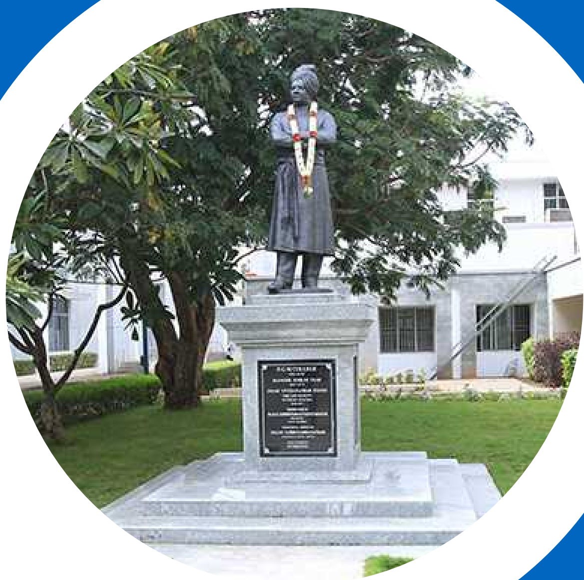
VOLUME X
ISBN No.: 978-81-953602-6-0
Physical Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,
Pollachi-642001



SUPPORTED BY

ABT Industries Ltd.,
Dairy Division

PROCEEDING

One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, Pollachi-642001.

Proceeding of the

One day International Conference on

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

ISBN No: 978-81-953602-6-0

978- 81- 953602- 6- 0

Nallamuthu Gounder Mahalingam College

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

www.ngmc.org

ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

EDITORIAL BOARD

Novel approach to Generate Topologies by using $(\alpha, \beta, \gamma) -$ Cuts Of Neutrosophic Sets

[1]E. Kungumaraj and [2]R.Santhi

[1]Assistant Professor, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore
kungum99522@gmail.com

[2]Assistant Professor Department of Mathematics, Nallamuthu Goundar Mahalingam College, Pollachi
santhifuzzy@yahoo.co.in

Abstract:

Neutrosophic sets play important role to develop the novel ideas in Mathematics particularly in Topological spaces. Neutrosophic topology provides the new class of closed sets to develop the theoretical concepts in Topological spaces.As a continuous development of Neutrosophic approach in Topological Spaces, in this paper we have introduced a novel method to generate the Topologies using the (α, β, γ) - cuts of Neutrosophic sets, which is the extension work of the topologies generated by fuzzy numbers and intuitionistic fuzzy numbers.

Keywords: Fuzzy Numbers, Intuitionistic Fuzzy Numbers, Neutrosophic sets,
(α, β, γ) – cuts of Neutrosophic sets.

1. Introduction:

Neutrosophic set is the generalization of classical set, fuzzy set(1965), intuitionistic fuzzy set (1986) which consists of membership values, non-membership values, indeterminant values of a set. The concept of "Neutrosophic set" was first given by F. Smarandache (2005). Bera and Mahapatra introduced the (α, β, γ) – cuts of Neutrosophic sets. Initially, topologies genetaed by the open sets. Then the topologies generate by the basis and subbasis. It is very easy when the number of elements are less. To overcome the difficulty of these methods, in this paper a novel approach to generate topologies using Neutrosophic numbers and Neutrosophic Basis. This novel approach is based on topologies generated by fuzzy numbers introduced by Padmapriya in 2010 and Topologies generated by Intuitionistic fuzzy numbers by Santhi and Kungumaraj, 2020.

Dr.E.Kungumaraj, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore; kungum99522@gmail.com
Dr.R.Santhi, Department of Mathematics, NGM College, Pollachi, santhifuzzy@yahoo.co.in

2. Preliminaries:

Definition 2.1 Fuzzy Set: Let X be a nonempty set. A fuzzy set $\bar{A}$ of X is defined as $\bar{A} = \{(x, \mu_{\bar{A}}(x)); x \in X\}$ where $\mu_{\bar{A}}(x)$ is called the membership function which maps each element of X to a value between 0 and 1.

Definition 2.2 Fuzzy Number: A fuzzy number $\bar{A}$ is a convex normalized fuzzy set on the real line ℝ such that:

(i) There exist at least one $x \in \mathbb{R}$ with $\mu_{\bar{A}}(x) = 1$;

(ii) $\mu_{\bar{A}}(x)$is piecewise continuous.

