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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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Weakly δP_S-Continuous Functions

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ABSTRACT

The purpose of this paper is to introduce a new class of functions called weakly δP_S-continuous functions by usingδP_S-open sets in topologicalspaces. Some properties and characterizations ofweaklyδP_S-continuous functions arefound.

KEYWORDS: AlmostδP_S-continuousand WeaklyPre-continuousFunctions

1. INTRODUCTION

The class of δ-open subsets of a topological space was firsrt introduced by Veliko [22] in 1968. Munshi [11] initiated and studied the concept of supe continuous mappings in 1982. Masshour et al [10] introduced the concept of precontinuous and weak precontinuous mappings in 1982. Since then many authors defined the various forms of weakly continuous mappings.

In 2020, Vidhyapriya et al., [23] defined a new class of open sets namely δP_S-open sets, combining the concepts of δ-preopen and semi-closed sets. In this paper the author defined weakly δP_S-continuous functions using δP_S-continuous [24], almost δP_S-continuous [25] and precontinuous functions. Further their properties and comparisons are studied.

2. PRELIMINARIES

Definition 2.1. A subset A of a space X is said to be

a) Preopen [10] if A ⊆int (cl(A))
b) Semi-open [8] if A ⊆ cl (int(A))
c) Regular open [21] if A = int (cl(A))
d) θ-open [22] if for each x ϵ A there exists an open set G such that x∈ G ⊆ clG ⊆ A
e) θ-semi-open [4] if for each x ϵ A, there exists an semi-open set G such that x ∈ G ⊆ clG ⊆ A
f) δ-preopen [17] if A ⊆ Int(δcl(A))

- The closure and interior of A with respect to X are denoted by cl(A) and int(A) respectively.
- The intersection of particular class of closed sets of Xcontaining A is called the corresponding closure of A.
- The union of particular class of open sets of X contained in A is called the corresponding interior of A.
- The family of all preopen (resp. Semi-open, regular open, θ-open, θ-semi-open,) subsets of X is denoted by PO(X) (resp. SO(X), RO(X), θO(X),θSO(X), δPO(X)).
- The complement of a preopen (resp. resp. Semi-open, regular open, δ-open, θ-open, δ-preopen, θ-semi-open, δ-preopen) is said to be preclosed (resp. resp. Semi-closed, regular closed,θ-closed,θ-semiclosed, δ-preclosed).

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- The family of all preclosed [10] (resp. Semi-closed, regular closed, θ-closed, θ-semiclosed, δ-preclosed) subsets of X is denoted by PC(X) (resp. SC(X), RC(X), θC(X), θSC(X), δPC(X)).

Definition 2.2[23]:A δ- preopen subset A of a space X is called a δP_S-open set if for each x ϵ A, there exists a semi-closed set F such that x ϵ F⊆ A.

Definition 2.3:A function $f:(X,\tau)\to(Y,\sigma)$ is said to be precontinuous [10] (resp. super continuous [11]) if the inverse image of each open subset of Y is preopen (resp. δ-open) in X.

Definition 2.4: A function $f:(X,\tau)\to(Y,\sigma)$issaidtobealmostP_S-continuous[5](resp.almostprecontinuous[12],almostα-continuous [14]andalmostcontinuousinthesenseofSingal and Singal [20]) if for each $x\in X$ andeachopensetVofYcontaining*f*(x),thereexistsaδP_S-open(resp.preopen,α-openandopen)set$U$of$X$containing$x$suchthat$f(U)\subseteq intcl(V)$.

Definition 2.5: A function $f:(X,\tau)\to(Y,\sigma)$ is said to be almostδP_S-continuous[25](resp.,almoststronglyθ-continuous[13]andθ -irresolute[6])iftheinverse image of eachregular open subset ofYisδP_S-open(resp.,θ -openandintersectionofregular opensets) inX.

Definition 2.6:A function $f:(X,\tau)\to(Y,\sigma)$ is saidto be δP_S-continuous [24] (resp. precontinuous [10]and semi-continuous [8]) if the inverseimageofeachopensubsetofYisδP_S-open(resp.preopen and semi-open) in X.

Definition 2.7: Afunction$f:(X,\tau)\to(Y,\sigma)$issaid to be δ-continuous [15] (resp. θ-continuous[3]) if for each x ∈ X and each open set V of Ycontaining f(x), there exists an open set U of Xcontaining x such that f (intclU) ⊆ IntClV (resp.f(clU) ⊆ clV).

Definition 2.8:A function $f:(X,\tau)\to(Y,\sigma)$ is said to beweaklycontinuous[12](resp.weaklyα-continuous [16], weakly pre-continuous [7] and weakly δ-precontinuous[15]) if for each x ∈ X andeachopensetVofYcontaining*f*(x), there exists a open (resp. α-open, preopen andδ-preopen) set U of X containing x such that *f*(U) ⊆ clV.

Definition 2.9: Afunction $f:(X,\tau)\to(Y,\sigma)$issaidtobeS-continuous [27] if for every F ∈ RC(Y), f^{-1}(F) isthe union of regular closed sets of X.

Definition 2.10: Afunction $f:(X,\tau)\to(Y,\sigma)$ is almost open [19] if *f*(U) $\subseteq int(cl(f(U))$ for each opensubsetUof X.

Definition2.11. AspaceXissaidtobe

a) Extremally disconnected [1] if the closure ofeveryopen setof Xisopenin X.

b) Locally indiscrete [2] if every open subset ofXis closed.

c) Semi-T_1 [9] if to each pair of distinct pointsx, y of X, there exists a pair of semi-opensets,one containingxbutnotyandtheothercontainingybutnotx.

d) Semi-regular [28] if for any open set U of Xand each point x ∈ U, there exists a regular open setVof X such thatx∈V⊆U.

e) Almost regular[18] if forany regularclosedsetFofXandapointx∉F,thereexistdisjointopen setsU andVsuchthatF⊆U and x∈V.

Lemma 2.12[23].A subset A of a space X is δP_S-open if and only if A is a δ-preopen set and A is a union of semi-closed sets.

Proposition2 . 1 3 [23]. IfaspaceXissemi-T_1, then $\delta P_S O(X)=PO(X)$.

Proposition2.14[23]. Ifatopologicalspace(X,τ)islocallyindiscrete, then $\delta P_S O(X)=\tau$.

Lemma 2.15[23]. For any subset A of a space X. If $A \in \theta SO(X)$ and $A \in PO(X)$, then $A \in \delta P_S O(X)$

Lemma 2.16[23]. Let (X, τ) be any extremally disconnected space. If $A \in \theta SO(X)$ then $A \in \delta P_S O(X)$

Proposition 2.17[23]. Let (Y,τ_Y) be a subspace of aspace (X,τ). If $A \in \delta P_S O(Y,\tau_Y)$ and $Y \in RO(X,\tau)$, then $A \in \delta P_S O(X,\tau)$.

Corollary 2.18[23]. If $A \in \delta P_S O(X)$ and B is eitheropen or regular semi-open subset of X, then $A \cap B \in \delta P_S O(B)$.

Proposition2.19. LetAbeasubsetofatopologicalspace(X,τ),thenthefollowingstatements are true:

a) If $A \in SO(X)$, then $\delta P_S Cl(A) = cl(A)$[26]

b) If $A \in \tau$, then $cl_\theta(A) = cl(A)$ [22].

Proposition2.20[24]. If $f: (X, \tau) \to (Y, \sigma)$ is a continuous and an open function and V is a δP_S-open set of Y, then $f^{-1}(V)$ is a δP_S-open set ofX.

