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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

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FUZZY rpsI-CLOSED SETS AND FUZZY gprI-CLOSED SETS IN FUZZY IDEAL TOPOLOGICAL SPACES

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Abstract - The aim of this paper is to investigate the concept of fuzzy rpsI-closed sets, fuzzy gprI-closed sets and discuss their properties and obtain relations with existing fuzzy closed sets in fuzzy ideal topological spaces.

Keywords *fuzzy rpsI-closed sets, fuzzy pgprI-closed sets, fuzzy gprI-closed sets, fuzzy semi pre-I-closed sets, fuzzy pre-I-closed sets.*
2010 Subject classification: *54A20*

1 Introduction

The concept of ideal topological spaces was introduced by R.Vaidyanathaswamy [14] in 1945. Kuratowski[7] has introduced local function of a set with respect to a topology τ and an ideal.
The notion of fuzzy set theory and fuzzy set operation was formalized by Lofti A.Zadeh[18] and since then many eminent researches used this notion of fuzzy topology. The concepts of fuzzy semi pre-I-closed sets, fuzzy pre-I-closed sets, fuzzy I_{rg}-closed sets, fuzzy I_g-closed sets have been introduced and studied in fuzzy ideal topological spaces.Authors [3], [8] introduced weakly fuzzy pre I-open and fuzzy α I-open sets obtained a new decomposition of fuzzy continuity via ideals. In this paper we introduced fuzzy rpsI-closed set and fuzzy gprI-closed sets and investigate their properties in fuzzy ideal topological spaces.

2 Preliminaries

Let X be a nonempty set. A family τ of fuzzy sets of X is called a fuzzy topology[2] on X if the null fuzzy set 0 and the whole fuzzy set 1 belongs to τ and τ is closed with respect to any union and finite intersection. If τ is a fuzzy topology on X, then the pair (X, τ) is called a fuzzy topological space[18]. The members of τ are called fuzzy open sets of X and their complements are called fuzzy closed sets. The closure of a fuzzy set A of X denoted by Cl(A), is the intersection of all fuzzy closed sets which contains A. The interior[2] of a fuzzy set A of X denoted by Int(A) is the union of all fuzzy subsets contained in A. A fuzzy set A in $(X, \tau)$is said to be quasi-coincident[11] with a fuzzy set B, denoted by AqB, if there exists a point $x \in X$ such that $A(x) + B(x) > 1$[5]. A fuzzy set V in $(X, \tau)$is called a Q-neighbourhood[11] of fuzzy point x_β if there exists a fuzzy open set U of X such that $x_\beta q U \leq V$[5].

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A nonempty collection of fuzzy sets I of a set X satisfying the conditions
(i) if $A \in I$ and $B \leq A$, then $B \in I$,
(ii) if $A \in I$ and $B \in I$ then $A \cup B \in I$ is called a fuzzy ideal on X.
The triple (X, τ, I) denotes a fuzzy ideal topological space with a fuzzy ideal I and fuzzy topology τ[7, 10]. The local function for a fuzzy set A in X with respect to τ and I denoted by $A^*(\tau, I)$ (briefly A^*) in a fuzzy ideal topological space (X, τ, I) is the union of all fuzzy points x_β such that if U is a Q-neighbourhood of x_β and $E \in I$ then for at least one point $y \in X$ for which $U(y) + A(y) - 1 > E(y)$[12].The $*$-closure operator of a fuzzy set A denoted by $Cl^*(A)$ in (X, τ, I) defined as $Cl^*(A) = A \cup A^*$[13]. In (X, τ, I) the collection $\tau^*(I)$ is an extension of fuzzy topological space than τ via fuzzy ideal which is constructed by considering the class $\beta = \{U - E : U \in \tau, E \in I\}$ as a base[12].

Definition 2.1. *A fuzzy set A of a fuzzy topological space (X, τ) is called:*
(a) *fuzzy regular open [1] if $A = Int(cl(A))$.*
(b) *fuzzy regular closed [1] if $1 - A$ is fuzzy regular open.*

Definition 2.2. *A subset A of a fuzzy ideal topological space (X, τ, I) is called*
i. fuzzy I-open [9]if $A \leq int(A^*)$
ii. fuzzy pre-I-open [10] if $A \leq int(cl^*(A))$
iii. fuzzy semi-I-open [6] if $A \leq cl^*(int(A))$
iv. fuzzy α-I-open [17] if $A \leq int(cl^*(int(A)))$
v. fuzzy semi pre-I-open [17] if $A \leq cl(int(cl^*(A))$
The complement of the above mentioned open sets are their respective fuzzy closed sets.
The fuzzy semi pre-I-closure (resp. fuzzy semi-I-closure, fuzzy pre-I-closure,fuzzy α- I-closure, fuzzy I-closure) of a subset A of(X, τ, I) is the intersection of all fuzzy semi pre-I-closed(resp. fuzzy semi-I-closed, fuzzy pre- I-closed,fuzzyα-I-closed, fuzzy I-closed) sets containing A and is denoted by fuzzy spIcl(A)(resp. fuzzy sIcl(A), fuzzy pIcl(A),fuzzy αIcl(A),fuzzy Icl(A)). The following is useful in sequel.