Definition 2.4. α- cut of fuzzy number

The α- cut of a fuzzy number A(x) is defined as A(α) = {x/μ(x) ≥ A(x), α ∈ [0, 1]}.

Definition2. 5 Intuitionistic Fuzzy Set :

Let X be a nonempty set. An intuitionistic fuzzy set $\bar{A}^I$ of X is defined as $\bar{A} = \{(x, \mu_{\bar{A}^I}(x), \gamma_{\bar{A}^I}(x)); x \in X\}$ where $\mu_{\bar{A}^I}(x)$ and $\gamma_{\bar{A}^I}(x)$ are membership and nonmembership functions such that $\mu_{\bar{A}^I}(x)$, $\gamma_{\bar{A}^I}(x)$: X → [0, 1] and $0 \leq \mu_{\bar{A}^I}(x) + \gamma_{\bar{A}^I}(x) \leq 1$ for all x ∈ X.

Definition 2. 6 Intuitionistic Fuzzy Number:

An intuitionistic fuzzy subset $\bar{A}^I = \{(x, \mu_{\bar{A}^I}(x), \gamma_{\bar{A}^I}(x)); x \in A\}$ of the real line ℝis called an intuitionistic fuzzy number (IFN) if the following conditions hold:

(i) There exists x∈ℝsuch that $\mu_{\bar{A}^I}(x) = 1$ and $\gamma_{\bar{A}^I}(x) = 0$.

(ii) $\mu_{\bar{A}^I}(x)$is a continuous function from ℝ→ [0, 1] such that $0 \leq \mu_{\bar{A}^I}(x) + \gamma_{\bar{A}^I}(x) \leq 1$for all $x \in X$.

Definition2.7 (α-cut of a fuzzy number): The α-cut of A denoted by A^{α} is the crisp set $A^{\alpha} = (x \in X; A(x) \geq \alpha)$ and the strong α-cut of A is denoted by $A^{\alpha+}$ is the crisp set $A^{\alpha+} = x \in X; A(x) > \alpha)$.

Definition2.8(Topologies generated by the fuzzy subsets): If Ais a fuzzy subset of X then the topology generated by the α – cut of A is called the topology generated by the fuzzy subset A.

Dr.E.Kungumaraj, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore; kungum99522@gmail.com
Dr.R.Santhi, Department of Mathematics, NGM College, Pollachi, santhifuzzy@yahoo.co.in

Definition2.9 ((α, β) – Cut of Intuitionistic Fuzzy Subset): If A_I is an intuitionistic fuzzy number. A set of (α, β) – cut of A_I of X, is defined by$A_{\alpha,\beta} = \{x, \mu_{A^I}(x), \vartheta_{A^I}(x): x \in X;\ \mu_{A^I}(x) \geq \alpha, \vartheta_{A^I}(x) \leq \beta, \alpha, \beta \in [0,1]$ where α, β ∈ [0,1] are the fixed numbers and α +β ≤ 1.

Definition 2.10Topology Generated by (α ,β) – cut of Intuitionistic Fuzzy Numbers:

Let A is an Intuitionistic fuzzy subset of X. If τ be the collection of opens sets of (α, β)-cut of elements of A and it is a topology on X, then τ is called topology generated by Intuitionistic fuzzy subset A of X.

Definition2.11 Neutrosophic Set: Let X be a universe set. A neutrosophic set A on X is defined as $A = \{(T_A(x), I_A(x), F_A(x)): x \in X\}$, where $T_A(x), I_A(x), F_A(x): X \to ^{-}]0,1[^{+}$ represents the degree of membership, degree of indeterministic and degree of non-membership respectively of the element of $x \in X$, such that $0 \leq T_A(x), I_A(x), F_A(x) \leq 3$.

Definition 2.12 (α, β,γ)-cut of a Neutrosophic number: The Aof Neutrosophic set N is denoted by $N_{(\alpha,\beta,\gamma)}$, where $\alpha, \beta, \gamma \in [0,1]$ and are fixed numbers, such that $\alpha + \beta + \gamma \leq 3$ is defined as $N_{(\alpha,\beta,\gamma)} = \{(T_A(x), I_A(x), F_A(x)): x \in X, T_A(x) \geq \alpha, I_A(x) \leq \beta, F_A(x) \leq \gamma\}$.

