DECOMPOSITIONS OF n*-CONTINUITY VIA NANO IDEALS

[1]J. Jayasudha, [2]T. Rekhapriyadharsini
[1]Assistant Professor, [2]Research Scholar
Department of Mathematics,
Nallamuthu Gounder Mahalingam College, Pollachi 642001, Tamil Nadu, India.

Abstract: In this paper, we introduce and investigate the notions of NIω-continuous maps and NIω-irresolute maps in nano ideal topological spaces. Also we introduce some nano generalized locally closed sets namely NI-LC*-sets, weakly NI-LC*-sets, NI-slc sets, their continuous maps and obtained decompositions of n*-continuity.

2010 Mathematics Subject Classifications: 54A05, 54A10, 54B05.

Keywords: NI-LC*-set, weakly NI-LC*-set, NI-slc-set, ∧Ns-set, Nλs-I-closed set, NI-slc-continuous and Nλs-I-continuous.

I. INTRODUCTION AND PRELIMINARIES

Ideals in topological spaces have been considered since 1930. This topic gained its importance by the paper of Vaidyanathaswamy [20]. Hamlett and Jankovic [6] investigated further properties of ideal topological spaces. This initiated the generalization of some important properties in general topology via topological ideals. Later several authors have introduced and studied numerous generalized open sets in ideal topological spaces and also have obtained several decompositions of continuous maps and generalized continuous maps via ideals.

The notions of ω-closed sets and ω-continuity in topological spaces was introduced and studied by Sheik John [19]. Noiri et al. [14] introduced the notion of ω-closed sets in ideal topological spaces. Jafari et al. have obtained some decompositions of *-continuity via ideals in [4] and [5]. In 2013, Lellis Thivagar [10], [11] introduced the concept of nano topological spaces which was defined in terms of lower, upper approximations and boundary region of a subset of an universe using an equivalence relation on it. In 2016, Lellis Thivagar et al. [12] defined a nano local function for each subset with respect to I and $\tau_R(X)$ and thereby explored the field of nano ideal topological spaces. The notions of nano-I-open set and nano-I-continuous function was introduced by Parimala et al. in [15].

Definition 1.1 [10] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$. Then,

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. That is, $L_R(X) = \cup\{R(a): R(a) \subseteq X, a \in U\}$, where R(a) denotes the equivalence class determined by $a \in U$.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. That is, $U_R(X) = \cup\{R(a): R(a) \cap X \neq \phi, a \in U\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X)-L_R(X)$.

Property 1.1 [10] If (U, R) is an approximation space and X, Y ⊆ U, then

1. $L_R(X) \subseteq X \subseteq L_R(X)$.
2. $L_R(X) = U_R(X) = \phi$ and $L_R(U) = U_R(U) = U$.
3. $U_R(X\cup Y) = U_R(X)\cup U_R(Y)$.
4. $U_R(X\cap Y) \subseteq U_R(X)\cap U_R(Y)$.
5. $L_R(X\cup Y) \supseteq L_R(X)\cup L_R(Y)$.
6. $L_R(X\cap Y) = L_R(X)\cap L_R(Y)$.
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$.
8. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$.
9. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$.
10. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 1.2 [10] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{\phi, L_R(X), U_R(X), B_R(X), U\}$ where $X \subseteq U$. Then by Property 1.1, $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

This means that $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X and we call $(U, \tau_R(X))$ as a nano topological space. The elements of $\tau_R(X)$ are called nano-open sets and the complement of a nano-open set is a nano-closed set.
If $(U, \tau_R(X))$ is a nano topological space with respect to X, where $X \subseteq U$ and if $A \subseteq U$, then
(i) The nano interior of the set A is defined as the union of all nano-open subsets contained in A and is denoted by Nint(A).
(ii) The nano closure of the set A is defined as the intersection of all nano-closed subsets containing A and is denoted by Ncl(A).

Definition 1.3 [9] An ideal I on a topological space (X, τ) is a non-empty collection of subsets of X satisfying the following properties:

1. $A \in I$ and $B \subseteq A$ imply $B \in I$ (heredity).
2. $A \in I$ and $B \in I$ imply $A \cup B \in I$ (finite additivity).

Definition 1.4 [12] A nano topological space $(U, \tau_R(X))$ with an ideal I on U is called a nano ideal topological space or nano ideal space and denoted by $(U, \tau_R(X), I)$.

Definition 1.5 [12] Let $(U, \tau_R(X), I)$ be a nano ideal topological space. A a subset $A \subseteq U$, the set operator $A^*_n : P(U) \to P(U)$, is called the nano local function of A with respect to I and $\tau_R(X)$ and is defined as $A^*_n = \{x \in : U \cap A \notin I,$ for every $U \in \tau_R(X)\}$. The nano closure operator is defined as $Ncl^*(A) = A \cup (A^*_n)$.

