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Decompositions Of Nano R-*I*-Continuity

V. Inthumathi*, S. Narmatha*, S. Krishnaprakash*
[1]*Associate Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi.*
[2]*Assistant Professor, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore.*
[3]*Associate Professor, Department of Science and Humanities, Karpagam College of Engineering, Coimbatore.*

Abstract

In this paper , we introduce and study the notions of nano α-I-set, nano semi-I-regular, nano semi-pre-I-regular, nano A*B-I-sets, weak nano A*B-I-sets and investigate some of their basic properties. Also, we obtain the decompositions of nano R-I-continuity of nano ideal topological spaces.*

Keywords: *nano α*-I-set, nano semi-I-regular, nano semi-pre-I-regular, nano A*B-I-sets, weak nano A*B-I-sets and nano R-I-continuity.*

1. Introduction and Preliminaries

An Ideal I [6] on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following conditions : (1) $A \in I$ and $B \subseteq A$ imply $B \in I$ and (2) $A \in I$ and $B \in I$ imply $A \cup B \in I$. Given a topological space (X, τ) with ideal I on X. If $P(X)$ is the family of all subsets of X, a set operator $(.)^*$: $P(X) \rightarrow P(X)$, called a *local function* [3] of A with respect to τ and I is defined as follows : For $A \subseteq X$, $A^*(I, \tau) = \{x \in X: U \cap A \notin I \text{ for all } U \in \tau(x)\}$ where $\tau(x)=\{ U \in \tau : x \in U\}$. The closure operator defined by $Cl^*(A) = A \cup A^*(I, \tau)$ [6] is a *Kuratowski closure operator* which generates a topology τ^* called **-topology* finer than τ. Let U be a non-empty, finite universe of objects and R be an equivalence relation on U, $X \subseteq U$ and $\tau_R(X) = \{U, \varphi, U_R(X), L_R(X), B_R(X)\}$. Then $\tau_R(X)$ is a topology on U, called as the *nano topology* with respect to X. Elements of the nano topology are called as the *nano open sets* in U and the nano topological space is denoted by $(U, \tau_R(X))$ [4]. A nano topological space $(U, \tau_R(X))$ with an ideal I on U is called a nano ideal topological space [5] or nano ideal space and is denoted by $(U, \tau_R(X), I)$. Let $(U, \tau_R(X), I)$ be a nano topological space. A set operator $A^{*N} : P(U) \rightarrow P(U)$, is called the *nano local function* of I on U with respect to I on $\tau_R(X)$, is defined as $A^{*N} = \{x \in : U \cap A \notin I \text{ for all } U \in \tau(X)\}$ and is denoted by A^{*N}, where *nano closure operator* is defined as $NCl^*(A)=A \cup A^{*N}$. A subset A of a nano ideal topological space $(U, \tau_R(X), I)$ is said to be nano semi-*I*-open [5], (resp. nano α-*I*-open[5], nano regular-*I*-open(nano R-*I*-open)[5], nano t-*I*-open[1]), if $A \subseteq NCl^*(NInt(A))$, (resp. $A \subseteq NInt(NCl^*(NInt(A)))$, $A = NInt(NCl^*(A)$, $NInt(NCl^*(A)) = NInt(A))$. A subset A of a nano ideal topological space (U, $\tau_R(X)$, I) is said to be a nano A^*_R-*I*-set(briefly NA^*_R-I-set) [2], if $A = S \cap V$, where S is a nano closed set and V is a nano regular-I-open set. Throughout this paper U represents a nano topological space $(U, \tau_R(X))$ and U_I represents a nano ideal topological space $(U, \tau_R(X), I)$.