Proposition2.21[8]. Let $f: (X, \tau) \to (Y, \sigma)$ be a function andX is a locally indiscrete space. Then f is almostδP_S-continuousifandonlyif$f$isalmostcontinuous.

Corollary 2.22[25]. If $f: (X, \tau) \to (Y, \sigma)$is almost δP_S-continuous function if and only if f is almost continuous where X is locally indiscrete space.

Proposition2.23 [25]. If $f: (X, \tau) \to (Y, \sigma)$is an almost δP_S-continuous function and Y is semi-regular. Then f is δP_S-continuous.

Thefollowingresultscanbeproved easily.

Proposition2.24. If$f: (X, \tau) \to (Y, \sigma)$isalmost$\delta$-precontinuous and Y is semi-regular, then$f$isprecontinuous.

Proposition2.25. If $f: (X, \tau) \to (Y, \sigma)$ is almost continuousand Y is semi-regular, then$f$is continuous.

Proposition2.26[23]. Afunction$f: (X, \tau) \to (Y, \sigma)$isweaklycontinuousifandonlyif$clf^{-1}(V) \subseteq f^{-1}(clV)$for each opensubsetVof Y.

Proposition2.27[18]. A function $f: (X, \tau) \to (Y, \sigma)$ is almost-open if and only if $f^{-1}(clV) \subseteq clf^{-1}(V)$ for eachopen subsetVof Y.

Proposition2.28[18]. A function $f: (X, \tau) \to (Y, \sigma)$ is almost-openandalmostcontinuousifandonlyif$clf^{-1}(V) = f^{-1}(clV)$foreach opensubsetV of Y.

3. WEAKLY δP_S- CONTINUOUS FUNCTIONS

In this section, we introduce the conceptof weakly δP_S-continuous functions by using δP_S-opensets.Wegivesomecharacterizations of weakly δP_S-continuousfunctionswithseveralrelations between this function andothertypesofcontinuousfunctionsandspaces

Definition 3.1. A function $f: (X, \tau) \to (Y, \sigma)$ is calledweakly δP_S-continuous if for each$x \in X$ and each open set V of Y containing f(x), there exists aδP_S-opensetUofXcontainingxsuchthat$f(U) \subseteq \delta cl(V)$.[For an open set δ-closure and closure coincide[21]. Hence in the above definition we can have $f(U) \subseteq cl(V)$).

Lemma3.2. The following results supervene from their definitions directly:

a) Every almost δP_S-continuous functions is weakly δP_S-continuous.

b) Every weakly δP_S-continuous function is weakly δ-pre-continuous.

c) Every weakly P_S-continuous function is weakly δP_S-continuous.

Proof: a) Let $f:(X,\tau)\to(Y,\sigma)$ be almost δP_S-continuous. Let $x\in X$ and each open set V of Y containing $f(x)$. Since f is almost δP_S-continuous, there exists a δP_S-open set U of X contained in x such that $f(U)\subseteq$
$int(cl(V))$
We know that $int(cl(V))\subseteq cl(V)$
Hence $f(U)\subseteq cl(V)$
b) Let $f:(X,\tau)\to(Y,\sigma)$ be weakly δP_S-continuous. Let $x\in X$ and V be an open set in Y containing x. Since f is weakly δP_S-continuous, there exists a δP_S-open set V contained in $f(x)$ such that $f(U)\subseteq V$. Since every δP_S-open set is δP-open set f is weakly δ-precontinuous.
c) Follows from the fact that every P_S-open set is δP_S-open set.

Therefore from the above Proposition we have:

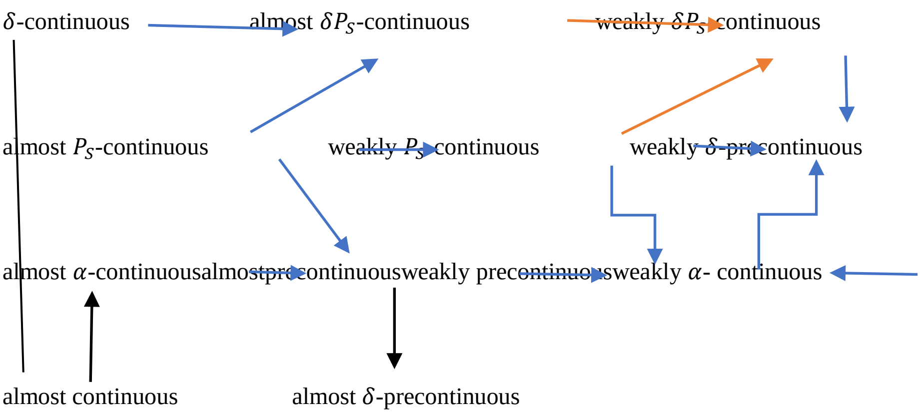

FIGURE 3.1

In the sequel, we shall show that none oftheimplicationsthatconcerningweaklyδP_S-continuityin Figure3.1 isreversible.

Example 3.3. Let X = {a, b, c, d} with the twotopologiesτ={ X, ∅, {c},{a,b}, {a,b,c}} andσ={ X, ∅, {a}, {c},{a,b},{a,c}, {a,b,c},{a,c,d}} then$\delta P_S O(X,\tau)$={∅, X, {c}, {a, b}, {a, b, c}}.Let $f:(X,\tau)\to(X,\sigma)$ be the identity function. Then$f$is weakly δP_S-continuous, but it is not almost δP_S-continuous, because {a} is an open set in (X,σ)containing*f*(a)=a,butthereexistnoδP_S-openset U in (X,τ) containing a such that a ∈*f* (U) ⊆IntCl{a} = {a}.

Example 3.4. Let X = {a, b, c, d} with the twotopologiesτ={∅,X,{a},{b},{a,b}}and{$\sigma=\{X,\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\},\{a,c,d\}\}$;then$\delta P_S O(X,\tau)=\{X,\emptyset,\{a\},\{b\},\{a,b\},\{a,b,c\},\{a,b,d\}\}$. Let $f:(X,\tau)\to(X,\sigma)$ be a function defined asfollows: *f* (a) =a,*f* (b) = *f*(c) = b and *f* (d) = d.Then*f*isweakly δP_S-continuous.However*f* is not weakly P_S-continuous since, an open set {a} in (X,σ) containing *f* ({a}) = a,butthereexistnoP_S-opensetUin(X,τ)containing a such that *f* ({d}) = a∈*f* (U)⊆int cl{a} = {a} as $P_S O(X,\tau)=\{X,\emptyset\}$.

Example 3.5. Let X,τ,σ be same as in Example 3.3. Then *f* isweakly δ-precontinuous,but itis notweakly δP_S-continuous.

Remark3.6. Wenoticethateveryidentityfunction is weakly δP_S-continuous and a function$f:(X,\tau)\to(Y,\sigma)$is weakly δP_S-continuous if either X isdiscrete orYisindiscrete.

Proof: Case-(i) X is discrete

Proof: When X is discrete, (ie.,)$\tau = \mathcal{P}(X)$.Hence for every $x \in X, \{x\}$ is a δP_S-open in X.

Therefore, the δP_S-open set $U = \{x\}$ containing x such that $f(U) = f(\{x\}) = f(x) \in V \subseteq cl(V)$. Thus f is weakly δP_S-continuous

Case –(ii) Y is indiscrete

Proof: When Y is indiscrete, $\sigma = \{Y, \sigma\}$ then $\delta P_S O(\sigma) = \{Y, \sigma\}$. Any open set V in σ is Y. and $cl(V) = Y$.

Hence for any $U, f(U) \subseteq Y = cl(V)$

$\therefore$ f is weakly δP_S-continuous.