Definition 1.6 A subset A of a nano topological space $(U, \tau_R(X))$ is said to be
1. nano semi-open [10] if $A \subset Ncl(Nint(A))$.
2. Ng-closed [1] if $Ncl(A) \subset G$ whenever G and G is nano-open.
3. Nω-closed [7] if $Ncl(A) \subset G$ whenever $A \subset G$ and G is nano semi-open in U.

Definition 1.7 Let $(U, \tau_R(X))$ and $(V, \psi_{R'}(Y))$ be nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \psi_{R'}(Y))$ is said to be
1. nano continuous [11] if $f^{-1}(A)$ is a nano closed in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \psi_{R'}(Y))$.
2. Ng-continuous [2] if $f^{-1}(A)$ is Ng-closed in $(U, \tau_R(X))$ for every nano-closed set A of $(V, \psi_{R'}(Y))$.

Definition 1.8 [3] A subset A of a nano topological space $(U, \tau_R(X))$ is said to be Nano locally closed (briefly NLC) if $A = G \cap F$, where G is nano-open and F is nano-closed.

Definition 1.9 [16] A subset A of a nano topological space $(U, \tau_R(X), I)$ is said to be nano ⋆-closed (briefly n⋆-closed) if $A^*_n \subset A$.

Definition 1.10 [16] A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be NIg-closed if $A^*_n \subset G$ whenever $A \subset G$ and G is nano-open.

Definition 1.11 [18] A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be NIω-closed (or NIĝ-closed) if $A^*_n \subset G$ whenever $A \subset G$ and G is nano semi-open.

Definition 1.12 [8] A function $f : (U, \tau_R(X), I) \to (V, \psi_{R'}(Y))$ is said to be n⋆-continuous if $f^{-1}(A)$ is n⋆-closed in U for every nano-closed set A in V.

Theorem 1.1 [18] Let $(U, \tau_R(X), I)$ be a nano topological space with an ideal I on U, and A is a subset of U. Then,
1. Every n⋆-closed is NIω-closed,
2. Every NIω-closed set is NIg-closed,
3. Every Nω-closed set is NIω-closed

II.NANO Iω-CONTINUITY AND NANO Iω-IRRESOLUTENESS

Definition 2.1 A function $f : (U, \tau_R(X), I) \to (V, \psi_{R'}(Y))$ is said to be NIω-continuous if $f^{-1}(A)$ is NIω-closed in U for every nano-closed set A in V.

Remark 2.1 If $I = \{\phi\}$ in the above definition, then the notion of NIω-continuity coincides with the notion of Nω-continuity.

Definition 2.2 A function $f : (U, \tau_R(X), I) \to (V, \psi_{R'}(Y), J)$ is said to be NIω-irresolute if $f^{-1}(A)$ is NIω-closed in $(U, \tau_R(X), I)$ for every NIω-closed set A in $(V, \psi_R(Y), J)$.

Definition 2.3 A function $f : (U, \tau_R(X)) \to (V, \psi_R(Y))$ is said to be Nω-continuous if $f^{-1}(A)$ is Nω-closed in U for every nano-closed set A in V.

Definition 2.4 A function $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ is said to be NIg-continuous if $f^{-1}(A)$ is NIg-closed in U for every nano-closed set A in V.

Theorem 2.2 For a function $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$, the following hold

1. Every nano continuous function is NIω-continuous.
2. Every n•-continuous function is NIω-continuous.
3. Every Nω-continuous function is NIω-continuous.
4. Every NIω-continuous function is NIg-continuous.

Proof. 1. Let f be a nano continuous function and A be a nano-closed set in $(V, \psi_R(Y))$. Then $f^{-1}(A)$ is nano-closed in $(U, \tau_R(X), I)$. Since every nano-closed set is n•-closed and hence NIω-closed, $f^{-1}(A)$ is NIω-closed in $(U, \tau_R(X), I)$. Therefore, f is NIω-continuous.
2. Let f be a n•-continuous function and A be a nano-closed set in $(V, \psi_R(Y))$. Then $f^{-1}(A)$ is n•-closed in $(U, \tau_R(X), I)$. Since every n•-closed set is NIω-closed [Theorem 1.1(1)], $f^{-1}(A)$ is NIω-closed in $(U, \tau_R(X), I)$. Therefore, f is NIω-continuous.
3. Let f be a Nω-continuous function. Then $f^{-1}(A)$ is Nω-closed in $(U, \tau_R(X), I)$ for every nano-closed set A in $(V, \psi_R(Y))$. Since every Nω-closed set is NIω-closed [Theorem 1.1(3)], $f^{-1}(A)$ is NIω-closed in $(U, \tau_R(X), I)$. Therefore, f is NIω-continuous.
4. Let f be a NIω-continuous function and A be a nano-closed set in $(V, \psi_R(Y))$. Then $f^{-1}(A)$ is NIω-closed in $(U, \tau_R(X), I)$. Since every NIω-closed set is NIg-closed [Theorem 1.1(2)], $f^{-1}(A)$ is NIg-closed in $(U, \tau_R(X), I)$. Therefore, f is NIg-continuous.

