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Department of Biological Science, Physical Science and Computational Science

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Irresolute topological simple ring

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ABSTRACT: In this paper, we introduced a generalized form of topological simple ring, namely irresolute topological simple ring by using semi-open sets which itself is a generalized form of open sets. We investigated their properties and establish their difference from topological simple ring. Examples of irresolute topological simple ring which fails to be topological simple ring are also provided.

KEYWORDS: Semi-open set, topological simple ring, irresolute mapping

1. INTRODUCTION

The concept of topological ring was introduced by D. Van Dantzig. Later the concept of topological ring was developed and studied by S. Warner [6] and I. Kalpanasy [3].In 2018, Haval M.Mohammed Salih[1] introduced the concept of irresolute topological ring.

A mapping $f: X \to Y$ is semi-continuous(irresolute)[4] if each open(semi-open-set) O in Y, then f^{-1}(O) is semi-open in X. A non-zero ring S whose only (two sided) ideals are S itself and zero is called simple ring [2].

2. IRRESOLUTE TOPOLOGICAL SIMPLE RING

Definition 2.1: An irresolute topological simple is a simple ring which is also a topological space if the following conditions are satisfied:

(i) for each $s, t \in S$ and each semi-open neighbourhood L of $s - t$ in S , there exist semi-open neighbourhood J and K of s and t respectively in S such that $J - K \subseteq L$.

(ii) for each $s, t \in S$ and each semi-open neighbourhood L of st^{-1} in S , there exist semi-open neighbourhood J and K of s and t in S such that $JK^{-1} \subseteq L$.

Example 2.2: Let R be a ring of real number. Then R with its usual topology τ is a topological simple ring as well as an irresolute topological simple ring.

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Example 2.3: Any simple ring is topological simple ring as well as irresolute topological simple ring with its discrete and indiscrete topology.

Remark 2.4: Any topological simple ring is an irresolute topological simple ring .But the converse is not true by following examples.

Example 2.5: Let $S = \{ 0,\ e\}$ be simple ring with order 2 and $\tau = \{ \emptyset,\ \{0\},\ S\}$ be topology. Then $(s,\ \tau)$ is an irresolute topological simple but not topological simple ring.

Example 2.6: Let S be simple ring of real number with topology τ = $\{\emptyset,\ Q,\ R\}$ where Q be set of all rational umbers. Then (S, τ) is an irresolute topological simple ring. But it fails to be topological simple ring. For Q be an open set containing $0 + {}^{1}\!/_{2} = {}^{1}\!/_{2}$, there do not exist open sets J containing 0 and K containing ${}^{1}\!/_{2}$ in S satisfying $J + K \subseteq Q$.

Theorem 2.7: Let (S, τ) be an irresolute topological simple ring and $R \in SO(S)$. Then the following condition holds.

(i) $-R,\ R^{-1} \in SO(S)$

(ii) $t + R, R + t, tR$ and $Rt \in SO(S)$ for each $t \in S$

Proof: (i)Let $u \in -R$. Then $u = -r$ for some $r \in R$. By definition 3.1, there exist an semi-open set J in S containing u such that $-J \subseteq R \Rightarrow J \subseteq -R$. Therefore u is an semi- interior point of $-R$. Thus $u \in s(-R)^{\circ}$. Hence $-R \in SO(S)$. Analogously $R^{-1} \in SO(S)$.

(ii) Let $u \in s + R$. Then there exist $J, K \in SO(S)$ such that $-t \in J, u \in K$ and $J + K \subseteq R \Rightarrow -t + K \subseteq R \Rightarrow K \subseteq t + R \Rightarrow u \in s(t + R)^{\circ}$. Therefore $s(t + R)^{\circ} = t + R$. Hence $t + R \in SO(S)$. Analogously $R + t, tR$ and $Rt \in SO(S)$.

Corollary 2.8: Let R be any semi-open set in irresolute topological simple ring. Then $R + T \in SO(S)$ for any $T \subseteq S$.

Corollary 2.9: Let G be any semi-closed set in an irresolute topological simple ring. Then

(i) $-G, G^{-1} \in$ SO(C)

(ii) $t + G, G + t, Gt$ and $tG \in SO(C)$ for each $t \in$ S

Theorem 2.10: Suppose that (i) the sum of two semi-compact subsets of an irresolute topological simple ring is semi-compact.

(ii) the product of two semi-compact subsets of an irresolute topological simple ring is semi-compact.

(iii) M^{-1}, $-M$ are semi-compact.

Proof: (i) Let M and N are semi-compact subsets of an irresolute topological simple ring S. Then $M \times N$ is semi-compact subset of $S \times S$. Since addition mapping **a** is an irresolute, the restriction mapping of **a** onto a subspace $M \times N$ of $S \times S$ is an also irresolute. Then $M + N$ is semi-compact.

(ii) Since multiplication mapping **m** is an irresolute, then the restriction mapping of **m** onto a subspace $M \times N$ of $S \times S$ is an also irresolute. Then MN is semi-compact.

