set relations and its applications in decision making, *International Journal of Pure and Applied Mathematics*, 77(4), 549-563, (2012).
6. K. Alhazaymeh, N. Hassan, Application of Generalized Vague soft expert set in decision making, *International Journal of Pure and Applied Mathematics*, 93(3), 361-367, (2014).
7. K. Atanassov, Intuitiontistic Fuzzy sets, *Fuzzy Sets and Systems*, 20, 87-96, (1986).
8. Chang Wang, An-jing Qu, The Applications of vague soft sets and generalized vague soft sets, *Acta Mathematicae Applicatae Sinica English Series*, 31(4) 977-990, (2015).
9. Chang Wang, V. Inthumathi, M. Pavithra, Vague soft separation axioms and vague soft continuous functions in vague soft topological spaces, Submitted.,
10. Chang Wang, Y. Li, Topological Structure of Vague Soft Sets, *Abstract and Applied Analysis*, Vol, Article ID 504021, 8 pages (2014).
11. W.L. Gau, and D.J. Buehrer, Vague sets, *IEEE Transactions on Systems Man and Cybernetics*, 23(2), 610-614,(1993).
12. E. Fayad, H. Mahdi, Soft βc-open sets and Soft βc-Continuity, *International Mathematical Forum*, Vol. 12 (1), 9-26, (2017).
13. V. Inthumathi, M. Pavithra, Decomposition of vague α-soft open sets in vague soft topological spaces, *Global Journal of Pure and Applied Mathematics*, 14(3), 501-515, (2018).
14. V. Inthumathi, M. Pavithra, On Vague α-soft Continuous Functions, Submitted...
15. A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, A.M. ABD El-Latif, γ-operation and decompositions of some forms of soft continuity in soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, 7(2), 181- 196, (2014).
16. P.K. Maji, R. Biswas, A.R. Roy, On Soft set theory, *Computers and Mathematics with Applications*, 45, 555-562,(2003).
17. D. Molodtsov, Soft set theory-first results, *Computers and Mathematics with Applications*, 37(4-5), 19-31, (1999).
18. M. Shabir and M Naz, On soft topological spaces, *Computers and Mathematics with Applications*, 61, 1786-1799, (2011).
19. W. Xu, J. Ma,S. Wang, and G. Hao, Vague soft set and their properties, *Computers and Mathematics with Application*, 59, 787-794, (2010).
20. L.A. Zadeh, Fuzzy sets, *Information and Control*, 8(3), 338-353, 1965.