Definition 2.3. *[1] A fuzzy set A of a fuzzy ideal topological spaces (X, τ, I) is called*
(a) fuzzy I_g closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is fuzzy open.
(b) fuzzy I_g open if its complement 1-A is fuzzy I_g closed.
(c) fuzzy I_{rg} closed if $A^* \subseteq U$ whenever $A \subseteq U$ and U is fuzzy regular open.
(d) fuzzy I_{rg} open if its complement 1-A is fuzzy $I_r g$ closed.

Corollary 2.4. *For any subset A of a fuzzy ideal topological space (X, τ, I) ,the following results hold:*
fuzzy $sIcl(A) = A \cup int(cl^*(A))$
fuzzy $pIcl(A) = A \cup cl^*(int(A))$
fuzzy $spIcl(A) = A \cup int(cl^*(int(A)))$

Remark 2.5. *A is open if and only if $int(A) = A$ and A is $*$-open if and only if $A = int^*(A)$.*

Definition 2.6. *[15] A space X is called extremally disconnected if the closure of each open subset of X is open.*

Definition 2.7. *[4] A subset A of (X, τ) is called generalized pre regular closed (briefly gpr-closed) if $pcl(A) \subset U$ whenever $A \subset U$ and U is regular open in (X, τ).*

Corollary 2.8. *Let (X, τ, I) be an fuzzy ideal topological space and $A \leq X$. If $A \leq A^*$, then $A^* = cl(A) = cl^*(A)$[6].*

3 Fuzzy rpsI-closed sets

In this section, we introduce new class of sets namely fuzzy rpsI-closed set, fuzzy gprI-closed set and discuss some of their properties in fuzzy ideal topological spaces.

Definition 3.1. *A subset A of a fuzzy ideal topological space* (X, τ, I) *is called fuzzy regular pre semi-I-closed(fuzzy rpsI-closed)if fuzzy* $spIcl(A) \leq U$ *whenever* $A \leq U$ *and U is fuzzy* I_{rg}*-open.*
The complement of the above mentioned fuzzy closed set is their respective fuzzy open set.

Theorem 3.2. *Every fuzzy semi pre- I-closed set is fuzzy rpsI-closed.*
Proof: *Let A be a fuzzy semi pre-I-closed set in X. Let* $A \leq U$ *and U be fuzzy* I_{rg}*-open. Since A is fuzzy semi pre-I-closed, we have fuzzy* $spIcl(A) = A \leq U$ *and U is fuzzy* I_{rg}*-open. Therefore A is fuzzy rpsI-closed.*
The following example shows that the converse of the above theorem is not true.

Example 3.3. *Let X={a,b,c} and the fuzzy sets* α_1,α_2,α_3 *and* α_4 *of X are defined as follows:*
$\alpha_1(a) = 0.4, \alpha_1(b) = 0.5, \alpha_1(c) = 0.3$
$\alpha_2(a) = 0.6, \alpha_2(b) = 0.4, \alpha_2(c) = 0.5$
$\alpha_3(a) = 0.6, \alpha_3(b) = 0.5, \alpha_3(c) = 0.5$
$\alpha_4(a) = 0.4, \alpha_4(b) = 0.4, \alpha_4(c) = 0.3$
Let $\tau = \{0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, 1\}$ *be a fuzzy topology and* $I = \{0\}$ *be a fuzzy ideal on X. Then the fuzzy set {0.6,0.4,0.7} is fuzzy rpsI-closed but not fuzzy semi pre-I-closed.*

Remark 3.4. *A subset of a fuzzy rpsI-closed set need not be fuzzy rpsI-closed set.*

Theorem 3.5. *Every fuzzy closed set is fuzzy rpsI-closed set.*
Proof: *Let A be a closed set in X. Let* $A \leq U$ *and U be fuzzy* I_{rg}*-open. Since A is closed we have* $A = cl(A), cl(A) \leq U$*. But fuzzy* $spIcl(A) \leq cl(A) \leq U$*. Therefore A is fuzzy rpsI-closed.*