Proposition3.7. Ifafunction$f: (X, \tau) \to (Y, \sigma)$isweakly δP_S-continuous,thenforeachx∈ X andeachθ-openset V of Y containing f(x), there exists a δP_S-open setUinXcontainingxsuch that$f(U) \subseteq V$..

Proof. Let$x \in X$andletVbeanyθ-opensetofY containing$f$(x).Thenforeach$f$(x)∈V,thereexistsanopensetGcontaining$f$(x)suchthat$G \subseteq Cl(G) \subseteq V$. Since f is weakly δP_S-continuous,there exists a δP_S-open set U of X containing xsuch that $f(U) \subseteq Cl(G) \subseteq V$..

This completes theproof.

Corollary 3.8. If a function $f: (X, \tau) \to (Y, \sigma)$ is weaklyδP_S-continuous, then for each$x \in X$ and each θ-open set V of Y containing f(x), there exists asemi-closedsetFinXcontainingxsuchthat$f(F) \subseteq V$.

Proof. Let $x \in X$and let V be any θ-open set ofYcontaining$f$(x).Since$f$isweaklyδP_S-continuous, then by Proposition3.7, there exists aδP_S-opensetUinXcontainingx suchthat $f(U) \subseteq V$.SinceUisaδP_S-opensetinX,thenforeachx∈ U, there exists a semi-closed set F of X suchthat$x \in F \subseteq U$.Therefore,weobtain$f(F) \subseteq f(U) \subseteq V$. Hence $f(F) \subseteq V$.

Proposition3.9. Let $f: (X, \tau) \to (Y, \sigma)$ be a function. If foreach $x \in X$ and each regular closed set R of Ycontaining $f(x)$, there exists a δP_S-open set U in Xcontaining x such that $f(U) \subseteq R$, then f is weaklyδP_S-continuous.

Proof. Let$x \in X$ and let V be any open set of Ycontaining f (x). Then put $R = cl(V)$which is aregularclosedsetofYcontaining$f$(x).Byhypothesis, there existsa δP_S-open setU inXcontainingxsuchthat$f(U) \subseteq R = cl(V)$.Hence$f$isweaklyδP_S-continuous.

Proposition3.10. If a function $f: (X, \tau) \to (Y, \sigma)$ is weaklyδP_S-continuous, then the inverse image of each θ-open setof $Y$is a δP_S-open setin X.

Proof. Let V be any θ-open set in Y. We have to show that $f^{-1}(V)$is a δP_S-open set in X. Let $x \in f^{-1}(V)$Then$f(x) \in V$. Since f is weakly δP_S-continuous, then by Proposition3.7, there exists aδP_S-opensetUofXcontainingxsuchthat$f(U) \subseteq V$, whichimpliesthat$x \in U \subseteq f^{-1}(V)$. Therefore,$f^{-1}(V)$)isa$\delta P_S$-opensetin$X$.

Corollary3.11. Ifafunction$f: (X, \tau) \to (Y, \sigma)$isweakly$\delta P_S$-continuous,thentheinverseimageofeachθ-closed setof Y is a δP_S-closedsetin X.

Proposition3.12. Let$f: (X, \tau) \to (Y, \sigma)$beafunction.If$f^{-1}(cl(V))isa\delta P_S$-opensetinXforeachopen set Vin Y, then$f$is weaklyδP_S-continuous.

Proof. Let $x \in X$and let V be any open set of Ycontaining f(x). Then $x \in f^{-1}(V) \subseteq f^{-1}(cl(V))$. Byhypothesis, we have $f^{-1}(cl(V))$ is a δP_S-open set inXcontainingx.Therefore,weobtain$f(f^{-1}(cl(V))) \subseteq cl(V)$. Hence f is weakly δP_S-continuous.

Corollary3 .13. Let$f:(X,\tau)\to(Y,\sigma)$beafunction.If$f^{-1}(int(F))$isaδP_S-closedsetinXforeach closed set F in Y, then $f$is weaklyδP_S-continuous.

Proposition3.14. Let $f:(X,\tau)\to(Y,\sigma)$ be a function. If theinverse image of each regular closed set of Y is aδP_S-open set in X, then f is weakly δP_S-continuous.

Proof. LetVbeanyopensetofY.Then$cl(V)$ is a regular closed set in Y. By hypothesis, we have $f^{-1}(cl(V))$isaδP_S-opensetinX.Therefore,byProposition3.12, $f$is weaklyδP_S-continuous.

Corollary 3.15. Let $f:(X,\tau)\to(Y,\sigma)$ be a function. If theinverse image of each regular open set of Y is aδP_S-closedsetinX,then$f$isweaklyδP_S-continuous.

Proposition3.16. If a function $f:(X,\tau)\to(Y,\sigma)$ is weaklyδP_S-continuous, then for each $x\in X$ and eachopensetVofYcontaining$f$(x),thereexistsa semi-closedsetFinXcontainingxsuchthat$f(F)\subseteq cl(V)$.

Proof. Let $x\in X$ and let V be any open set of Ycontaining f (x). Since f is weakly δP_S-continuous,then there exists a δP_S-open set U of X containing x such that $f(U)\subseteq cl(V)$. Since U is δP_S-open set,then for each $x\in U$, there exists a semi-closedset F of X such that $x\in F\subseteq U$.Therefore, wehave $f(F)\subseteq Cl(V)$.

The following result is a characterizationofweaklyδP_S-continuous functions:

Proposition3.17. Forafunction$f:(X,\tau)\to(Y,\sigma)$,thefollowingstatements are equivalent:

a) f is weaklyδP_S-continuous.

b) $\delta P_S cl f^{-1}(intcl(B))\subseteq f^{-1}(cl(B))$ for each $B\subseteq Y$

c) $f^{-1}(int(B))\subseteq \ \delta P_S int f^{-1}(cl(int(B)))$ for each $B\subseteq Y$

d) $f^{-1}(int(cl\,V))\subseteq \ \delta P_S int f^{-1}(clV)$ foreachopensetVofY

e) $f^{-1}(V)\subseteq \delta P_S int(f^{-1}(cl(V))$ for each regular open set V of Y.

f) $\delta P_S(cl\left(f^{-1}(int(F))\right)\subseteq f^{-1}(F)$,foreachregular closed setF ofY.

g) $\delta P_S(cl(f^{-1}(int(F)))\subseteq f^{-1}(cl(int(F)))$,foreachclosed set F of Y.

h) $\delta P_S(cl(f^{-1}(V)))\subseteq f^{-1}(cl(V))$,foreachopenset$V$ of Y.

i) $f^{-1}(int(F))\subseteq \delta P_S(int(f^{-1}(F))$,for each closed set F of Y.

Proof. (a) ⇒(b). LetB be any subset ofY.Assume that$x\notin f^{-1}(cl(B))$. Then$f(x)\notin cl(B)$ andthere exists an open set V containing $f(x)$suchthat$V\cap B=\emptyset$,hence$cl(V)\cap int(cl(B))=\emptyset$.By(a),thereexistsa$\delta P_S$-opensetUofXcontainingxsuchthat$f(U)\subseteq cl(V)$.Therefore,wehave$f(U)\cap Int(Cl(B))=\emptyset$whichimpliesthat$U\cap f^{-1}(int(cl(B))=\emptyset$ and hence$x\notin \delta P_S(cl(f^{-1}(int(cl(B))))$.Therefore, weobtain$\delta P_S(cl(f^{-1}(int(cl(B)))\subseteq f^{-1}(cl(B))$.