2. Nano A*B-*I*-sets and weak nano A*B-*I*-sets.

Definition 2.1: Let A be a subset of a nano ideal topological space U_I. Then A is said to be a :

1. nano pre-*I*-open, if $A \subseteq NInt(NCl^*(A))$.
2. nano β–*I*-open, if $A \subseteq NCl(NInt(NCl^*(A)))$.
3. nano α*-*I*-set , if $NInt(A) = NInt(NCl^*(NInt(A)))$.
4. nano semi-*I*-regular, if it is both nano semi-*I*-open and nano t-*I*-set.
5. nano semi-pre-*I*-regular, if it is both nano β–*I*-open and nano α*-*I*-set.

6. nano A*B-*I*-set. (briefly NA*B-*I*-set) if $A = S \cap V$, where S is nano closed and V is nano semi-*I*-regular.
7. weak nano A*B-*I*-set. (briefly weak NA*B-*I*-set) if $A = S \cap V$, where S is nano closed and V is nano semi-pre-I-regular.

Proposition 2.1. For the nano ideal topological space U_I, the following hold :

1. Every nano regular-*I*-open set is nano semi-*I*-regular.
2. Every nano semi-*I*-regular set is a NA*B-*I*-set.
3. Every NA*$_R$-*I*-set is a NA*B-*I*-set.

Proof. (1). Let A be a nano regular-*I*-open set in U_I. Then $A = NInt(NCl^*(A))$ and so $NInt(A)=NInt(NCl^*(A))$. Therefore A is a nano t-*I*-set in U_I. Since every nano regular-*I*-open set is a nano semi-*I*-open in U_I, A is both nano semi-*I*-open and nano t-*I*-set in U_I, which implies A is a nano semi-*I*-regular set in U_I .
(2). Let A be a nano semi-*I*-regular set in U_I. Then $A = A \cap U$, where U is nano closed in U_I Thus A is a NA*B-*I*-set in U_I.
(3). Let A be a NA*$_R$-I-set in U_I. Then $A = S \cap V$, where S is nano closed and V is nano regular-I-open in U_I. By above proof (1), V is nano semi-*I*-regular in U_I. Thus, A is a NA*B-*I*-set in U_I.

Remark 2.1. The following examples shows that the converses of the above Proposition need not be true.

Example 2.1. Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$, $X = \{b, d\}$, $\tau_R(X) =\{\varphi, U, \{a,b,d\},\{d\},\{a, b\}\}$ and $I = \{\varphi, \{a\}\}$. Then the set $\{c, d\}$ is a nano semi-*I*-regular set but not a nano regular-*I*-open set in U_I and the set {c} is a NA*B-*I*-set but not a nano semi-*I*-regular set in U_I.

Example 2.2. Let $U = \{a, b, c, d, e\}$, $U/R = \{\{a\}, \{b, c\}, \{d, e\}\}$, $X = \{a, d\}$, $\tau_R(X) = \{\varphi, U, \{a\}, \{a, d, e\}, \{d, e\}\}$ and $I = \{\varphi, \{a\}\}$. Then the set $\{c, d, e\}$ is a NA*B-*I*-set but not a NA*$_R$-*I*-set in U_I.

Proposition 2.2. The intersection of two nano α*-*I*-sets in U_I is a nano α*-*I*-set in U_I.

Proof. Let A and B be two nano α*-*I*-sets in U_I. Then, $NInt(A) = NInt(NCl^*(NInt(A)))$ *and* $NInt(B) = NInt(NCl^*(NInt(B)))$. Now, $NInt(A \cap B) = NInt(A) \cap NInt(B) = NInt(NCl^*(NInt(A))) \cap NInt(NCl^*(NInt(B))) = NInt(NCl^*(NInt(A)) \cap NCl^*(NInt(B))) \supseteq NInt(NCl^*(NInt(A) \cap NInt(B) = NInt (NCl^* (NInt(A \cap B))) \supseteq NInt(A \cap B)$. Thus, $NInt(A \cap B) \supseteq NInt(NCl^*(NInt((A \cap B))) \supseteq NInt(A \cap B)$, which implies $NInt(A \cap B)=NInt(NCl^*(NInt(A \cap B)))$. Hence $A \cap B$ is a nano α*-*I*-set in U_I.