Definition 2.13 Strong (α, β, γ)-cut of a Neutrosophic number: The Strong $(\alpha, \beta, \gamma)^{+} - cut$ of neutrosophic set N is denoted by $N_{(\alpha,\beta,\gamma)^{+}}$, where $\alpha, \beta, \gamma \in [0,1]$ and are fixed numbers, such that $\alpha + \beta + \gamma \leq 3$ is defined as $N_{(\alpha,\beta,\gamma)^{+}} = \{(T_A(x), I_A(x), F_A(x)): x \in X, T_A(x) > \alpha, I_A(x) < \beta, F_A(x) < \gamma\}$.

Definition 2.14 Crisp Basis: Let X be non-empty set. B ⊂ P(X) is called a base if

(i) ∪ {B / B ∈ $\mathcal{B}$ } = X

(ii) U, V ∈ $\mathcal{B}$ and x∈ U ∩ V implies there exist W ∈ $\mathcal{B}$ such that x ∈ W ⊂ U ∩ V.

Let τ be the collection of all union of finite number of elements of $\mathcal{B}$. Then τ is a topology and $\mathcal{B}$ is a base for the topology.

3. Topology Generated by(α , β ,γ) $- cut$ Neutrosophic Sets:

Definition 3.1: Let A is a Neutrosophic subset of X. If τ be the collection of opens sets of (α , β ,γ) $- cut$ of elements of A and it is a topology on X, then τ is called the topology generated by the Neutrosophic Subset A of X.

Dr.E.Kungumaraj, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore; kungum99522@gmail.com
Dr.R.Santhi, Department of Mathematics, NGM College, Pollachi, santhifuzzy@yahoo.co.in

Example 3.2: Let X = (a, b, c) and A = $\left\{\frac{0.1}{a}+\frac{0.2}{b}+\frac{0.7}{c};\frac{0.8}{a'}+\frac{0.7}{b'}+\frac{0.2}{c'};\frac{0.3}{a''}+\frac{0.4}{b''}+\frac{0.6}{c''}\right\}$.

Then $A^{(\alpha,\beta,\gamma)}$ is the whole set for α =0, β =1, γ =1, and is empty set for α =1, β =0, γ =0.

Also $A^{(\alpha,\beta,\gamma)} = X$ for 0<α ≤ 0.1, 0.8 < β ≤ 1, 0.8 < γ ≤ 1, $A^{(\alpha,\beta,\gamma)} = \{c\}, for\ 0.2 < \alpha \leq 0.7;$ $0.2 < \beta \leq 0.7;\ 0.2 < \gamma \leq 0.7, A^{(\alpha,\beta,\gamma)} = \phi\ for\ 0.7 < \alpha \leq 1, 0.2 < \beta \leq 0$. Then $\tau_N(A)$ = {X, {b, c}, {c}, ϕ }. Clearly $\tau_N(A)$ is topology on X.

Example 3.3: : Let X = (p, q, r) and A = $\left\{\frac{0.8}{p}+\frac{0.9}{q}+\frac{1}{r};\frac{0.2}{p'}+\frac{0.1}{q'}+\frac{0}{r'};\frac{0.2}{p''}+\frac{0.1}{q''}+\frac{0}{r''}\right\}$.

Then$\tau_N(A)$ = {X, {r, s}, {s}, ϕ } when α =0.9, β =0.3, γ =0.4. Clearly $\tau_N(A)$ is not topology on X.

Theorem 3.4: If A = (a, a', a'') is a neutrosophic subset of X = {x} then $\tau_A = \{X, \phi\}$.

Proof: Let X = {x} and A = $\frac{a}{x};\frac{a'}{x}+\frac{a''}{x}; 0 \leq a \leq 1$. Then $\tau_N(A) = X$.

Topology generated by $\tau_N(A)$ = Topology generated by {X} = {X, ϕ}. Therefore $\tau_N(A) = \{X, \phi\}$.

Theorem 3.5: If A = $(a, a', a''; b, b', b'')$ is a Neutrosophic subset of X ={x, y} then $\tau_N(A) = \{X, \phi\}$ or $\tau_N(A) = \{X, \phi, \lambda\}$ where $\lambda \in \{\{x\}, \{y\}\}$.

Proof: Let X ={x, y}. There are four topologies on X

They are given by $\tau_{N_1}(A) = \{X, \phi\}$, $\tau_{N_2}(A) = \{X, \phi, \{x\}\}$, $\tau_{N_3}(A) = \{X, \phi, \{y\}\}$, $\tau_{N_4}(A) = \{X, \phi, \{x\}, \{y\}\}$.