(b)⇒(c). LetBbeanysubsetofY.Thenapply (b) toY\Bweobtain$\delta P_S cl f^{-1}$(intcl(Y\B))⊂f^{-1}(cl(Y\B))⇒$\delta P_S cl f^{-1}$(int(Y\intB))⊆f^{-1}(Y\intB)⇒$\delta P_S cl f^{-1}$(Y\$cl$intB)⊆$f^{-1}$(Y\intB)⇒$\delta P_S cl$(X\$f^{-1}$($cl$intB))⊆X\$f^{-1}$(intB)⇒X\ δP_Sint(f^{-1}(clintB)) ⊆ X\$f^{-1}$(intB) ⇒$f^{-1}$(intB) ⊆ δP_Sintf^{-1}(clintB).Therefore,weobtainf^{-1}(intB)⊆ δP_Sintf^{-1}(clintB).

(c) ⇒ (d). Let V be any open set of Y. Then apply (c) to $cl(V)$we obtain f $^{-1}$(IntclV) ⊆ δP_Sintf^{-1}(clintclV) = $\delta P_S$$i$ntf $^{-1}$(clV). Therefore, we obtain f $^{-1}$(intclV) ⊆ δP_Sintf^{-1}(clV).

(d) ⇒ (e). Let V be any regular open set of Y. Then V is an open set of Y. By (d), we have f $^{-1}$(V) = f $^{-1}$(intclV)⊆ δP_Sintf^{-1}(clV)). Therefore, we obtain f $^{-1}$(V) ⊆ δP_Sintf^{-1}(clV).

(e)⇒ (f). Let F be any regular closed set of Y.Then Y\F is a regular open set of Y. By (e), wehave f^{-1}(Y\F) ⊆

δP_Sintf^{-1}(cl(Y\F)) ⇒ X\$f^{-1}$(F) ⊆ δP_Sintf^{-1}(Y\intF)⇒X\$f^{-1}$(F)⊆ δP_Sint(X\$f^{-1}$(intF)) ⇒X\$f^{-1}$(F)⊆X\ $\delta P_S clf^{-1}$(intF)⇒$\delta P_S clf^{-1}$(intF)⊆f^{-1}(F). Hence $\delta P_S clf^{-1}$(intF) ⊆f^{-1}(F).

(f)⇒ (g). Let F be any closed set of Y. Then$cl\ int(F)$ is a regular closed set of Y. By (f) wehave$\delta P_S clf^{-1}$(intclintF)=$\delta P_S clf^{-1}$(intF)⊆f^{-1}(clintF). Therefore, we obtain $\delta P_S clf^{-1}$(intF) ⊆f^{-1}(clintF).

(g)⇒(h). LetVbeanyopensetofY.Thenby (g)wehave$\delta P_S clf^{-1}$(V)⊆ $\delta P_S clf^{-1}$(intclV)⊆f^{-1}(clintclV)=f^{-1}(clV).Therefore,$\delta P_S clf^{-1}$(V)⊆f^{-1}(clV).

(h)⇒ (i). Let F be any closed set of Y. ThenY\FisanopensetofY.By(h),wehave$\delta P_S clf^{-1}$(Y\F) ⊂f^{-1}(cl(Y\F)) ⇒$\delta P_S cl$(X\$f^{-1}$(F)) ⊆$f^{-1}$(Y\intF)⇒X\ δP_Sintf^{-1}(F)⊆X\$f^{-1}$(intF)⇒$f^{-1}$(intF)⊆ δP_Sintf^{-1}(F).Therefore,f^{-1}(intF)⊆ δP_Sintf^{-1}(F).

(i)⇒ (a). Let x be any point of X and let V beany open set in Y containing f (x). Then x ∈f^{-1}(V) and clV is a closed set in Y. By (i), wehave x ∈f^{-1}(V) ⊂f^{-1}(intclV) ⊆ δP_Slntf^{-1}(clV).Put U = δP_SIntf^{-1}(clV). Then we obtain x ∈ U ∈δP_SO(X) and f (U) ⊆ clV. Therefore, f is weaklyδP_S-continuous.

Proposition3.18. Ifafunction$f: (X,\tau) \rightarrow (Y,\sigma)$iscontinuous, then$f$is weakly$\delta P_S$-continuous.

Proof. Let V be any open set of Y. Since$f$ iscontinuous,thenf^{-1}(V)isanopensetandhenceit is a semi-open set. By Proposition2.19(a), wehave$\delta P_S clf^{-1}$(V)=clf^{-1}(V).Also,since$f$iscontinuous, then clf^{-1}(V) ⊆$f^{-1}(cl(V)$. Therefore,we obtain that $\delta P_S clf^{-1}$(V) ⊆f^{-1}(clV) and henceby Proposition3.17,$f$is weaklyδP_S-continuous.

Anothercharacterization theoremofweaklyδP_S-continuous functions is the following:

Proposition3.19. Forafunction$f: (X,\tau) \rightarrow (Y,\sigma)$,thefollowingstatements are equivalent:

a) f is weaklyδP_S-continuous.
b) f($\delta P_S cl$A)⊆ $cl_\theta f$(A), foreach subsetAofX.
c) int$_\theta f$(A)⊆f (δP_SintA), for eachsubsetAofX.
d) f^{-1}(int$_\theta$B)⊆ δP_Sintf^{-1}(B),foreachsubsetBofY.
e) $\delta P_S cl$f $^{-1}$(B) ⊆ f $^{-1}$(cl_θB), for each subset B of Y.

Proof. (a) ⇒ (b). Let A be a subset of X. Suppose that f($\delta P_S cl$A) ⊈ cl_θf(A). Then there exists y ∈f($\delta P_S cl$A) such that y ∉cl_θf(A), then there exists an open set G in Y containing y such that clG ∩ f(A) = ∅. If f $^{-1}$(y) = ∅, then there is nothing to prove. Suppose that x be any arbitrary point of f^{-1}(y), so f(x) ∈ G. Since G is anopenset in Y, by (a), there exists a δP_S-open set H in X containing x such that f(H) ⊆ clG. Therefore, wehave$f$(H)∩f(A)=∅.Then$y \notin \delta P_S cl(f(A)) \Rightarrow x \notin \delta P_S cl(A)$. Hence$y \notin \delta P_S cl(A)$ which is a contradiction. Therefore,wehave f($\delta P_S cl$A)⊆ $cl_\theta f$(A).

(b)⇒(c). LetAbeanysubsetofX.Thenapply (b) to X\A we obtain f($\delta P_S cl$(X\A)) ⊆ cl_θf(X\A) ⇒ f(X\ $\delta P_S i$ntA) ⊆ cl_θ(Y\ f(A)) ⇒ Y\ f(δP_SintA) ⊆ Y\int$_\theta$f(A) ⇒int$_\theta$f(A) ⊆ f(δP_SintA). Therefore, we obtain that int$_\theta$f(A) ⊆ f($\delta P_S i$ntA).

(c) ⇒ (d). Let B be a subset of Y. Then f $^{-1}$(B) is a subset of X. By (c), we have Int$_\theta$f(f $^{-1}$(B)) ⊆ f(δP_Sintf^{-1}(B)). Then intθB⊆f(δP_SIntf $^{-1}$(B)) and hence f $^{-1}$(int$_\theta$B) ⊆ δP_Sintf^{-1}(B).