Remark 2.7 The relationships defined above, are shown in the following diagram.

nano continuity	→	Nω-continuity	→	Ng-continuity
↓		↓		↓
n•-continuity	→	NIω-continuity	→	NIg-continuity

None of these implications is reversible as shown by the following examples.

Example 2.8 Let U = {a, b, c} be the universe, X = {a, b} ⊆ U with U/R = {{a}, {b, c}}, $\tau_R(X)$ = {ϕ, {a}, {b, c}, U}, the ideal I = {ϕ, {c}} and let V = {a, b, c}, Y = {a} ⊆ V, V/R' = {{a}, {b, c}} and $\psi_R(Y)$ = {ϕ, {a}, V}. Define $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ as f(a) = b, f(b) = a, f(c) = c. Then f is NIω-continuous but not nano continuous.

Example 2.9 Let U = {a, b, c, d} be the universe, X = {a, d} ⊆ U with U/R = {{a, c}, {b}, {d}}, $\tau_R(X)$ = {ϕ, {d}, {a, c}, {a, c, d}, U}, the ideal I = {ϕ, {d}} and let V = {a, b, c, d} be the universe, Y = {a, b} ⊆ V, V/R' = {{a}, {c}, {b, d}} and $\psi_R(Y)$ = {ϕ, {a}, {b, d}, {a, b, d}, V}. Define $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ as f(a) = a, f(b) = b, f(c) = d, f(d) = c. Then f is NIω-continuous but not n•- continuous.

Example 2.10 Let U = {a, b, c, d} be the universe, X = {a, d} ⊆ U with U/R = {{b, c}, {a}, {d}}, $\tau_R(X)$ = {ϕ, {a, d}, U}, the ideal I = {ϕ, {d}} and let V = {a, b, c, d} be the universe, Y = {b, d} ⊆ V, V/R' = {{a}, {b}, {c, d}} and $\psi_R(Y)$ = {ϕ, {b}, {c, d}, {b, c, d}, V}. Then the identity map $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ is NIg-continuous but not NIω-continuous.

Theorem 2.4 A function $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ is NIω-continuous if and only if $f^{-1}(A)$ is NIω-open in $(U, \tau_R(X), I)$ for every nano-open set A in $(V, \psi_R(Y))$.

Proof. Let A be a nano-open set in $(V, \psi_R(Y))$ and $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ be NIω-continuous. Then A^c is nano-closed in $(V, \psi_R(Y))$ and $f^{-1}(A^c)$ is NIω-closed in $(U, \tau_R(X), I)$. But $f^{-1}(A^c) = (f^{-1}(A))^c$ and so $f^{-1}(A)$ is NIω-open in $(U, \tau_R(X), I)$.
Conversely, suppose that $f^{-1}(A)$ is NIω-open in $(U, \tau_R(X), I)$ for each nano-open set A in $(V, \psi_R(Y))$. Let F be a nano-closed set in $(V, \psi_R(Y))$. Then F^c is nano-open in $(V, \psi_R(Y))$ and by hypothesis $f^{-1}(F^c)$ is NIω-open in $(U, \tau_R(X), I)$. Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is NIω-closed in $(U, \tau_R(X), I)$ and so f is NIω-continuous.

Remark 2.5 The composition of two NIω-continuous maps need not be NIω-continuous as seen from the following example.

Example 2.11 Let U = {a, b, c, d} be the universe, X = {a, c} ⊆ U with U/R = {{b, c}, {a}, {d}}, $\tau_R(X)$ = {ϕ, {a}, {b, c}, {a, b, c}, U}, the ideal I = {ϕ, {d}} and let V = {a, b, c, d} be the universe, Y = {a, d} ⊆ V with V/R' = {{a}, {c}, {b, d}} and $\psi_R(Y)$ = {ϕ, {d}, {a, c}, {a, c, d}, V} and W = {a, b, c, d} be the universe, Z = {a} ⊆ W, W/R'' = {{a}, {b, c}, {d}}, $\sigma_R(Z)$ = {ϕ, {a}, W}. Define $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ and $g : (V, \psi_R(Y), J) \to (W, \sigma_R(Z))$ to be the identity maps. Then f and g are NIω-continuous but $g \circ f$ is not NIω-continuous.

Theorem 2.6 Let $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ be NIω-continuous and $g : (V, \psi_R(Y), J) \to (W, \sigma_R(Z))$ be nano continuous. Then $g \circ f : (U, \tau_R(X), I) \to (W, \sigma_R(Z))$ is NIω-continuous.