Example 3.6. *In Example 3.3 , {0.6,0.5,0.5} is fuzzy rpsI-closed but the subset {0.6,0.4,0.5} is not fuzzy rpsI-closed. Also {0.6,0.5,0.5} is not fuzzy closed.*

Definition 3.7. *A subset A of a fuzzy ideal topological space* (X, τ, I) *is called fuzzy pre generalized pre regular I-closed(fuzzy pgprI-closed) if fuzzy* $pIcl(A) \leq U$ *whenever* $A \leq U$ *and U is fuzzy* I_{rg}*-open.*

Theorem 3.8. *Every fuzzy pgprI-closed set is fuzzy rpsI-closed.*
Proof: *Let A be a fuzzy pgprI-closed set in X. Let* $A \leq U$ *and U be fuzzy* I_{rg}*-open. Since A is fuzzy pgprI-closed we have fuzzy* $pIcl(A) \leq U$*. Also fuzzy* $spIcl(A) \leq$*fuzzy* $pIcl(A) \leq U$*. Therefore A is fuzzy rpsI-closed.*
The following example shows that the converse of the above theorem is not true.

Example 3.9. *In Example 3.3,{0.6,0.5,0.5} is fuzzy rpsI-closed but not fuzzy pgprI-closed.*

Theorem 3.10. *Every fuzzy pre-I-closed set is fuzzy rpsI-closed.*
Proof: *Let A be a fuzzy pre-I-closed set in X. We know that fuzzy pre-I-closure of A is the smallest fuzzy pre-I-closed containing A. Therefore fuzzy* $pIcl(A) \leq A$*. Suppose* $A \leq U$ *and U be fuzzy* I_{rg}*-open. Then fuzzy* $pIcl(A) \leq U$ *and U be fuzzy* I_{rg}*-open. Therefore A is fuzzy pgprI-closed. By Theorem 3.8, A is fuzzy rpsI-closed.*
The following example shows that the converse of the above theorem is not true.

Example 3.11. *Let X={a,b,c} and the fuzzy set* α_1*and* α_2 *are defined as follows:*
$\alpha_1(a) = 0.5$,$\alpha_1(b) = 0.5$,α_1=*0.6*
$\alpha_2(a) = 0.4$,$\alpha_2(b) = 0.5$,α_2=*0.4*
Let $\tau = \{0, \alpha_1, \alpha_2, 1\}$ *be a fuzzy topology and* $I = \{0\}$ *be a fuzzy ideal on X.*
Then, {0.5, 0.5,0.6} is fuzzy rpsI-closed but not fuzzy pre-I-closed.

Theorem 3.12. *Every fuzzy* α*-I-closed set is fuzzy rpsI-closed.*
Proof: *Let A be a fuzzy* α*-I-closed set in X. We know that every fuzzy* α*-I-closed set is fuzzy pre-I-closed set. By Theorem 3.10, A is fuzzy rpsI-closed.*
The following example shows that the converse of the above theorem is not true.

Example 3.13. *In Example 3.11, {0.6,0.4,0.4} is fuzzy rpsI-closed but not fuzzy* α*-I-closed.*

Theorem 3.14. *Every fuzzy rI-closed set is fuzzy rpsI-closed.*
Proof: *Let A be a fuzzy rI-closed subset of X. Let* $A \leq U$ *and U be fuzzy* I_{rg}*-open. Since A is fuzzy rI-closed, we have* $A = cl^*(int(A))$*. Therefore* $cl^*(int(A)) \leq U$ *and U be fuzzy* I_{rg}*-open implies* $int(cl^*(int(A))) \leq int(U) \leq U$ *and U be fuzzy* I_{rg}*-open.* $A \cup int(cl^*(int(A))) \leq A \cup U = U$ *and U be fuzzy* I_{rg}*-open. By corollary 2.4(iii),we have fuzzy* $spIcl(A) \leq U$ *and U be fuzzy* I_{rg}*-open. Hence A is fuzzy rpsI-closed.*
The following example shows that the converse of the above theorem is not true.

Example 3.15. *Let X={a,b,c} and the fuzzy set* α_1*,and* α_2 *are defined as follows:*
$\alpha_1(a) = 0.4$,$\alpha_1(b) = 0.5$,$\alpha_1 = 0.5$
$\alpha_2(a) = 0.6$,$\alpha_2(b) = 0.7$,$\alpha_2 = 0.5$
Let $\tau = \{0, \alpha_1, \alpha_2, 1\}$ *be a fuzzy topology and* $I = \{0\}$ *be a fuzzy ideal on X.*
Then,{0.4,0.5,0.5} is fuzzy rpsI-closed but not fuzzy rI-closed.