Remark 2.2. The following example shows that the union of two nano α*-*I*-set in U_I need not be a nano α*-*I*-set in U_I.

Example 2.3. Let $U = \{a, b, c, d, e\}$, $U/R = \{\{a\}, \{b, c\}, \{d, e\}\}$, $X = \{a, d\}$, $\tau_R(X) = \{\varphi, U, \{a\}, \{a, d, e\}, \{d, e\}\}$ and $I = \{\varphi, \{a\})$. Then the sets $A = \{a, c, d\}$ and $B = \{a, c, e\}$ are nano α*-*I*-set in U_I but the set $A \cup B = \{a, c, d, e\}$ is not a nano α*-*I*-set in U_I.

Proposition 2.3. In a nano ideal topological space U_I the following hold :

1. Every nano semi-*I*-regular set is a nano semi-pre-*I*-regular set.
2. Every NA*B-*I*-set is a weak N A*B-*I*-set.

Proof. (1). Let A be a nano semi-I-regular set in $U_{\mathcal{I}}$. Then A is both nano semi-I- open and nano t-I-set in $U_{\mathcal{I}}$. Since every nano semi-I-open set is a nano β-I-open set in $U_{\mathcal{I}}$. A is nano β-I-open set in $U_{\mathcal{I}}$. Further since every nano t- I-set is a nano α∗-I-set in $U_{\mathcal{I}}$, A is a nano α∗-I-set in $U_{\mathcal{I}}$. Thus, A is a nano semi-pre-I-regular set in $U_{\mathcal{I}}$.

(2). Let A be a NA*B-I-set in $U_{\mathcal{I}}$. Then $A = S \cap V$, where S is nano closed and V is nano semi-I-regular in $U_{\mathcal{I}}$. By above proof, every nano semi-I-regular set is a nano semi-pre-I-regular set in $U_{\mathcal{I}}$. Thus V is a nano semi-pre-I-regular set in $U_{\mathcal{I}}$. Therefore A is a weak NA*B-I-set in $U_{\mathcal{I}}$.

Remark 2.3 The converses of the above Proposition need not be true as shown in the following example.

Example 2.4. Let $U = \{a, b, c, d\}$, $U/R = \{\{a, b\}, \{c\}, \{d\}\}$, $X = \{b, d\}$, $\tau_R(X) =\{\varphi, U, \{a, b, d\}, \{d\}, \{a, b\}\}$ and $\mathcal{I} = \{\varphi, \{a\}\}$. Then,

1. The set $\{a, c, d\}$ is a nano semi-pre-I-regular set but not a nano semi-I-regular set in $U_{\mathcal{I}}$.
2. The set $\{a, c, d\}$ is a weak NA*B-I-set but not a NA*B-I-set in $U_{\mathcal{I}}$.

Theorem 2.1. Let $U_{\mathcal{I}}$ be a nano ideal topological space. Then a subset A of $U_{\mathcal{I}}$ is a nano regular-I-open set in $U_{\mathcal{I}}$ if and only if A is nano open and a NA*$_R$-I-set in $U_{\mathcal{I}}$.

Proof. Necessity : Let A be a nano regular-I-open set in $U_{\mathcal{I}}$. By proposition 2.6 [2], A is a nano open set in $U_{\mathcal{I}}$, Further by Proposition 2.1 [2], A is a NA*$_R$-I-set in $U_{\mathcal{I}}$. Thus A is both nano open and a NA*$_R$-I-set in $U_{\mathcal{I}}$.