(d) ⇒ (e). Let B be any subset of Y. Then apply (d) to Y\B we obtain f $^{-1}$(Int$_\theta$(Y\B)) ⊆ δP_Sintf^{-1}(Y\B) ⇒ f $^{-1}$(Y\$cl_\theta$B) ⊆ $\delta P_S i$nt(X\f $^{-1}$(B)) ⇒ X\f $^{-1}$(cl_θB) ⊆ X\ $\delta P_S cl$f $^{-1}$(B) ⇒$\delta P_S cl$f $^{-1}$(B) ⊆ f $^{-1}$(cl_θB). Therefore, we obtain $\delta P_S cl$f $^{-1}$(B) ⊆f $^{-1}$(cl_θB).

(e) ⇒ (a). Let V be any open set of Y. By (e), we have $\delta P_S cl$f $^{-1}$(V) ⊆f $^{-1}$(cl_θV). By Proposition2.19 (b), we have $\delta P_S cl f^{-1}(V) \subseteq f^{-1}(cl(V))$.

Therefore, by Proposition3.17, f is weakly δP_S-continuous.

Proposition3.20. A function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous if and only if $\delta P_S cl f^{-1}(intcl(B)) \subseteq f^{-1}(cl(B))$ for each subset B of Y

Proof. Necessity. Let B be any subset of Y. Assume that x $\notin$ f $^{-1}(cl_\theta$B). Then f (x) $\notin cl_\theta$B and hence there exists an open set H containing f (x) such that B $\cap cl$H = $\emptyset$. This implies that cl_θB $\cap$ H = $\emptyset$ and so H $\subseteq$ Y\\$cl_\theta$B and hence clH $\subseteq$ cl(Y\\$cl_\theta$B). Since f is weakly δP_S-continuous, there exists a δP_S-open set U of X containing x such that f (U) $\subseteq$ clH $\subseteq$ cl(Y\\$cl_\theta$B) = Y\\Int$cl_\theta$B. This implies that f (U) $\cap int(cl_\theta$B) = $\emptyset$ and hence U $\cap$ f $^{-1}$(intcl_θB) = $\emptyset$. Then x $\notin \delta P_S cl$f $^{-1}$(intcl_θB). Therefore, $\delta P_S cl$f $^{-1}$(intcl_θB) $\subseteq$ f $^{-1}(cl_\theta$B).

Sufficiency. Let V be any open set of Y. Then by hypothesis and Proposition2.19(b), we have $\delta P_S cl$(f^{-1}(intclV))=$\delta P_S cl$f^{-1}(Int$int\big(cl_\theta(V)\big) \subseteq f^{-1}\big(cl_\theta(V)\big)$. Therefore, $\delta P_S cl(f^{-1}(intcl(V) \subseteq f^{-1}(cl(V))$. Hence by Proposition3.17(b), f is weakly δP_S-continuous.

From Proposition3.20, we obtain that:

Corollary 3.21. A function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous if and only if $f^{-1}(int_\theta(B)) \subseteq \delta P_S(int(f^{-1}(cl\big(int_\theta(B)\big))$ for each subset B of Y.

Proposition3.22. A function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous if and only if $f^{-1}(V) \subseteq \delta P_S(int(f^{-1}(cl(V))))$ for each open set V of Y.

Proof. Necessity. Let f be weakly δP_S-continuous and let V be any open set of Y. Then $V \subseteq int(cl(V))$. Therefore, by Proposition3.17(b), $f^{-1}(V) \subseteq f^{-1}(int(cl(V)) \subseteq \delta P_S int(f^{-1}\big(cl(V)\big))$. Hence f $^{-1}$(V) $\subset f^{-1} \subseteq \delta P_S(int(f^{-1}(cl(V))$.

Sufficiency. Let V be any regular open set of Y. Then V is an open set of Y. By hypothesis, we have $f^{-1}(V) \subseteq \delta P_S f^{-1}(cl(V))$. Therefore, by Proposition3.17(c), f is weakly δP_S-continuous.

From Proposition3.22, we obtain that:

Corollary 3.23. A function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous if and only if $\delta P_S cl(f^{-1}(int(F))) \subseteq f^{-1}(F)$ for each closed set F of Y.

Proposition3.24. A function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous if and only if $\delta P_S cl(f^{-1}(V)) \subseteq f^{-1}(\delta P_S\big(cl(V)\big))$ for each open set V of Y.

Proof. Necessity. Let V be any open set of Y. Since f is weakly δP_S-continuous, then by Proposition3.17(h), we have $\delta P_S cl(f^{-1}(V)) \subseteq f^{-1}(\delta P_S\big(cl(V)\big))$. Since V is an open set and hence V is a semi-open set. Therefore, by Proposition2.19(a), we obtain $\delta P_S(cl\big(f^{-1}(V)\big)) \subseteq f^{-1}(\delta P_S(cl(V))$.

Sufficiency. Let F be any closed set of Y. Then $int(F)$ is an open set in Y. By hypothesis, we have $\delta P_S(cl(f^{-1}\big(int(F)\big)) \subseteq f^{-1}(\delta P_S(cl\big(int(F)\big))$. Since $int(F)$ is a semi-open set, then by Proposition2.19(a), $\delta P_S(cl(f^{-1}\big(int(F)\big)) \subseteq f^{-1}(cl\big(int(F)\big))$. Therefore, by Proposition3.17(g), f is weakly δP_S-continuous.

From Proposition3.24, we obtain that:

Corollary 3.25. A function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous if and only if f $^{-1}(\delta P_S$intF) $\subseteq \delta P_S int(f^{-1}(F))$ for each closed set F of Y.

Proposition3.26. If a function $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous, then $f:(X,\tau) \to (Y,\sigma_\theta)$ is δP_S-continuous.

Proof. Let H $\in \sigma_\theta$, then H is θ-open set in (Y,σ). Since $f:(X,\tau) \to (Y,\sigma)$ is weakly δP_S-continuous, then by

Proposition3.10, f^{-1}(H) is a δP_S-open set in X. Therefore,$f: (X, \tau) \to (Y, \sigma_\theta)$ is δP_S-continuous.

Proposition3.27. Let X be a locally indiscrete space. Then the function $f: (X, \tau) \to (Y, \sigma)$ is weakly δP_S-continuous if and only if $f: (X, \tau) \to (Y, \sigma_\theta)$is continuous.

Proof. Let H $\in \sigma_\theta$, then H is θ-open set in (Y,σ). Since $f: (X, \tau) \to (Y, \sigma)$is weakly δP_S-continuous, then by Proposition3.10, f^{-1}(H) is a δP_S-open set in X. Since X is a locally indiscrete space, then by Proposition2.14, f^{-1}(H) is open set in X. Therefore, $f: (X, \tau) \to (Y, \sigma_\theta)$is continuous.

Proposition3.28. Let $f: (X, \tau) \to (Y, \sigma)$be a function. Let $\mathcal{B}$ be any basis for τ_θ in Y. If f is weakly δP_S-continuous, then for each$B \in \mathcal{B}$, f^{-1}(B) is a δP_S-open set of X.

Proof. Suppose that f is weakly δP_S-continuous. Since each $B \in \mathcal{B}$is a θ-open subset of Y, therefore, by Proposition3.10, f^{-1}(B) is a δP_S-open subset of X.

4. PROPERTIES AND COMPARISONS

In this section, we give some properties of weakly δP_S-continuous functions and we compare them with other types of continuous functions.

Proposition4.1. Let $f: (X, \tau) \to (Y, \sigma)$ be weakly δP_S-continuous function. If A is a regular semi-open subset of X, then the restriction$f|A: A \to Y$ is weakly δP_S-continuous in the subspace A.