Case 1: If $A = \frac{a}{x};\frac{a'}{x};\frac{a''}{x}$ where $0 \leq a, a', a'' \leq 1$ then $\tau_N(A) = X$

Topology generated by $\tau_N(A)$ = Topology generated by {X} = {X, ϕ} = τ_{N_1}

Case 2: If $A = \frac{a}{x}+\frac{b}{y};\frac{a'}{x}+\frac{b'}{y};\frac{a''}{x}+\frac{b''}{y}$ where $0 \leq a'' \leq a' \leq a < b \leq b' \leq b'' \leq 1$ then $\tau_N(A) = \{X, \{x\}\}$

Dr.E.Kungumaraj, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore; kungum99522@gmail.com
Dr.R.Santhi, Department of Mathematics, NGM College, Pollachi, santhifuzzy@yahoo.co.in

Topology generated by $\tau_N(A)$ = Topology generated by $\{X, \{x\}\}$ = {X,{x}, ϕ} = τ_{N_2}

Case 3: If A = $A = \frac{a}{x} + \frac{b}{y}; \frac{a'}{x} + \frac{b'}{y}; \frac{a''}{x} + \frac{b''}{y}$ where $0 \leq b'' \leq b' \leq b < a \leq a' \leq a'' \leq 1$ then

$\tau_N(A) = \{X, \{y\}\}$.

Topology generated by $\tau_N(A)$ = Topology generated by $\{X, \{x\}\}$ = {X,{y}, ϕ} = τ_{N_3}

Theorem 3.6: The discrete topology on {x, y} is not generated by the $(\alpha, \beta, \gamma) - cut$ of any Neutrosophic subset of {x, y}.

Proof: Let A be a Neutrosophic subset of {x.y}. Then $A = \left(\frac{a}{x} + \frac{b}{y}; \frac{a'}{x} + \frac{b'}{y}; \frac{a''}{x} + \frac{b''}{y}\right)$

If a = b then using Theorem 3.4, $\tau_N(A) = \tau_{N_1}$

If a < b then using Theorem 3.5, $\tau_N(A) = \tau_{N_2}$

If a > b then using Theorem 3.5 $\tau_N(A) = \tau_{N_3}$

4. Neutrosophic Basis in Topological Spaces:

Definition 4.1:Neutrosophic Basis

Let X be a nonempty set and B ⊂ P(X) . A function $f_\mu, f_\kappa, f_\vartheta: P(X) \to [0,1]$ is called Neutrosophic Basis if

(i) $\cup \{B \mid f_\mu(B) = 1, f_\vartheta(B) = 0, f_\gamma(B) = 0\} = X$.

(ii) For each $(\alpha, \beta, \gamma) \in (0, 1]$ with $\alpha + \beta + \gamma \leq 3 . f_\mu(U) \geq \alpha, f_\kappa(U) \leq \beta, f_\vartheta(U) \leq \gamma$ and $f_\mu(V) \geq \alpha, f_\kappa(V) \leq \beta, f_\vartheta(V) \leq \gamma$ and x ∈ U ∩ V implies there exist $f_\mu(W) \geq \alpha, f_\kappa(W) \leq \beta, f_\vartheta(W) \leq \gamma$ such that x ∈ W ⊂ U ∩ V.

If τ_N is the topology of X based on the basis B, then it is called the topology generated by the Neutrosophic Basis.

Dr.E.Kungumaraj, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore; kungum99522@gmail.com
Dr.R.Santhi, Department of Mathematics, NGM College, Pollachi, santhifuzzy@yahoo.co.in

Definition 4.2: Strong Neutrosophic Basis

Let X be a nonempty set. A function $f_\mu, f_\kappa, f_\vartheta : P(X) \to [0,1]$ is called Strong Neutrosophic Basis if

(i) $\cup \{B \mid f_\mu(B) = 1, f_\vartheta(B) = 0, f_\gamma(B) = 0\} = X.$

(ii) $f_\mu(U \cap V) \geq \min\{f_\mu(U), f_\mu(V)\}$, $f_\kappa(U \cup V) \leq \max\{f_\kappa(U), f_\kappa(V)\}$, $f_\vartheta(U \cup V) \leq \max\{f_\vartheta(U), f_\vartheta(V)\}$, for U, V ⊂ X with U ∩ V = ϕ.