Proof. Let A be a nano-closed in $(W, \sigma_R(Z))$. Suppose that $f : (U, \tau_R(X), I) \to (V, \psi_R(Y))$ be NIω-continuous and $g : (V, \psi_R(Y), J) \to (W, \sigma_R(Z))$ be nano continuous. Then $g^{-1}(A)$ is nano-closed in $(V, \psi_R(Y), J)$.
Since $f : (U, \tau_R(X), I) \to (V, \psi_R(Y), J)$ is NIω-continuous, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is NIω-closed in $(U, \tau_R(X), I)$. Thus $g \circ f$ is NIω-continuous.

Theorem 2.7 A function $f:(U, \tau_R(X), I) \to (V, \psi_{R'}(Y), J)$ is NIω-irresolute if and only if the inverse image of every NIω-open set in $(V, \psi_{R'}(Y), J)$ is NIω-open in $(U, \tau_R(X), I)$.

Proof. Let B be a NIω-open set in $(V, \psi_{R'}(Y), J)$. Suppose that $f:(U, \tau_R(X), I) \to (V, \psi_{R'}(Y), J)$ is a NIω-irresolute map. Then B^c is NIω-closed in $(V, \psi_{R'}(Y), J)$ and $f^{-1}(B^c)$ is NIω-closed in $(U, \tau_R(X), I)$. But $f^{-1}(B^c) = (f^{-1}(B))^c$ and so $f^{-1}(B)$ is NIω-open in $(U, \tau_R(X), I)$.

Conversely, suppose that $f^{-1}(B)$ is NIω-open in $(U, \tau_R(X), I)$ for each NIω-open set B in $(V, \psi_{R'}(Y), J)$. Let D be a NIω-closed set in $(V, \psi_{R'}(Y), J)$. Then D^c is NIω-open in $(V, \psi_{R'}(Y), J)$ and by hypothesis $f^{-1}(D^c)$ is NIω-open in $(U, \tau_R(X), I)$. Since $f^{-1}(D^c) = (f^{-1}(D))^c$, we have $f^{-1}(D)$ is NIω-closed in $(U, \tau_R(X), I)$ and so f is NIω-irresolute.

Theorem 2.8 Let $f:(U, \tau_R(X), I) \to (V, \psi_{R'}(Y), J)$ and $g:(V, \psi_{R'}(Y), J) \to (W, \sigma_{R''}(Z), K)$ be NIω-irresolute. Then $(g \circ f):(U, \tau_R(X), I) \to (W, \sigma_{R''}(Z), K)$ is NIω-irresolute.

Proof. Let $g:(V, \psi_{R'}(Y), J) \to (W, \sigma_{R''}(Z), K)$ be NIω-irresolute and A be any NKω-open set in $(W, \sigma_{R''}(Z), K)$. Then $g^{-1}(A)$ is NJω-open in $(V, \psi_{R'}(Y), J)$. Since $f:(U, \tau_R(X), I) \to (V, \psi_{R'}(Y), J)$ is NIω-irresolute, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is NIω-open in $(U, \tau_R(X), I)$. Hence $g \circ f$ is NIω-irresolute.

Theorem 2.9 If $f:(U, \tau_R(X), I) \to (V, \psi_{R'}(Y), J)$ is NIω-irresolute and $g:(V, \psi_{R'}(Y), J) \to (W, \sigma_{R''}(Z))$ is n∗-continuous. Then $(g \circ f):(U, \tau_R(X), I) \to (W, \sigma_{R''}(Z))$ is NI -continuous.

Proof. Let $f:(U, \tau_R(X), I) \to (V, \psi_{R'}(Y), J)$ be NIω-irresolute, $g:(V, \psi_{R'}(Y), J) \to (W, \sigma_{R''}(Z))$ be n∗-continuous and let A be any nano-closed set of $(W, \sigma_{R''}(Z))$. Then $g^{-1}(A)$ is n∗-closed in $(V, \psi_{R'}(Y), J)$. Since every n∗-closed set is NIω-closed, $f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A)$ is NIω-closed in $(U, \tau_R(X), I)$. Hence $g \circ f$ is NIω-continuous.

III.NI-slc SETS

Definition 3.1 A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is called

1. NI-LC*-set if $A = G \cap F$ where G is nano regular open and F is n∗-closed.
2. weakly NI-LC*-set if $A = G \cap F$ where G is nano-open and F is n∗-closed.
3. NI-slc-set if $A = G \cap F$ where G is nano semi-open and F is n∗-closed.

Proposition 3.2 Let $(U, \tau_R(X), I)$ be a nano ideal topological space and $A \subseteq U$. Then the following hold.

1. If A is n∗-closed, then A is a NI-LC*-set.
2. If A is n∗-closed, then A is a weakly NI-LC*-set.
3. If A is a NI-LC*-set, then A is a weakly NI-LC*-set.

Proof. 1. Follows from Definition 3.1 (1).

2. Follows from Definition 3.1 (2).
3. Let A be a NI-LC*-set. Then $A = G \cap F$, where G is nano regular open and F is n∗-closed. Since every nano regular open set is nano-open [13], A is a weakly NI-LC*-set.