Definition 3.16. *A subset A of a fuzzy ideal topological space* (X, τ, I) *is called fuzzy SI set if* $cl^*(int(A)) = int(A)$*.*

Theorem 3.17. *Every fuzzy SI set is fuzzy rpsI-closed.*
Proof: *Let A be a fuzzy SI set of X. Let* $A \leq U$ *and U be fuzzy* I_{rg}*-open. Since A is fuzzy SI set we have* $cl^*(int(A)) = int(A)$*.*
Now, $A \leq U \Rightarrow int(A) \leq int(U) \leq U \Rightarrow cl^*(int(A) \leq U \Rightarrow int(cl^*(int((A))) \leq int(U) \leq U \Rightarrow A \cup int(cl^*(int(A)) \leq A \cup U = U \Rightarrow$ *fuzzy* $spIcl(A) \leq U$*. Hence A is fuzzy rpsI-closed.*

Example 3.18. *In Example 3.15,{0.5,0.5,0.6} is fuzzy rpsI-closed but not fuzzy SI-closed.*

Definition 3.19. *A subset A of a fuzzy ideal topological space* (X, τ, I) *is said to be fuzzy semi***-I-open if* $A \leq cl(int^*(A))$*.*

Corollary 3.20. *Let* (X, τ, I) *be fuzzy ideal topological space and K be subset of X. Then the following properties are equivalent:*
i. K is fuzzy rI-closed .
ii. K is fuzzy semi**-I-open and closed.*
Proof: *i)→ ii)*
Let K be an fuzzy rI-closed set in X. Then we have, $K = cl(int^*(K))$*. It follows that K is fuzzy semi***-I-open and closed.*
ii)→ i)
Suppose that K is a fuzzy semi**-I-open and closed . From the definition of fuzzy semi***-I-open and closed, we have K is fuzzy rI-closed set.*

Theorem 3.21. *Let (X, τ, I) be fuzzy ideal topological space and K be subset of X. If K is fuzzy semi*-I-open and closed then K is fuzzy rpsI-closed.*
Proof: *Follows from corollary 3.20 and Theorem 3.14, we have K is fuzzy rpsI- closed.*

Corollary 3.22. *Let (X, τ, I) be fuzzy ideal topological space and K be subset of X, the following properties are equivalent:*
i. K is fuzzy rI-closed .
ii. There exist a $*$-open set L such that $K = cl(L)$.
Proof: *Suppose that there exists fuzzy $*$-open set L such that $K = cl(L)$. Since $L = int^*(L)$, then we have $cl(L) = cl(int^*(L))$. It follows that,*
$cl(int^*(cl(L)) = cl(int^*(cl(int^*(L))))$
$= cl(int^*(L))$
$= cl(L)$
This implies, $K = cl(L) = cl(int^*(cl(L)))$
$= cl(int^*(K))$.
i)$\rightarrow$ ii)
Suppose that K is a fuzzy rI-closed in X. We have $K = cl(int^*(K))$. We take $L = int^*(K)$. It follows that, L is a fuzzy $*$-open and $K = cl(L)$.

Theorem 3.23. *Let (X, τ, I) be a fuzzy ideal topological space and K be a subset of X. Suppose there exist a $*$-open set L such that $K = cl(L)$ then K is fuzzy rpsI-closed set.*
Proof: *Follows from corollary 3.22 and Theorem 3.14, we have K is fuzzy rpsI-closed.*

Definition 3.24. *A subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy weakly semi-I-open if $A \leq cl^*(int(cl(A)))$. The complement of fuzzy weakly semi-I-open is fuzzy weakly semi-I-closed.*

Remark 3.25. *If a subset A of a fuzzy ideal topological space (X, τ, I) is fuzzy weakly semi-I-closed then A is fuzzy semi pre-I-closed.*

Theorem 3.26. *Every fuzzy weakly semi-I closed set is fuzzy rpsI-closed.*
Proof: *Let A be fuzzy weakly semi-I-closed. By corolary 3.25, A is fuzzy semi pre-I-closed. By Theorem 3.2, we have A is fuzzy rpsI-closed.*
The following example shows that the converse of the above theorem is not true.

Example 3.27. *Let X={a,b} and the fuzzy set α_1 and α_2 are defined as follows:*
$\alpha_1(a) = 0.5$,$\alpha_1(b) = 0.3$
$\alpha_2(a) = 0.6$,$\alpha_2(b) = 0.5$
Let $\tau = \{0, \alpha_1, \alpha_2, 1\}$ be a fuzzy topology and $I = \{0\}$ be a fuzzy ideal on X.
Then, {0.4,0.5} is fuzzy rpsI-closed but not fuzzy weakly semi-I-closed .