Proof. Let x ∈ A and V be an open set of Y containing f(x). Since f is weakly δP_S-continuous, there exists a δP_S-open set U of X containing x such that f(U) ⊆ clV. Since A is a regular semi-open subset of X, by Corollary 2.18, A ∩ U is a δP_S-open subset of A containing x and (f|A)(A ∩ U) = f(A ∩ U) ⊆ f(U) ⊆ clV. This show that f|A is weakly δP_S-continuous.

Corollary 4.2. Let $f: (X, \tau) \to (Y, \sigma)$ be a weakly δP_S-continuous function. If A is a regular open subset of X, then the restriction f|A:A→Y is weakly δP_S-continuous in the subspace A.

Proof. Since every regular open set is regular semi-open, this is an immediate consequence of Proposition4.1.

Proposition4.3. Let $f: (X, \tau) \to (Y, \sigma)$ be a function. If for each x ∈ X, there exists a regular open set A of X containing x such that the restriction f|A:A→Y is weakly δP_S-continuous, then f is weakly δP_S-continuous.

Proof. Let x ∈ X, then by hypothesis, there exists a regular open set A containing x such that f|A:A→Y is weakly δP_S-continuous. Let V be any open set of Y containing f(x), there exists a δP_S-open set U in A containing x such that (f|A)(U) ⊆ clV. Since A is regular open set, by Proposition 2.17, U is δP_S-open set in X and hence f(U) ⊆ clV. This shows that f is weakly δP_Scontinuous.

As an immediate consequence of Corollary 4.2 and Proposition4.3, we obtain that:

Corollary 4.4. Let {Uα : α ∈ Δ} be a regular open cover of a topological space X. A function $f: (X, \tau) \to (Y, \sigma)$is weakly δP_S-continuous if and only if the restriction f|Uα :Uα →Y is weakly δP_Scontinuous for each α ∈ Δ.

Remark 4.5. If $f: (X, \tau) \to (Y, \sigma)$ is a weakly δP_Scontinuous function and A, B are any subsets of X. Then the restriction f|A:A→f(A) need not be weakly δP_S-continuous in general. Moreover, f|(A ∪ B): A∪B→f(A ∪ B) is not always weakly δP_S-continuous even if f|A:A→f(A), f|B:B→f(B) and f are all weakly δP_S-continuous.

Proposition4.6. If X = R ∪ S, where R and S are regular open sets and$f: (X, \tau) \to (Y, \sigma)$ is a function such that both f|R and f|S are weakly δP_S-continuous, then f is weakly δP_S-continuous.

Proof. Let x ∈ X and V be an open set of Y containing f (x). Since f|R and f|S are weakly δP_S-continuous, there exist δP_S-open sets U of R and W of S with x ∈ U and x ∈ W, such that (f|R)(U) ⊆ clV and (f|S)(W) ⊆ clV.

Then f(U ∪ W) = (f|R)(U) ∪ (f|S)(W) ⊆ clV. Since R and S are regular open sets in X, then by Proposition 2.17, U and W are δP_S-open sets in X. Since union of two δP_S-open sets isδP_S-open, then U ∪ W is a δP_S-open set of X containing x. Therefore, f is weakly δP_S-continuous. In general, if X = ∪{Kα : α ∈ Δ}, where each Kα is a regular open set and $f:(X,\tau) \to (Y,\sigma)$ is a function such that the restriction f|Kα is weakly δP_S-continuous for each α, then f is weakly δP_S-continuous.

Proposition4.7. Let X = $R_1 \cup R_2$, where R_1 and R_2 are regular open sets in X. Let f:$R_1 \to$Y and g:$R_2 \to$Y be weakly δP_S-continuous. If f(x) = g(x) for each x ∈ $R_1 \cap R_2$, then h:$R_1 \cup R_2 \to$Y such that

$$h(x) = \begin{cases} f(x) & if\ x \in R_1\ and\ x \notin R_2 \\ g(x) & if\ x \in R_1 and\ x \notin R_2 \\ f(x) = g(x) & if\ x \in R_1 \cap R_2 \end{cases}$$

is weakly δP_S-continuous.

Proof. Let x ∈ X and V be an open set of Ycontaining *h* (x). Then x ∈ $R_1 \cup R_2$ and V is anopensetofYcontaining*f*(x)and*g*(x).Since*f*is weakly δP_S-continuous, there exists a δP_S-openset U of X containing x such that *f* (U) ⊆ clV.Then$f^{-1}(cl$V)isaδP_S-opensetofR$_1$containingx. But R_1 is a regular open set in X, then byProposition 2.17, $f^{-1}(cl$V) is a δP_S-open set of Xcontaining x. Similarly, $f^{-1}(cl$V) is a δP_S-open setin R_2 and hence, a δP_S-open set in X. Since unionoftwoδP_S-opensetsisδP_S-open.Therefore,$h^{-1}(cl$V) = $f^{-1}(cl$V) ∪$g^{-1}(cl$V) is a δP_S-open setin X and it is clear that$h$ ($h^{-1}(cl$V)) ⊆ clV.Hence$h$ is weaklyδP_S-continuous.

Proposition4.8. Let$f:(X,\tau) \to (Y,\sigma)$beweakly$\delta P_S$-continuous surjection and A be aregular semi-open subset of X. If *f* is an openfunction, then the function g:A→f (A), definedby g (x) = f (x) for each x ∈ A, is weakly δP_S-continuous.

Proof. PuttingH=*f*(A).Letx∈AandV be any open set in H containing g (x). Since H isopeninYandVisopeninH,thenVisopeninY. Since *f* is weakly δP_S-continuous, there exists aδP_S-open set U in X containing x such that f (U) ⊆ clV. Taking W = U ∩ A, since A is either openoraregularsemi-opensubsetofX,thenbyCorollary 2.18, W is a δP_S-open set in A containingx and g (N) ⊆ cl_YV ∩ H = cl_HV. Then g (W) ⊆ cl_HV.Thisshowsthat$g$isweaklyδP_S-continuous.

Proposition4.9. Let$f:(X,\tau) \to (Y,\sigma)$beaweakly$\delta P_S$-continuous function and for each x ∈ X. If Y isany subset of Z containing *f* (x), then $f:(X,\tau) \to (Z,\eta)$isweaklyδP_S-continuous.

Proof. Let x ∈ X and V be any open set of Zcontaining*f*(x).ThenV∩YisopeninYcontaining*f*(x).Since$f:(X,\tau) \to (Y,\sigma)$isweakly$\delta P_S$-continuous, there exists a δP_S-open set U of Xcontaining x such that $f(U) \subseteq cl(V \cap Y)$ andhence $f(U) \subseteq cl(V)$. Therefore, $f:(X,\tau) \to (Z,\eta)$is weaklyδP_S-continuous.

We shall obtain some conditions forwhichthe composition of two functions is weakly δP_S-continuous:

Proposition4.10. Let$f:(X,\tau) \to (Y,\sigma)andg:(Y,\sigma) \to (Z,\eta)$befunctions.Thenthecompositionfunction$g \circ f:(X,\tau) \to (Z,\eta)$isweakly$\delta P_S$-continuousif*f*and*g*satisfyoneofthefollowingconditions:

a) *f*isδP_S-continuous and*g*is weaklycontinuous.
b) *f*isweaklyδP_S-continuousand*g*isalmoststronglyθ-continuous.
c) *f*isweaklyδP_S-continuousand*g*isθ-continuous.
d) *f*isweaklyδP_S-continuousand*g* iscontinuous.
e) *f*iscontinuousandopenand*g*isweaklyδP_S-continuous.