The converse of Proposition 3.2 is not true in general.

Example 3.3 Let U = {a, b, c, d} be the universe, X= {a, b} ⊆ U with U\R= {{a}, {c}, {b, d}}, $\tau_R(X)$ = {ϕ, {a}, {b, d}, {a,b,d}, U} and the ideal I = {ϕ, {a}}. Then

1. A = {b, d} is a NI-LC*-set but not a n∗-closed set.
2. A = {b, d} is a weakly NI-LC*-set but not a n∗-closed set.
3. A = {a, b, d} is a weakly NI-LC*-set but not a NI-LC*-set.

Proposition 3.4 For a subset A of a nano ideal topological space $(U, \tau_R(X), I)$ the following hold.

1. If A is n∗-closed then A is NI-slc-set.
2. If A is nano semi-open then A is NI-slc-set.
3. If A is weakly NI-LC*-set then A is NI-slc-set.

Proof. 1. Follows from Definition 3.1 (3).

2. Follows from Definition 3.1 (3).
3. Let A be weakly NI-LC*-set. Then $A = G \cap F$, where G is nano-open and F is n∗-closed. Since every nano-open set is nano semi-open [17], A is NI-slc-set.

The converse of proposition 3.4 is not true in general as shown by the following examples.

Example 3.5 Let U = {a, b, c} be the universe, X = {a, b} ⊆ U with U\R = {{a}, {b, c}}, $\tau_R(X)$ = {ϕ, {a}, {b, c}, U} and the ideal I = {ϕ, {c}}. Then the set A = {c} is a NI-slc-set but not nano semi-open.

Example 3.6 Let U = {a, b, c, d} be the universe, X= {a, d} ⊆ U with U\R = {{a, c}, {b}, {d}}, $\tau_R(X)$ = {ϕ, {d}, {a, c}, {a, c, d}, U} and the ideal I = {ϕ, {d}}. Then the set A = {a, c} is a NI-slc-set but not n∗-closed.

Example 3.7 Let U = {a, b, c, d}be the universe, X = {a, c}⊆ U with U\R = {{a},{b, c},{d}}, $\tau_R(X)$ = {ϕ ,{a},{b, c},{a, b, c}, U} and the ideal I = {ϕ, {d}}. Then the set A = {a, d}is a NI-slc-set but not a weakly NI-LC*-set.

Remark 3.1 From Proposition 3.2 and Proposition 3.4, we have the following implications.

n∗-closed → NI-LC*-set → weakly NI-LC*-set → NI-slc-set

Remark 3.2

1. The notions of NIω-closed sets and NI-LC*-sets are independent.
2. The notions of NIω-closed sets and weakly NI-LC*-sets are independent.
3. The notions of NIω-closed sets and NI-slc-sets are independent.

Example 3.8 Let U = {p, q, r, s, t} be the universe, X={p, s}⊆U with U\R = {{p, q},{r, t},{s}}, $\tau_R(X)$={ϕ ,{p, q},{s},{p,q,s}, U} and the ideal I = {ϕ,{p}}. Then

1. A = {s} is a NI-LC*-set but nota NIω-closed set.
2. A = {q, r, t} is a NIω-closed set but not a NI-LC*-set.
3. A = {s} is a weakly NI-LC*-set but not a NIω-closed set.
4. A = {q, r, s, t} is a NIω-closed set but not a weakly NI-LC*-set.

Example 3.9 Let U = {a, b, c} be the universe, X = {a, b} ⊆ U with U\R = {{a}, {b, c}}, $\tau_R(X)$ = {ϕ, {a}, {b, c}, U} and the ideal I = {ϕ, {c}}. Then the set A = {b} is a NIω-closed set but not NI-slc-set.

Example 3.10 Let U = {a, b, c, d} be the universe, X = {a, b}⊆U with U\R={{b, d},{a},{c}}, $\tau_R(X)$={ ϕ,{a},{b, d},{a, b, d},U} and the ideal I = {ϕ, {a}}. Then {b, d}is NI-slc-set but not a NIω-closed set.

Theorem 3.3 A subset of a nano ideal topological space (U, $\tau_R(X)$, I) is n∗-closed if and only if it is both NIω-closed and a NI-slc-set.

Proof. Necessity follows from Theorem 1.1(1) and Proposition 3.4(1). To prove the sufficiency, assume that A is both NIω-closed and a NI-slc-set. Then A = G ∩ F, where G is nano semi-open and F is n∗-closed. Therefore, A ⊂ G and A ⊂ F and so by hypothesis, $A^*_n \subset G$ and $A^*_n \subset F$. Thus $A^*_n \subset G \cap F = A$. Hence A is n∗-closed.

Theorem 3.4 For a subset A of a nano ideal topological space (U, $\tau_R(X)$, I), the following are equivalent.