Remark 3.28. *The concepts of fuzzy I_g-closed set and fuzzy rpsI-closed set are independent.*

Example 3.29. *Let X={a,b} and the fuzzy set α_1 and α_2 are defined as follows:*
$\alpha_1(a) = 0.5$,$\alpha_1(b) = 0.4$
$\alpha_2(a) = 0.6$,$\alpha_2(b) = 0.7$
Let $\tau = \{0, \alpha_1, \alpha_2, 1\}$ be a fuzzy topology and $I = \{0\}$ be a fuzzy ideal on X.
Then, {0.6,0.8} is fuzzy I_g-closed but not fuzzy rpsI-closed and {0.6,0.5}is fuzzy rpsI-closed but not fuzzy I_g-closed.

Theorem 3.30. *If A is fuzzy rpsI-closed and $cl^*(int(A))$ is open. Then A is fuzzy pgprI-closed.*
Proof: *Let $A \leq U$ and U be fuzzy I_{rg}-open. Since A is fuzzy rpsI-closed, fuzzy $spIcl(A) \leq U$ whenever $A \leq U$ and U is fuzzy I_{rg}-open. By corollary 2.4(iii),$A \cup int(cl^*(int(A)))) \leq U$ which implies $A \cup (cl^*(int(A))) \leq U$. Again by corollary 2.4(ii), fuzzy $pIcl(A) \leq U$ whenever $A \leq U$ and U is fuzzy I_{rg}-open. Therefore A is fuzzy pgprI-closed.*

Remark 3.31. *The union of two fuzzy rpsI-closed sets need not be a fuzzy rpsI-closed set.*

Example 3.32. *Consider the fuzzy ideal topological space in Example 3.29. In this fuzzy ideal topological space the sets {0.6,0.5} and {0.5,0.7} are fuzzy rpsI-closed sets, but their union {0.6,0.7} is not fuzzy rpsI-closed set.*

Remark 3.33. *The intersection of two fuzzy rpsI-closed sets need not be a fuzzy rpsI-closed set.*

Example 3.34. *Consider the fuzzy ideal topological space in Example 3.3. In this fuzzy ideal topological space the sets {0.6,0.5,0.5} and {0.7,0.4,0.5} are fuzzy rpsI-closed sets, but their intersection {0.6,0.4,0.5} is not fuzzy rpsI-closed set.*

Theorem 3.35. *Suppose A is fuzzy I_{rg}-open and A is fuzzy rpsI-closed then A is fuzzy semi pre-I-closed.*
Proof: *Since A is fuzzy I_{rg}-open and A is fuzzy rpsI-closed and $A \leq A$, we have fuzzy $spIcl(A) \leq A$. Therefore A is fuzzy semi pre-I-closed.*

Theorem 3.36. *If A is fuzzy rpsI-closed, then fuzzy $spIcl(A)\backslash A$ does not contain a non empty fuzzy I_{rg}-closed set.*
Proof: *Suppose A is fuzzy rpsI-closed set. Let F be a fuzzy I_{rg}-closed subset of fuzzy $spIcl(A)\backslash A$. Then $F \leq$ fuzzy $spIcl(A) \cap (X\backslash A) \leq X\backslash A$ and $A \leq X\backslash F$. But A is fuzzy rpsI-closed and since $X\backslash F$ is fuzzy I_{rg}-open, we have fuzzy $spIcl(A) \leq X\backslash F$. Therefore $F \leq X\backslash$ fuzzy $spIcl(A)$. Since $F \leq$ fuzzy $spIcl(A)$, we have $F \leq (X\backslash$ fuzzy $spIcl(A)) \cap$ fuzzy $spIcl(A) = \phi$ implies $F = \phi$. Therefore fuzzy $spIcl(A)\backslash A$ does not contain a non empty fuzzy I_{rg}-closed set.*

Theorem 3.37. *If A is fuzzy rpsI-closed and if $A \leq B \leq$ fuzzy $spIcl(A)$ then*
i) B is fuzzy rpsI-closed.
ii)fuzzy $spIcl(B)\backslash B$ contains no non-empty fuzzy rpsI-closed sets.
Proof: *i. Given $A \leq B \leq$ fuzzy $spIcl(A)$.Then fuzzy $spIcl(A)$ =fuzzy $spIcl(B)$. Suppose that $B \leq U$ and U is fuzzy I_{rg}-open. Since A is fuzzy rpsI-closed and $A \leq B \leq U$, fuzzy $spIcl(A) \leq U$ we have fuzzy $spIcl(B) \leq U$. Therefore B is fuzzy rpsI-closed.*
ii.Proof follows from Theorem 3.36.