Proof. a) Let x ∈ X and W be an open set of

Zcontaining$g$(f(x)).Since$g$isweaklycontinuous,thereexistsanopensetVofYcontaining f (x) such that $g(V) \subseteq cl(W)$(i.e.,) $f(x) \in V \subseteq g^{-1}(cl(W))$. Hence$g^{-1}(cl(W))$ is open in Y containing f (x). Since f isweakly δP_S-continuous, there exists a δP_S-open setU of X containing x such that $f(U) \subseteq g^{-1}(cl(W))$, from Definition 2.6. Therefore, we obtain (gof)(U) = g(f (U)) ⊆ clW.Hence$g \circ f$is weaklyδP_S-continuous.

b) LetWbeanyregularopensubsetofZ. Since g is almost strongly θ-continuous from the Definition 2.5, g^{-1}(W)is θ-opensubsetofY.Since$f$isweaklyδP_S-continuous, then by Proposition3.10, $(g \circ f)^{-1}$(W) =f^{-1}(g^{-1}(W)) is a δP_S-open subset in X. Therefore,$g \circ f$isalmostδP_S-continuous, [byProposition 3.10(e) of 24]andhenceitisweaklyδP_S-continuous, by Proposition 3.2(a).

c) Let$x \in X$,andWbeanopensetofZcontaining g(f(x)). Since g is θ-continuous, there exists an open set V of Y containing f (x) suchthat$g(cl(V)) \subseteq cl(W)$, by definition 2.7.Since$f$isweaklyδP_S-continuous, there exists a δP_S-open set U of Xcontaining x such that$f(U) \subseteq cl(V)$. Hence$g(f(U)) \subseteq g(cl(V)) \subseteq cl(W)$). Therefore, $g \circ f$is weaklyδP_S-continuous.

d) Letx∈XandWbeanopensetofZcontaining g(f(x)). Since g is continuous, g^{-1}(W)is an open set of Y containing f (x). Since f isweakly δP_S-continuous, there exists a δP_S-open setU of X containing x such that f (U) ⊆ clg^{-1}(W).Also, since g is continuous, then we have f (U) ⊆g^{-1}(clW).Thisimpliesthat$g$(f(U))⊆ clW.Therefore,go$f$is weaklyδP_S-continuous.

e) Letx∈XandWbeanopensetofZcontaining$g$(f(x)).Since$g$isweaklyδP_S-continuous, there exists a δP_S-open set U of Ycontaining f (x) such that g (U) ⊆ clW. It is clearthat g^{-1}(clW) is a δP_S-open set of Y containing f(x).Since$f$iscontinuousandopen,thenbyProposition2.20, f^{-1}(g^{-1}(clW)) = $(g \circ f)^{-1}$(clW) is a δP_S-open set in X containingxandclearly$(g \circ f)$($(g \circ f)^{-1}$(clW)) ⊂clW. Hence f is weaklyδP_S-continuous.

Proposition4.11. If$f: (X, \tau) \to (Y, \sigma)$isaweakly$\delta P_S$-continuousfunctionandYisalmostregular,then $f$is almostδP_S-continuous.

Proof. Let x ∈ X and let V be any open set of Ycontaining f (x). By the almost regularity of Y,thereexistsaregularopensetGofYsuchthat $f(x) \in G \subseteq clG \subseteq int(cl\, V)$ [18, Proposition 2.2].Since $f$is weakly δP_S-continuous, there exists aδP_S-opensetUofXcontainingxsuchthat$f(U) \subseteq cl(G) \subseteq int(cl\, V)$. Therefore,$f$isalmost$\delta P_S$-continuous, from Definition 2.4.

Proposition4.12. If$f: (X, \tau) \to (Y, \sigma)$isaweakly$\delta P_S$-continuousfunctionandYisanextremallydisconnectedspace,then$f$isalmostδP_S-continuous.

Proof. Let$x \in X$ and let V be any open set of Ycontaining f (x). Since f is weakly δP_S-continuous,there exists a δP_S-open set U of X containing xsuch that f (U) ⊆ clV. Since Y is extremallydisconnected,from Definition 2.11(a) $cl(V)$ is open, (i.e.,)$cl(V) = int(cl(V))$, then$f(U) \subseteq int(cl(V))$. Therefore,$f$isalmost$\delta P_S$-continuous.

Corollary 4.13. A function $f: (X, \tau) \to (Y, \sigma)$is almost δP_S-continuous if and only if f is weakly δP_S-continuous and itsatisfies oneofthe followingproperties:

a) Yis almostregular.
b) Y isextremallydisconnected.

Proof. The proof follows from Proposition 3.2(a). The converse is proved in Proposition4.11 andProposition4.12.

Corollary4.14. Let$f: (X, \tau) \to (Y, \sigma)$beafunctionandXisalocallyindiscretespace.Then$f$isweakly δP_S-

continuousifandonlyif$f$isweaklycontinuous.

Proof. Follows fromProposition2.14.

Corollary 4.15. If X is a locally indiscrete spaceand Y is either almost regular or an extremallydisconnected space, the following statements areequivalentfor afunction$f:(X,\tau)\to(Y,\sigma)$:

a) f is almost δP_S-continuous.
b) f is weaklyδP_S-continuous.
c) $f$isweaklycontinuous.
d) $f$isalmostcontinuous.

Proof.(a) $\Rightarrow(b)$Followsfrom Proposition 3.2

(b) $\Rightarrow(c)$Follows fromCorollary 4.14

(c) $\Rightarrow(d)$Follows from Corollary 4.12

(d) $\Rightarrow(a)$ Since X is locally indiscrete, $\delta P_S O(X)=\tau$. Hence almost continuous function is a almost δP_S-continuous function, from Proposition 2.21.

Corollary4.16. IfYisaregularspace,thefollowingstatementsareequivalentforafunction $f:(X,\tau)\to(Y,\sigma)$:

a) $f$is δP_S-continuous.
b) f is almost δP_S-continuous.
c) f is weaklyδP_S-continuous.

Proof. FollowsfromProposition4.11andProposition2.23andthefactthateveryregular space is almostregularand semi-regularspace.

Corollary4.17. IfXisalocally indiscretespace andYisaregularspace,thefollowingstatementsareequivalentforafunction$f:(X,\tau)\to(Y,\sigma)$:

a) $f$is δP_S-continuous.
b) $f$is almost δP_S-continuous.
c) $f$is weaklyδP_S-continuous.
d) $f$isweaklycontinuous.
e) $f$isalmostcontinuous.
f) $f$iscontinuous.

Proof. FollowsfromCorollary4.15,Corollary 4.16 and Proposition2.25 and the fact that everyregular space is almost regular and semi-regularspace.

Proposition4.18. Let$f:(X,\tau)\to(Y,\sigma)$be a function and Xisasemi-T_1space.Then$f$isweaklyδP_S-continuousifandonlyif$f$isweaklyδ-precontinuous.

Proof. FollowsfromProposition2.13.

Corollary 4.19. If X is a semi-T_1 space and Y iseitheralmostregularoranextremallydisconnected space, the following statements areequivalentfor afunction$f:(X,\tau)\to(Y,\sigma)$:

a) $f$is almost δP_S-continuous.
b) $f$is weaklyδP_S-continuous.
c) $f$isweaklyδ-precontinuous.
d) $f$isalmostδ-precontinuous.

Proof. FollowsfromCorollary4.13,Proposition4.18 andProposition2.22.

Corollary 4.20. If X is a semi-T_1 space and Y isaregularspace,thefollowingstatementsareequivalentfor afunction$f:(X,\tau)\to(Y,\sigma)$:

a) f is δP_S-continuous.
b) f isalmostδP_S-continuous.
c) f is weaklyδP_S-continuous.
d) f isweaklyδ-precontinuous.
e) f isalmostδ-precontinuous.
f) f isδ-precontinuous.