1. A is a n∗-closed set.
2. A is a NI-LC*-set and NIω-closed set.
3. A is a weakly NI-LC*-set and NIω-closed set.
4. A is a NI-slc-set and NIω-closed set.

Proof. (1) ⇒ (2): Let A be a n∗-closed set. Then by Proposition 3.2 (1), it follows that A is a NI-LC*-set. Also, we know that every n∗-closed set is NIω-closed. Hence A is a NI-LC*-set and NIω-closed set.

(2) ⇒ (3): Follows from Proposition 3.2 (3).

(3) ⇒ (4): Follows from Proposition 3.4 (3).

(4) ⇒ (1): This is obvious from Theorem 3.3.

Theorem 3.5 For a subset A of a nano ideal topological space (U, $\tau_R(X)$, I), the following are equivalent.

1. A is a n∗-closed set.
2. A is a weakly NI-LC*-set and NIω-closed set.
3. A is a weakly NI-LC*-set and NIg-closed set.

Proof. (1) ⇒ (2): Let A be a n∗-closed set. We know that every n∗-closed set is NIω-closed [Theorem 1.1 (1)]. Hence A is a NIω-closed set. On the other hand, A can be written as A = U ∩A, where U is nano-open and A is n∗-closed. Hence A is a weakly NI-LC*-set.

(2) ⇒ (3): This is obvious from Theorem 1.1 (2).

(3) ⇒ (1): Let A be a weakly NI-LC*-set and a NIg-closed set. Since A is weakly NI-LC*-set, A = G ∩F, where G is nano-open and F is n∗-closed. Now, A ⊂ G and A is NIg-closed set implies $A^*_n \subset G$. Also, A ⊂ F and F is n∗-closed implies $A^*_n \subset F$. Thus $A^*_n \subset G \cap F = A$. Hence A is n∗-closed.

IV.A New Subset of a Nano Topological Space

Definition 4.1 Let A be a subset of a nano topological space (U, $\tau_R(X)$). Then the nano s-kernel of the set A, denoted by Ns-ker(A) is the intersection of all nano semi-open supersets of A.

Definition 4.2 A subset A of a nano topological space (U, $\tau_R(X)$) is called ∧Ns-set if A = Ns-ker(A).

Definition 4.3 A subset A of a nano ideal topological space (U, $\tau_R(X)$, I) is called Nλs-I-closed if A = G∩F where G is a ∧Ns-set and F is n∗-closed

Proposition 4.4 In a nano ideal topological space (U, $\tau_R(X)$, I), every n∗-closed set is Nλs-I-closed.
Proof. It is obvious from Definition 4.3.

The converse of Proposition 4.4 need not be true as seen from the following example.

Example 4.5 Let U = {a, b, c, d} be the universe, X = {a, d} ⊆ U with U\R = {{a, c}, {b}, {d}}, $\tau_R(X)$ = {ϕ, {d}, {a, c}, {a,c,d},U} and the ideal I = {ϕ ,{d}}. Then the set A = {a, c} is Nλs-I-closed but not n∗-closed.

Lemma 4.1 For a subset A of a nano ideal topological space (U, $\tau_R(X)$, I), the following are equivalent.
1. A is Nλs-I-closed.
2. A = P ∩ Ncl*(A) where P is a ∧Ns-set.
3. A = Ns-ker(A) ∩ Ncl*(A).

Proof. (1) ⇒ (2) : Let A be a Nλs-I-closed set. Then A = P ∩ Q, where P is a Nλs-I-set and Q is n∗-closed. Clearly, A ⊂ P ∩ Ncl*(A). Since Q is n∗-closed, Ncl*(A) ⊂ Ncl*(Q)=Q and so P ∩ Ncl*(A) ⊂ P∩Q = A. Therefore, A = P ∩ Ncl*(A).
(2) ⇒ (3) : Let A = P ∩ Ncl*(A), where P is a ∧Ns-set. Since P is a ∧Ns-set, we have A = Ns-ker(A) ∩ Ncl*(A).
(3) ⇒ (1) : Let A = Ns-ker(A) ∩ Ncl*(A). By Definition 4.2 and the notion of n∗-closed set, we get A is Nλs-I-closed.

Lemma 4.2 A subset A of a nano ideal topological space (U, $\tau_R(X)$, I) is NIω-closed if and only if Ncl*(A) ⊂ Ns-ker(A).
Proof. Necessity follows from Definition 4.2. To prove the sufficiency, let Ncl* (A) ⊆ Ns-ker(A). If P is any nano semi-open set containing A, then Ncl*(A) ⊆ Ns-ker(A) ⊆ P. Therefore, A is NIω-closed.

Remark 4.3 The notions of NIω-closed sets and Nλs-I-closed sets are independent.