If τ_N is the topology of X based on the basis B, then it is called the topology generated by the Neutrosophic Strong Basis.

Theorem 4.3: Every crisp basis induces a Neutrosophic basis.

Proof: Let X be a non-empty set. Let **B** be a crisp basis. Define $f: P(X) \to [0,1]$ as $f(A) = 1 if\ A \in B$ and $iff(A) = 0$ if A does not belong to **B**.

(i) ∪ {B | (B) = 1} = ∪ {B / B ∈ **B** } = X, by definition of crisp basis

(ii) Take α, β, γ ∈ (0,1] with $\alpha + \beta + \gamma \leq 3$. Let $f_\mu(U) \geq \alpha, f_\kappa(U) \leq \beta, f_\vartheta(U) \leq \gamma$, and $x \in U \cap V,\ f_\mu(V) \geq \alpha, f_\kappa(V) \leq \beta, f_\vartheta(V) \leq \gamma$ implies $f_\mu(U) = 1, f_\kappa(U) = 0, f_\vartheta(U) = 0$ and $f_\mu(V) = 1, f_\kappa(V) = 0, f_\vartheta(V) = 0$. This implies that U, V ∈**B.** Since B is a crisp basis ∃ W∈**B** , x ∈ W ⊂ U ∪ V. Since W∈**B,** $f(W) = 1$and hence $f_\mu(W) \geq \alpha$, $f_\kappa(U) = 0, f_\vartheta(U) = 0$. Hence f is Neutrosophic basis. Thus, every crisp basis is a Neutrosophic basis

Dr.E.Kungumaraj, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore; kungum99522@gmail.com
Dr.R.Santhi, Department of Mathematics, NGM College, Pollachi, santhifuzzy@yahoo.co.in

References:

1. Atanassov, K. (1986). Intuitionistic Fuzzy Sets, Fuzzy Sets and System, 20 (1), 87–96.
2. Bera. T and N. K. Mahapatra, (α, β, γ)-cut of neutrosophic soft set and it's application to neutrosophic soft groups, Asian Journal of Math. and Compt. Research, 12(3),160-178, 2016.
3. Florentin Smarandache, Neutrosophic Set – A Generalization of the Intuitionistic Fuzzy Set; in International Journal of Pure and Applied Mathematics, Vol. 24, No. 3, 287-297, 2005.
4. Padmapriya, R., & Thangavelu, P., Topologies Generated by the cuts of a Fuzzy set, International Journal of Mathematical Archive, 2, 1–10, 2014.
5. Padmapriya, R., & Thangavelu, P., Characterization of Nearly open sets using Fuzzy sets, Global Journal of pure and applied Mathematics, 11, 18–20, 2015.
6. Santhi. R and Kungumaraj. E, Topologies generated by intuitionistic fuzzy numbers, Notes on Intuitionistic Fuzzy Sets, ISSN 1310-4926, Vol.26, No.1, doi.10.7546/nifs.2020.26.1.36-45, 36 -45, 2020.
7. Sharma, P. K. (α, β)-Cut of Intuitionistic fuzzy modules, International Journal of Mathematical Sciences and Applications, 1 (3), 1489–1492, 2011.
8. Sumathi. I.R, Antony Crispin Sweety.C, New approach on Differential equations via trapezoidal neutrosophic number, Complex & Intelligent Systems, 5, doi.10.1007/s40747-019-00117-3,417-424, 2019.
9. Zadeh, L. Fuzzy sets, Journal of Information and Controls, 8, 338–353, 1965.

Dr.E.Kungumaraj, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore; kungum99522@gmail.com
Dr.R.Santhi, Department of Mathematics, NGM College, Pollachi, santhifuzzy@yahoo.co.in

BIOGRAPHY:

I have completed M.Sc., B.Ed., Ph.D. I have gained my doctoral degree in April 2021 in the field of Topology. I have published 15 papers both in National and International Reputed Journals. Among the fifteen, two are published in Scopus indexed journals. Currently I'm working as an Assistant Professor of Mathematics in Sri Krishna Arts and Science College, Coimbatore. In total I have ten years of teaching experience in Arts and Science Colleges.

Dr.E.Kungumaraj, Department of Mathematics, Sri Krishna Arts and Science College, Coimbatore; kungum99522@gmail.com
Dr.R.Santhi, Department of Mathematics, NGM College, Pollachi, santhifuzzy@yahoo.co.in