Theorem 3.38. *Let A be fuzzy rpsI-closed. Then A is fuzzy semi pre-I-closed iff fuzzy $spIcl(A)\backslash A$ is fuzzy I_{rg}-closed.*
Proof: *Let A be fuzzy semi pre-I-closed, then fuzzy $spIcl(A) = A$.Therefore fuzzy $spIcl(A)\backslash A = \phi$ which is fuzzy I_{rg}-closed. Conversely, suppose that fuzzy $spIcl(A)\backslash A$ is fuzzy I_{rg}-closed, by Theorem 3.36 we have fuzzy $spIcl(A)\backslash A = \phi$. Thus fuzzy $spIcl(A) = A$. Hence A is fuzzy semi pre-I-closed.*

Definition 3.39. *A subset A of a fuzzy ideal topological space (X, τ, I) is fuzzy quasi I-open if $A \leq cl(int(A^*))$.*

Remark 3.40. *Every fuzzy open set is fuzzy quasi I-open set and every fuzzy quasi I-open set is fuzzy rpsI-open.*
fuzzy I-open→fuzzy quasi I-open→ fuzzy rpsI-open.
The following example shows that the reverse implications need not be true.

Example 3.41. *In Example 3.3, {0.4,0.4,0.4} is fuzzy rpsI-open but not fuzzy quasi-open and {0.4,0.5,0.6} is fuzzy quasi-open but not fuzzy I-open.*

Theorem 3.42. *In an extremally disconnected space X, every fuzzy rpsI-closed set is fuzzy pgprI-closed.*
Proof: *In an extremally disconnected space X,$cl^*(int(A))$ is open for every subset A of X. Then the proof follows from Theorem 3.30.*

Theorem 3.43. *For every point x of a space X,$X\backslash\{x\}$ is fuzzy rpsI-closed or fuzzy I_{rg}-open.*
Proof: *Suppose $X\backslash\{x\}$ is not fuzzy I_{rg}-open. Then X is the only fuzzy I_{rg}-open set containing $X\backslash\{x\}$. This implies that fuzzy $spIcl(X\backslash\{x\}) \leq X$. Hence $X\backslash\{x\}$ is fuzzy rpsI-closed set in X.*

4 Fuzzy generalized pre regular-I-closed sets

Definition 4.1. *A subset A of (X, τ, I) is called fuzzy generalized pre regular I-closed(briefly fuzzy gprI-closed) if fuzzy $pIcl(A) \leq U$ whenever $A \leq U$ and U is fuzzy regular open in (X, τ, I).*

Definition 4.2. *A subset A of a fuzzy ideal topological space (X, τ, I) is called*
i. a fuzzy generalized pre-I-closed set(briefly fuzzy gpI-closed) if fuzzy $pIcl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open.
ii. a fuzzy generalized semi pre-I-closed set(briefly fuzzy gspI-closed) if fuzzy $spIcl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open.

Theorem 4.3. *Every fuzzy I_{rg}-closed set is fuzzy gprI-closed.*
Proof: *Let $A \leq X$ be fuzzy I_{rg}-closed. Let $A \leq U$ and U be fuzzy regular open. Then $A^* \leq U$ because A is fuzzy I_{rg}-closed. Since every closed set is fuzzy pre-I-closed, fuzzy $pIcl(A) = A \cup cl^*(int(A)) \leq A \leq U$. Therefore fuzzy $pIcl(A) \leq U$. Hence A is fuzzy gprI-closed.*

Example 4.4. *Let X={a,b,c} and the fuzzy set α_1,α_2,α_3 and α_4 are defined as follows:*
$\alpha_1(a) = 0.5$,$\alpha_1(b) = 0.5$,$\alpha_1(c) = 0.4$
$\alpha_2(a) = 0.4$,$\alpha_2(b) = 0.6$,$\alpha_2(c) = 0.5$
$\alpha_3(a) = 0.4$,$\alpha_3(b) = 0.5$,$\alpha_3(c) = 0.4$
$\alpha_4(a) = 0.5$,$\alpha_4(b) = 0.6$,$\alpha_4(c) = 0.5$
Let $\tau = \{0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, 1\}$ be a fuzzy topology and $I = \{0\}$ be a fuzzy ideal on X. Then the fuzzy set {0.5,0.5,0.3} is fuzzy gprI-closed but not fuzzy I_{rg}-closed.