Sufficiency : Assume that A is nano open and a NA*$_R$-I-set in $U_{\mathcal{I}}$. Since A is nano open and $A \subseteq NCl^*(A)$, $A \subseteq NInt(NCl^*(A))$. Since A is a NA*$_R$-I-set, $A = S \cap V$, where S is nano closed and V is nano regular-I-open in $U_{\mathcal{I}}$. Since S is nano closed, $NInt(NCl^*(S)) \subseteq S$. Since V is nano regular-I-open in $U_{\mathcal{I}}$, we have $NInt(NCl^*(V)) = V$. Thus, $NInt(NCl^*(A)) = NInt(NCl^*(S \cap V)) \subseteq Int(NCl^*(S) \cap NCl^*(V)) \subseteq S \cap V = A$. Therefore, $A \subseteq NInt(NCl^*(A)) \subseteq A$ and so A is a nano regular-I-open set in $U_{\mathcal{I}}$.

Theorem 2.2. For a subset A of a nano ideal topological space $U_{\mathcal{I}}$, the following are equivalent :

1. A is nano regular-I-open.
2. A is nano open and a NA*B-I-set.
3. A is nano α-I-open and a NA*B-I-set.
4. A is nano pre-I-open and a NA*B-I-set.

Proof. (1) → (2) Let A be a nano regular-I-open in $U_{\mathcal{I}}$. By Proposition 2.6 [2], A is nano open in $U_{\mathcal{I}}$. By Proposition 2.1 (1) and (2), A is NA*B-I-set in $U_{\mathcal{I}}$. Thus A is both nano open and a NA*B-I-set in $U_{\mathcal{I}}$.

The proof of (2) → (3) and (3) → (4) are obvious by Proposition 3.1 [2].

(4) → (1) Let A be a nano pre-I-open set and a NA*B-I-set in $U_{\mathcal{I}}$. Since A is a NA*B-I-set in $U_{\mathcal{I}}$, $A = S \cap V$, where S is nano closed and V is nano semi-I-regular in U_I. Since S is nano closed in $U_{\mathcal{I}}$, $NInt(NCl^*(S)) \subseteq S$. Further, Since V is nano semi-I-regular in $U_{\mathcal{I}}$, V is a nano t-I-set in $U_{\mathcal{I}}$. Therefore $NInt(NCl^*(V)) = NInt(V) \subseteq V$. Now, $NInt(NCl^*(A)) = NInt(NCl^*(S \cap V) \subseteq NInt(NCl^*(S) \cap NCl^*(V)) = NInt (NCl^*(S)) \cap NInt(NCl^*(V)) \subseteq (S \cap V) = A$. Moreover, Since A is a nano pre-I-open in $U_{\mathcal{I}}$, $A \subseteq NInt (NCl^* (A))$. Therefore $A = NInt(NCl^*(A))$. Hence A is a nano regular-I-open in $U_{\mathcal{I}}$.

Theorem 2.3. Let A be a subset in $U_{\mathcal{I}}$. Then A is a nano semi-I-regular set in $U_{\mathcal{I}}$ if and only if A is nano semi-I-open and a NA*B-I-set in $U_{\mathcal{I}}$.

Proof . Necessity : Let A be a nano semi-I-regular set in $U_{\mathcal{I}}$. Then A is a nano semi-I-open set in $U_{\mathcal{I}}$. By Proposition 2.1 (2), A is a NA*B-I-set in $U_{\mathcal{I}}$. Therefore A is both nano semi-I-open set and a NA*B-I-set in $U_{\mathcal{I}}$.

Sufficiency: Let A be a nano semi-I-open set and a NA*B-I-set in $U_{\mathcal{I}}$. Since A is a NA*B-I-set, $A = S \cap V$, where S is nano closed and V is nano semi-I-regular in $U_{\mathcal{I}}$. Thus, we have $NInt(NCl^*(A)) = NInt(NCl^*(S \cap V) \subseteq NInt(NCl^*(S) \cap NCl^*(V)) = NInt(NCl^*(S)) \cap NInt(NCl^*(V)) \subseteq (S \cap V) = A$.