Proof. FollowsfromCorollary4.16,Corollary 4.19 and Proposition2.24 and the fact that everyregular space is almost regular and semi-regularspace.

Proposition4.21. If $f:(X,\tau)\to(Y,\sigma)$is a semi-continuousfunction. Then$f$is weakly continuous if and onlyif*f*is weaklyδP_S-continuous.

Proof. Necessity. Let V be any open set of Y.Since f is weakly continuous, by Proposition2.26,clf^{-1}(V) $\subseteq f^{-1}(cl$V). Since f is semi-continuous,then f^{-1}(V) is a semi-open set in X. Hence byProposition2.19(a),$\delta P_S clf^{-1}$(V)=clf^{-1}(V).Therefore,weobtain$\delta P_S clf^{-1}$(V)$\subseteq f^{-1}(cl$V).ThusbyProposition3.17(h),f isweaklyδP_S-continuous.

Sufficiency. Let V be any open set in Y. Since *f*isweaklyδP_S-continuous,byProposition3.17(h),$\delta P_S clf^{-1}$(V)$\subseteq f^{-1}(cl$V).Since*f*issemi-continuous, then f^{-1}(V) is semi-open set of X.HencebyProposition2.19(a),wehave$\delta P_S clf^{-1}$(V)=clf^{-1}(V).Therefore,weobtainclf^{-1}(V)$\subseteq f^{-1}(cl$V).ThusbyProposition2.26,*f*isweaklycontinuous.

Corollary 4.22.A function$f:(X,\tau)\to(Y,\sigma)$isweaklyδP_S-continuousifandonlyif*f*isweakly continuousifitsatisfiesoneofthe following properties:

a) X islocallyindiscretespace.
b) *f*is semi-continuous.

Proof. FollowsfromCorollary4.14andProposition4.21.

Proposition4.23. If$f:(X,\tau)\to(Y,\sigma)$isweaklyθs-continuous and weakly δ-precontinuous, then f isweaklyδP_S-continuous.

Proof. Let x ∈ X and let V be any open set of Ycontaining f (x). Since f is weakly θs-continuousand weakly δ-pre-continuous, then there exists a θ-semi-open and a δ-preopen set U of X containing xsuch that*f*(U)⊆ clV, respectively. HencebyLemma 2.15, U is a δP_S-open set of X containing xsuch that f (U) ⊆ clV. Therefore, f is weakly δP_S-continuous.

Proposition4.24. Let $f:(X,\tau)\to(Y,\sigma)$be a function and Xbeanextremallydisconnectedspace.If*f*isweaklyθs-continuous,then*f*isweaklyδP_S-continuous.

Proof. FollowsfromLemma2.16.

Proposition4.25. If$f:(X,\tau)\to(Y,\sigma)$isweaklyδ-precontinuous and eitherS-continuous or aθ-irresolutefunction,then*f*isweaklyδP_S-continuous.

Proof. Let x ∈ X and V be any open set of Ycontaining f (x). Since f is weakly δ-precontinuous,there exists a δ-preopen set U of Y containing f (x)suchthat*f*(U)⊆ clV.Then$f^{-1}(cl$V)isaδ-preopen set of Y containing x. Since clV is aregularclosedsetofYand*f*iseitherS-continuous or θ-irresolute, then $f^{-1}(cl$V) is theunionofregularclosedsetsofXandhenceisthe union of semi-closed sets of X. By Lemma2.12, $f^{-1}(cl$V) is a δP_S-

open set of X containing xand clearly $f(f^{-1}(cl\text{V})) \subseteq cl\text{V}$. Hence f is weaklyδP_S-continuous.

Corollary4.26. Let$f: (X,\tau) \to (Y,\sigma)$beeitherS-continuous or a θ-irresolute function. Then $f$isweakly δP_S-continuous if and only if f is weaklyδ-precontinuous.

Proposition4.27. If a function $f: (X,\tau) \to (Y,\sigma)$is weakly$\delta P_S$-continuousandopen,then$f(\delta P_S cl\text{V}) \subseteq \delta P_S cl f(V)$ for eachopen setV of X.

Proof. Let V be any open set of X. Since f isopen, then f (V) is an open set in Y. Since f isweaklyδP_S-continuous,thenbyProposition3.24,weobtainthat$\delta P_S cl f^{-1}(f(\text{V})) \subseteq f^{-1}(\delta P_S cl f(\text{V}))$whichimplies that$f(\delta P_S cl\text{V}) \subseteq \delta P_S cl f(\text{V})$.

Corollary 4.28. If a function $f: (X,\tau) \to (Y,\sigma)$is weakly δP_S-continuous and open, then $\delta P_S int f(F) \subseteq f(\delta P_S int(F))$for each closed set F of X.

Proposition4.29. Ifafunction$f: (X,\tau) \to (Y,\sigma)$issemi-continuous and almost open, then$f$is weakly δP_S-continuous if and only if $\delta P_S cl f^{-1}(\text{V}) = f^{-1}(\delta P_S cl\text{V})$for each open setVof Y.

Proof. Necessity**.** Let V be any open set of Y.Since$f$isweaklyδP_S-continuous,thenbyProposition3.24, $\delta P_S cl f^{-1}(\text{V}) \subseteq f^{-1}(\delta P_S cl\text{V})$. Since Visopen,henceitissemi-open.ThenbyProposition2.19(a),$\delta P_S cl(V) = cl(V)$whichimpliesthat$\delta P_S cl(V) \subseteq cl(V)$ and hence $f^{-1}(\delta P_S cl\text{V}) \subseteq f^{-1}(cl\text{V})$. Since V is an open set of Y and f is almost open, then byProposition2.27, $f^{-1}(cl\text{V}) \subseteq cl f^{-1}(\text{V})$. Therefore,we have $f^{-1}(\delta P_S cl\text{V}) \subseteq f^{-1}(cl\text{V}) \subseteq f^{-1}(\text{V})$ andhence $f^{-1}(\delta P_S cl\text{V}) \subseteq cl f^{-1}(\text{V})$. Since V is an openset of Y and f is semi-continuous, then $f^{-1}(\text{V})$ is a semi-open set in X. Then by Proposition2.19(a)weobtainthat$f^{-1}(\delta P_S cl\text{V}) \subseteq \delta P_S cl f^{-1}(\text{V})$.Therefore,we have $\delta P_S cl f^{-1}(\text{V}) = f^{-1}(\delta P_S cl\text{V})$.

Sufficiency**.** Follows from Proposition3.24.

Corollary4.30. Ifafunction$f: (X,\tau) \to (Y,\sigma)$isweakly$\delta P_S$-continuous,semi-continuous and almost open,then$\delta P_S int f^{-1}(\text{F}) = f^{-1}(\delta P_S int\text{F})$foreach closed setF of Y.

Proof. Followsfrom Proposition4.29.

Corollary 4.31. If a function $f: (X,\tau) \to (Y,\sigma)$is weakly$\delta P_S$-continuous,semi-continuousand almost open, then $\text{cl} f^{-1}(\text{V}) = f^{-1}(\text{clV})$ for each open setVof Y.

Proof. FollowsfromProposition4.29 and Proposition 2.19(a).

Corollary 4.32. Let $f: (X,\tau) \to (Y,\sigma)$be an almost openfunction. If f is weakly δP_S-continuous and semi-continuous,then$f$isalmostcontinuousandhence $f$is weaklycontinuous.

Proof. FollowsfromProposition4.31 and Proposition2.28.

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