Theorem 4.5. *Every fuzzy gpI-closed set is fuzzy gprI-closed.*
Proof: *Let A be a fuzzy gpI-closed in (X, τ, I) and $A \leq U$ where U is fuzzy regular open. since every fuzzy regular open set is fuzzy open and A is fuzzy gpI-closed, fuzzy $pIcl(A) \leq U$. Hence A is fuzzy gprI-closed.*

Example 4.6. *In Example 4.4, {0.4,0.6,0.4} is fuzzy gprI-closed set but not fuzzy gpI-closed.*

Remark 4.7. *Fuzzy gprI-closed sets and fuzzy gspI-closed sets are independent of each other.*

Example 4.8. *In Example 4.4, {0.5,0.6,0.5} is fuzzy gprI-closed set but not fuzzy gspI-closed. {0.5,05,0.4} is fuzzy gspI-closed set but not fuzzy gprI-closed.*

Theorem 4.9. *Let A be a fuzzy gprI-closed set in (X, τ, I). Then fuzzy $pIcl(A) - A$ does not contain any non-empty fuzzy regular closed set.*
Proof: *Let F be a fuzzy regular closed set such that $F \leq$fuzzy $pIcl(A) - A$. Then $F \leq X - A$ implies $A \leq X - F$. A is fuzzy gprI-closed and $X - F$ is fuzzy regular open. Therefore fuzzy $pIcl(A) \leq X - F$. That is $F \leq X-$ fuzzy $pIcl(A)$. Hence $F \leq$ fuzzy $pIcl(A) \cap (X-$fuzzy $pIcl(A)) = \phi$. This shows $F = \phi$.*

Theorem 4.10. *Let A be fuzzy gprI-closed in (X, τ, I). Then A is fuzzy pre-I-closed if and only if fuzzy $pIcl(A) - A$ is fuzzy regular closed.*
Proof*:*Necessity: *Let A be fuzzy pre-I-closed. Then fuzzy $pIcl(A) = A$ and so fuzzy $pIcl(A) - A = \phi$ which is fuzzy regular closed.*
Sufficiency: *Suppose fuzzy $pIcl(A) - A$ is fuzzy regular closed.Then fuzzy $pIcl(A) - A = \phi$ since A is fuzzy gprI-closed. That is, fuzzy $pIcl(A) = A$ or A is fuzzy pre-I-closed.*

Theorem 4.11. *If A is fuzzy gprI-closed and $A \leq B \leq$fuzzy $pIcl(A)$, then B is fuzzy gprI-closed.*
Proof: *Let $B \leq U$ where U is fuzzy regular open. Then $A \leq B$ implies $A \leq U$. Since A is fuzzy gprI-closed, fuzzy $pIcl(A) \leq U$. $B \leq$ fuzzy $pIcl(A)$ implies fuzzy $pIcl(B) \leq$ fuzzy $pIcl(A)$. Thus fuzzy $pIcl(B) \leq U$ and shows that B is fuzzy gprI-closed.*

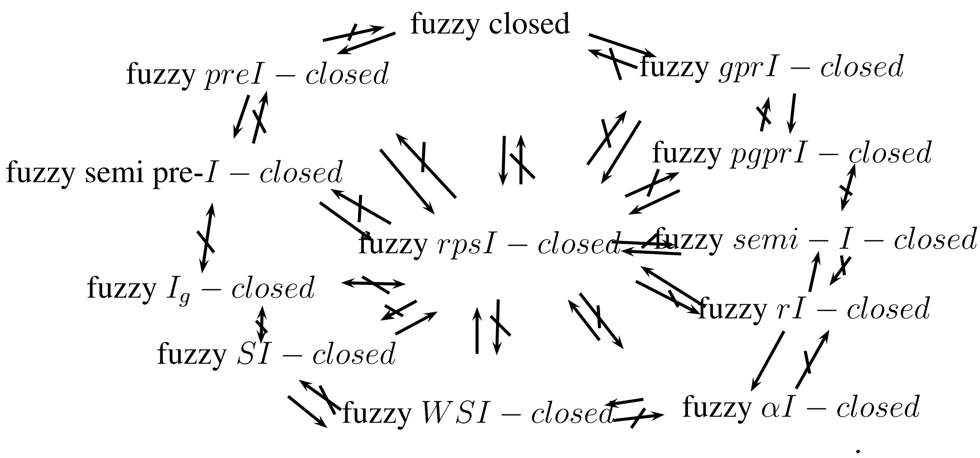
Theorem 4.12. *Every fuzzy gprI-closed set is fuzzy rpsI-closed set.*
Proof: *Let A be a fuzzy gprI-closed set. From the definition 4.1, fuzzy $pIcl(A) \leq U$ whenever $A \leq U$, U is regular open. Now, fuzzy $spIcl(A) \leq$ fuzzy $pIcl(A) \leq U$. We know that, every regular open set is fuzzy I_{rg}-open. Therefore fuzzy $spIcl(A) \leq U$, U is fuzzy I_{rg}- open.*
The converse need not be true.