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Therefore, NInt(NCl(A)) ⊆ A*, which implies *NInt(NCl*(A)) ⊆ NInt(A)*. Moreover, *NInt(A) ⊆ NInt(NCl*(A))*. Therefore, *NInt(A) = NInt(NCl*(A))* and this implies that A is a nano t-*I*-set in $U_{\boldsymbol{I}}$. Hence *A* is a nano semi-*I*-regular set in $U_{\boldsymbol{I}}$.

Theorem 2.4. Let *A* be a subset of $U_{\boldsymbol{I}}$. Then *A* is a nano semi-pre-*I*-regular set in $U_{\boldsymbol{I}}$ if and only if *A* is a nano β-*I*-open set and a weak NA*B-*I*-set in $U_{\boldsymbol{I}}$.

Proof. Necessity : Let *A* be a nano semi-pre-*I*-regular set in $U_{\boldsymbol{I}}$. Then *A* is a nano β-*I*-open set in $U_{\boldsymbol{I}}$. Since *A* is nano semi-pre-I-regular, we have *A* = *A* ∩ *U*, where *U* is nano closed. Thus *A* is a weak NA*B-*I*-set in $U_{\boldsymbol{I}}$. Therefore, *A* is both nano β-*I*-open set and a weak NA*B-*I*-set in $U_{\boldsymbol{I}}$.

Sufficiency : Let *A* be a nano β-*I*-open set and a weak NA*B-*I*-set in $U_{\boldsymbol{I}}$. Then *A* = *S* ∩ *V*, where *S* is a nano closed set and *V* is a nano semi-pre-*I*-regular set in $U_{\boldsymbol{I}}$. Since each nano closed set in $U_{\boldsymbol{I}}$ is a nano α*-*I*-set in $U_{\boldsymbol{I}}$. S is a nano α*-*I*-set in $U_{\boldsymbol{I}}$. Also since every nano semi-pre-*I*-regular set in $U_{\boldsymbol{I}}$ is a nano α*-*I*-set in $U_{\boldsymbol{I}}$, *V* is a nano α*-*I*-set in $U_{\boldsymbol{I}}$. Thus *A* = *S* ∩ *V* is a nano α*-*I*-set in $U_{\boldsymbol{I}}$. By Proposition 2.2, *A* is both nano β-*I*-open set and a nano α*-*I*-set in $U_{\boldsymbol{I}}$. Hence *A* is a nano semi-pre-*I*-regular set in $U_{\boldsymbol{I}}$.

3. Decomposition of nano R-*I*-Continuity

Definition 3.1. A mapping f: $U_{\boldsymbol{I}} \rightarrow U_1$ is said to be nano β-*I*-continuous (resp. nano α-*I*-continuous, nano pre-*I*-continuous, nano semi-*I*-continuous, nano R-*I*- continuous, nano semi-pre -*I*-regular continuous, NA*$_R$ -*I*-continuous,, NA*B-*I*- continuous, weak NA*B -*I*-continuous) if for every V ∈ $\tau_{R_1}(X_1)$, $f^{-1}(V)$ is a nano β-*I*-open set (resp. nano α-*I*-open set, nano pre-*I*-open set, nano semi-*I*-open set, nano regular-*I*-open set, nano semi-pre-*I*-regular, NA*$_R$ -*I* -set, NA*B -*I*-set, weak NA*B-*I*-set) in $U_{\boldsymbol{I}}$.

Remark 3.1. The following examples shows that the

1. Notion of NA*B-*I*-continuity is independent of notions of nano α-*I*-continuity, nano pre-*I*-continuity and nano semi-*I*-continuity.
2. Notion of weak N A*B-*I*-continuity is independent of notion of nano β-*I*-continuity.