Example 4.6 Let U = {a, b, c, d} be the universe, X = {a, d} ⊆ U with U\R ={{a, c}, {b}, {d}}, $\tau_R(X)$={ ϕ,{d},{a, c},{a, c, d},U} and the ideal I={ ϕ ,{d}}. Then
1. The set A = {a, c} is Nλs-I-closed but not NIω-closed.
2. The set A = {b, c} is NIω-closed but not Nλs-I-closed.

Theorem 4.4 A subset of a nano ideal topological space (U, $\tau_R(X)$, I) is n∗-closed if and only if it is both NIω-closed and Nλs-I-closed.
Proof. Necessity is obvious from every n∗-closed set is NIω-closed and Proposition 4.4. We shall prove sufficiency. Let A be a NIω-closed set and a Nλs-I-closed set. As A is a Nλs-I-closed set A = G ∩ F, where G is a ∧Ns-set and F is n∗-closed. Now A⊂G and A is NIω-closed set implies A_n^* ⊂ G. Also A ⊂ F and F is n ∗-closed set implies A_n^* ⊂ F.
Thus A_n^* ⊂ G∩F = A. Hence A is n∗ -closed.

V. DECOMPOSITIONS OF n∗ -CONTINUITY

Definition 5.1 A function f : (U, $\tau_R(X)$, I) ⟶ (V, $\psi_{R'}(Y)$) is said to be NI-LC*-continuous (resp. weakly NI-LC*-continuous) if f^{-1} (A) is a NI-LC*-set (resp. weakly NI-LC*-set) in (U, $\tau_R(X)$, I) for every nano-closed set A in (V, $\psi_{R'}(Y)$).

Definition 5.2 A function f : (U, $\tau_R(X)$, I) ⟶ (V, $\psi_{R'}(Y)$) is said to be NI-slc-continuous (resp. Nλs-I-continuous) if f^{-1}(A) is a NI-slc-set (resp. Nλs-I-closed set) in (U, $\tau_R(X)$, I) for every nano-closed set A in (V, $\psi_{R'}(Y)$).

Example 5.3 Let U={a, b, c, d} be the universe, X={a, c}⊆U, with U\R={{a},{b, c},{d}}, $\tau_R(X)$ = {ϕ, {a}, {b, c}, {a, b, c}, U}, the ideal I = {ϕ,{d}}, and let V ={a, b, c, d} be the universe, Y = {a, d}⊆V, with V\R'={{a, c},{b},{d}}, and $\psi_{R'}(Y)$ = {ϕ, {d}, {a, c}, {a, c, d}, V}. Define a function f : (U, $\tau_R(X)$, I) ⟶ (V, $\psi_{R'}(Y)$) to be the identity map. Then the map f is NI-slc-continuous.

Remark 5.1 Every n∗-continuous map is NI-slc-continuous, but the converse is not true.

Example 5.4 Let U={a, b, c, d} be the universe, X={a, c}⊆ U, with U\R={{a},{b, c},{d}}, $\tau_R(X)$={ϕ,{a}, {b, c}, {a, b, c}, U}, the ideal I = {ϕ, {d}} and let V = {a, b, c, d} be the universe, Y = {a, d} ⊆ V, with V\R' = {{a}, {d}, {b, c}} and $\psi_{R'}(Y)$ = {ϕ, {a, d}, V}. Then the identity map f : (U, $\tau_R(X)$, I) ⟶ (V, $\psi_{R'}(Y)$) is NI-slc-continuous but not n∗-continuous.

Remark 5.2 The concepts of NIω-continuity and NI-slc-continuity are independent as seen from the following examples.

Example 5.5 Let U = {a, b, c, d} be the universe, X={a, c}⊆U, with U\R={{a},{b, c},{d}}, $\tau_R(X)$ = {ϕ, {a}, {b, c}, {a, b, c}, U}, the ideal I = {ϕ, {d}} and let V = {a, b, c ,d} be the universe, Y = {a, d} ⊆ V, with V\R' = {{a}, {d}, {b, c}} and $\psi_{R'}(Y)$ = {ϕ, {a, d}, V}. Then the identity map f: (U, $\tau_R(X)$, I) ⟶ (V, $\psi_{R'}(Y)$) is NI-slc-continuous but not NIω-continuous.

Example 5.6 Let $U=\{a, b, c, d\}$ be the universe, $X=\{a, b\}\subseteq U$, with $U\backslash R=\{\{a\},\{c\},\{b, d\}\}$, $\tau_R(X)=\{\phi, \{a\}, \{b, d\}, \{a, b, d\}, U\}$, the ideal $I = \{\phi, \{a\}\}$ and let $V = \{a, b, c, d\}$ be the universe, $Y = \{a, d\}\subseteq V$, with $V\backslash R'=\{\{a, c\}, \{b\}, \{d\}\}$ and $\psi_{R'}(Y) = \{\phi, \{d\}, \{a, c\}, \{a, c, d\}, V\}$. Then the map $f : (U, \tau_R(X), I) \to (V, \psi_{R'}(Y))$ defined by $f(a) = a$, $f(b) = c$, $f(c) = b$, $f(d) = d$ is NIω-continuous but not NI-slc-continuous.