Example 4.13. *Let X={a,b} and the fuzzy set α_1 and α_2 are defined as follows:*
$\alpha_1(a) = 0.5$,$\alpha_1(b) = 0.4$,$\alpha_1(c) = 0.4$
$\alpha_2(a) = 0.6$,$\alpha_2(b) = 0.6$,$\alpha_2(c) = 0.7$
Let $\tau = \{0, \alpha_1, \alpha_2, 1\}$ be a fuzzy topology and $I = \{0\}$ be a fuzzy ideal on X.
Then, {0.5,0.7,0.7} is fuzzy rpsI-closed set but not gprI-closed.

Theorem 4.14. *Every fuzzy gprI-closed set is fuzzy pgprI-closed set.*
Proof: *Let A be a fuzzy gprI-closed set. From the definition 4.1, fuzzy $pIcl(A) \leq U$ whenever $A \leq U$, U is regular open. We know that, every regular open set is fuzzy I_{rg}-open. Therefore fuzzy $pIcl(A) \leq U$, U is fuzzy I_{rg}-open.*
The converse need not be true.

Example 4.15. *In Example 4.13, {0.5,0.7,0.7} is fuzzy pgprI-closed set but not gprI-closed .*

Summing up the above implications, we have following diagram. However, the reverse implications are not true as seen by the following examples.



Remark 4.16. *Let X={a,b} and the fuzzy set* α_1 *and* α_2 *are defined as follows:*
$\alpha_1(a) = 0.5, \alpha_1(b) = 0.4$
$\alpha_2(a) = 0.6, \alpha_2(b) = 0.7$
Let $\tau = \{0, \alpha_1, \alpha_2, 1\}$ *be a fuzzy topology and* $I = \{0\}$ *be a fuzzy ideal on X.*
Then, i.{0.5,0.5} is fuzzy semi-I-closed but not fuzzy closed.
ii. {0.5,0.5} is fuzzy semi-I-closed but not fuzzy rI-closed.
iii. Every fuzzy rI-closed set is fuzzy α*-I-Closed. But the converse need not be true, for example the set {0.4,0.3} is fuzzy* α*-I-closed but not fuzzy rI-closed.*
iv. {0.5,0.8} is fuzzy pre-I-closed but not fuzzy closed.
v. {0.6,0.5} is fuzzy weakly semi-I-closed but not fuzzy α*-I-closed.*
vi. {0.6,0.5} is fuzzy weakly semi-I-closed but not fuzzy SI-closed.
vii. fuzzy I_g*-closed and fuzzy SI-closed sets are independent to each other. For example, the set {0.5,0.5} is fuzzy* I_g*-closed but not SI closed and {0.4,0.7} is SI-closed set but not fuzzy* I_g*-closed.*
viii. {0.5,0.4} is fuzzy semi pre-I-closed but not fuzzy pre-I-closed.

Remark 4.17. *fuzzy g-closed and fuzzy semi pre-I-closed sets are independent to each other.*
In above Remark, the set {0.6,0.8}is fuzzy I_g*-closed but not semi pre-I-closed and {0.6,0.5} is semi pre-I-closed set but not fuzzy* I_g*-closed.*

Remark 4.18. *The concepts of fuzzy pgprI-closed set and fuzzy semi-I-closed set are independent.*

Example 4.19. *Let X={a,b} and the fuzzy set* α_1 *and* α_2 *are defined as follows:*
$\alpha_1(a) = 0.5, \alpha_1(b) = 0.4, \alpha_1(c) = 0.4$
$\alpha_2(a) = 0.6, \alpha_2(b) = 0.6, \alpha_2(c) = 0.7$
Let $\tau = \{0, \alpha_1, \alpha_2, 1\}$ *be a fuzzy topology and* $I = \{0\}$ *be a fuzzy ideal on X.*
Then, {0.5,0.7,0.7} is fuzzy pgprI-closed but not fuzzy semi-I-closed. {0.5,0.4,0.4}is fuzzy semi-I-closed but not fuzzy pgprI-closed.

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