Example 3.1. Let *U = {a, b, c, d}, U/R = {{a, c}, {b}, {d}}, X = {a, d}*, $\tau_R(X)$ *={φ, U, {a, c, d}, {d}, {a, c}}, I = {φ, {a}, {c}, {a, c}}* , U_1 = *{x, y, z, w}*, U_1/R_1 *={{x, y}, {z}, {w}}*, X_1 = *{x, y, z}* and $\tau_{R_1}(X_1)$ = *{φ,* U_1*, {x, y, z}}*. Now,

1. Define f : $U_{\boldsymbol{I}} \rightarrow U_1$ as f(a) = x, f(b) = w, f(c) = y, f(d) = z. Then f is nano α-*I*-continuous, nano pre-*I*-continuous, nano semi-*I*-continuous but not NA*B–*I*-continuous.
2. Define f : $U_{\boldsymbol{I}} \rightarrow U_1$ as f(a) = x, f(b) = w, f(c) = y, f(d) = z. Then f is nano β-*I* -continuous but not weak NA*B–*I*-continuous.

Example 3.2. Let *U = {a, b, c, d}, U/R = {{a, c}, {b}, {d}}, X = {a, d}*, $\tau_R(X)$ *={φ, U, {a, c, d}, {d}, {a, c}}* , $\boldsymbol{I}$ = *{φ, {a}, {c}, {a, c}}*, U_1 = *{x, y, z, w}*, U_1/R_1 *={{x, y}, {z}, {w}}*, X_1 = *{z}*, and $\tau_{R_1}(X_1)$ = *{φ,* U_1*, {z}}*. Now,

1. Define f : $U_{\boldsymbol{I}} \rightarrow U_1$ as f(a) = x, f(b) = z, f(c) = y, f(d) = w. Then f is NA*B-*I*-continuous, but it is neither nano α-*I*-continuous nor nano pre-*I*-continuous and nano semi-*I*-continuous.
2. Define f : $U_{\boldsymbol{I}} \rightarrow U_1$ as f(a) = x, f(b) = z, f(c) = y, f(d) = w. Then f is weak NA*B-*I*-continuous but not nano β-*I*-continuous.

Theorem 3.1. A mapping f : $U_{\boldsymbol{I}} \rightarrow U_1$ is nano R-*I*-continuous if and only if it is nano continuous and NA*$_R$ -*I*-continuous.

Theorem 3.2. For a mapping f : $U_{\boldsymbol{I}} \rightarrow U_1$, the following are equivalent :

1. nano R-*I*-continuous.
2. nano continuous and NA*B-*I*-continuous.
3. nano α-*I*-continuous and NA*B-*I*-continuous.
4. nano pre-*I*-continuous and NA*B-*I*-continuous.

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Theorem 3.3. A mapping f : $U_{\boldsymbol{I}} \longrightarrow U_1$ is nano semi-*I*-regular continuous if and only if it is nano semi-*I*-continuous and NA*B-*I*-continuous.

Theorem 3.4. A mapping f : $U_{\boldsymbol{I}} \longrightarrow U_1$ is nano semi-pre-*I*-regular continuous if and only if it is nano β-*I*-continuous and weak NA*B-*I*-continuous.

References:

[1] Illangovan Rajasekaran, Ochanan Nethaji and Rajendran Prem Kumar, "Perceptions of several sets in Ideal Nano Topological Spaces", Journal of New Theory, (23), **(2018**), pp. 78–84.
[2] V. Inthumathi, S. Narmatha and S. Krishnaprakash, "A decomposition of nano semi-I- continuity", communicated.
[3] K. Kuratowski, "Topology", Academic Press, New York, vol.1, **(1966).**
[4] M. Lellis Thivagar and Carmel Richard, "On nano forms of weakly open sets", International Journal of Mathematics and Statistics Invention, 1(1) **(2013),** pp.31–37.
[5] M. Lellis Thivagar and V.Sutha devi, "New sort of operators in nano ideal topology", Ultra Scientist, 28 (1)(1), **(2016)**, pp.51–64.
[6] R.Vaidyanathaswamy, "The localization in set topology", Proc. Indian Acad. Sci. 20, **(1945),** pp.51–61.

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