Theorem 5.3 For a function $f : (U, \tau_R(X), I) \to (V, \psi_{R'}(Y))$, the following are equivalent.
1. f is n⋆-continuous.
2. f is NI-LC*-continuous and NIω-continuous.
3. f is weakly NI-LC*-continuous and NIω-continuous.
4. f is NI-slc-continuous and NIω-continuous.

Proof. This is an immediate consequence of Theorem 3.4.

Theorem 5.4 For a function $f : (U, \tau_R(X), I) \to (V, \psi_{R'}(Y))$, the following are equivalent.
1. f is n⋆-continuous.
2. f is weakly NI-LC*-continuous and NIω-continuous.
3. f is weakly NI-LC*-continuous and NIg-continuous.

Proof. This is an immediate consequence of Theorem 3.5.

Theorem 5.5A function $f:(U, \tau_R(X), I) \to (V, \psi_{R'}(Y))$ is n⋆-continuous if and only if it is both NIω-continuous and Nλs-I-continuous.
Proof. This is an immediate consequence of Theorem 4.4.

REFERENCES

[1] Bhuvaneswari, K. and Mythili Gnanapriya, K. 2014. Nano Generalized Closed sets, International Journal of Scientific and Research Publications, 14 (5):1-3.
[2] Bhuvaneswari, K. and Mythili Gnanapriya, K. 2015. On nano generalized continuous function in nano topological spaces, International Journal of Mathematical Archive, 4 :182-186.
[3] Bhuvaneswari, K. and Mythili Gnanapriya, K. 2016. Nano Generalized Locally Closed sets and NGLC-Continuous Functions in Nano Topological Spaces, Int. J. Math. And Appl., 4(1).
[4] Jafari, S. Viswanathan, K. and Jayasudha, J. 2012. G-I-LC*-sets and decompositions of ⋆-continuity, IOSR-Jour. of Math., 2(2): 43-46.2340 -2345.
[5] Jafari, S. Viswanathan, K. and Jayasudha, J. 2013. Another decomposition of ⋆-continuity via ideal topological spaces, Jordan Journal of Mathematics and statistics (JJMS), 6(4): 285- 295.
[6] Jankovic, D. and Hamlett, T.R. 1990. New topologies from old via ideals, Amer. Math. Monthly, 97(1990).
[7] Jayalakshmi, A. and Janaki, C. 2017. A New Form of Nano Locally Closed Sets in Nano Topological Space, Global Journal of Pure and Applied Mathematics, 13(9): 599-6006.
[8] Jayasudha, J. Rekhapriyadharsini, T. 2019. On some decompositions of nano ⋆-continuity, International Journal of Mathematics and statistics Invention, 7(1) :1-6.
[9] Kuratowski, K. 1966. Topology (Vol. 1, Academic press, New York.)
[10] Lellis Thivagar, M. and Carmel Richard, 2013. On nano forms of weakly open sets, International Journal of Mathematics and statistics Invention, 1(1):31-37.
[11] Lellis Thivagar, M. and Carmel Richard, 2013. On nano forms of continuity, Mathematical Theory and Modeling, 3(7): 32-37.
[12] Lellis Thivagar, M. and Sutha Devi, V.2016. New sort of operators in Nano Ideal Topology, Ultra Scientist, 28(1):51- 64.
[13] Nasaf, A. A. Aggour, A. I. and Darwesh, S. M. 2008. On some classes of nearly open sets in nano topological spaces, Journal of the Egyptian Mathematical Society, 119: 365-371.
[14] Noiri,T. Viswanathan, K. Rajamani, M. and Krishnaprakash, S. On some ω-closed sets in ideal topological spaces(submitted).
[15] Parimala, M. Jafari, S. 2018. On Some New Notions in Nano Ideal Topological Spaces, International Balkan Journal of Mathematics1(3): 85-92.
[16] Parimala, M. Jafari, S. and Murali, S.Nano ideal generalized closed sets in Nano Ideal Topological Spaces, (communicated).
[17] Rajasekaran, I. Meharin, M. and Nethaji, O. 2017. On new class of some nano open sets, International Journal of Pure and Applied Mathematical Sciences, 2 (10): 147-155.
[18] Rajendran, V. Sathishmohan, P. and Lavanya, K. 2018. On NIĝ -Closed sets in Nano Ideal Topological Spaces, Int. J. Math. And Appl., 6(2): 193-199.
[19] Sheik John, M. 2002. A study on generalized closed sets and continuous maps in topological and bitopological spaces, Ph.D. Thesis, Bharathiar University, Coimbatore.
[20] Vaidyanathaswamy, R. 1960. Set topology (Chelsea Publishing Company, New York.)