(iii) Since additive and multiplicative inverse are an irresolute, then $-M, M^{-1}$ are semi-compact.

Theorem 2.11: Let S be an irresolute topological simple ring and J be a semi-closed subsets of S. If L is semi-compact, then $J + L, L + J, JL$ and LJ are semi-closed in S.

Proof: Assume that $J + L$ is not semi-closed. Let $u \in s\overline{J + L}$ .Then $\notin J + L \Rightarrow (u - L) \cap J = \emptyset$. Since J is semi-closed, $S - J$ is semi-open containing $u - l$ where $l \in L$. Then there exist semi-open sets G_u and H_l of u and l respectively such that $G_u - H_l \subseteq S - J$. Hence { H_l / $l \in L$ } is a semi-open cover for L , there exist finite sub-cover $H_{l_1}, H_{l_2}, H_{l_{3,}}, \dots \dots, H_{l_n}$ such that L $\subseteq \bigcup_{i=1}^{n} H_{l_i}$ = H and J is semi-open set of u . Therefore $G - H \subseteq S - J$. Since $L \subseteq H$, $G - L \subseteq S - J \Rightarrow g - L \subseteq S - J$ for any $g \in G$. Thus $(g - L) \cap J = \emptyset \Rightarrow g \notin J + L \Rightarrow G \cap (J + L) = \emptyset$. Thus $u \notin s\overline{J + L}$ which is contradiction, $J + L$ is semi-closed. Similarly $JL, L + J$ and LJ are semi-closed.

Theorem 2.12: Let H and I be any subset of an irresolute topological simple ring S. Then

(i) s$\overline{H}$ + s$\overline{I} \subseteq$ s$\overline{H + I}$ (ii) -s$\overline{H} \subseteq s\overline{(-H)}$

(iii) s$\overline{H}$ s$\overline{I} \subseteq s\overline{HI}$ (iv) $s\overline{H}^{-1} \subseteq s\overline{H^{-1}}$

(v) If I is semi-compact subset of semi- Hausdorff S. Then s$\overline{H + I}$ = s$\overline{H}$ +s$\overline{I}$ = s$\overline{H}$ + I

Proof: (i) Let $u \in s\overline{H}$ and $v \in s\overline{I}$. Let L be a semi-open neighbourhood of $u + v$ in S. By definition 3.1, then there exist a semi-open neighbourhood J of u and K of v such that $J + K \subseteq L$. By assumption $H \cap J \neq \emptyset$, $I \cap K \neq \emptyset \Rightarrow h \in H \cap J$ and $i \in I \cap K \Rightarrow h + i \in (H + I) \cap L \Rightarrow (H + I) \cap L \neq \emptyset$. Thus $u + v \in s\overline{(H + I)}$.

(ii) Let g$\in -s\overline{H}$ and J be a semi-open neighbourhood of g. Since the inverse mapping is an irresolute, $-J$ is semi-open neighbourhood of $-g$. By assumption $-g \in s\overline{H}$

$\Rightarrow -J \cap H \neq \emptyset \Rightarrow g \in s\overline{(-H)}$.

(iii) Let $h \in s\overline{H}$ and $i \in s\overline{I}$. Let L be semi-open neighbourhood of hi. Then there exist a semi-open neighbourhood J of h and K of i such that $JK \subseteq L$. By assumption$H \cap J \neq \emptyset$, $I \cap K \neq \emptyset \Rightarrow h \in H \cap J$ and $i \in I \cap K \Rightarrow hi \in (HI) \cap L \Rightarrow HI \cap L \neq \emptyset$. Thus $hi \in s\overline{HI}$.

(iv) Analogously we can prove that (ii)

(v)Let s$\overline{H} + I$ = s$\overline{H}$ + $s\overline{I} \subseteq s\overline{H + I}$. By theorem 3.11, $s\overline{H} + I$ is semi-closed subset of S. Since $H + I \subseteq s\overline{H} + I$, $s\overline{H + I} \subseteq s\overline{H} + I$. Hence the result.

Theorem 2.13: Let R be any subset of an irresolute topological simple ring S. For each t$\in S$, then (i) $t + s\overline{R} = s\overline{t + R}$ (ii)$t + sR^{\circ}$ = $s(t + R)^{\circ}$

Proof: (i) Let $v \in t + s\overline{R}$ and $v = t + u$ for some $\in s\overline{R}$. Let L be semi-open neighbourhood of v in S. Since S is an irresolute topological simple ring, then there exist semi-open neighbourhood J and K of t and u in S such that $J + K \subseteq L$. Since $u \in s\overline{R}$, then there exist w of R and K such that $t + w \in (t + R) \cap L \Rightarrow (t + R) \cap L \neq \emptyset$. Hence $v \in s\overline{t + R}$. Let $u \in s\overline{t + R}$ and L be semi-open neighbourhood in S containing $-t + u$. Then there exist semi-open neighbourhood J of –t and K of u such that $J + K \subseteq L$. By assumption $u \in s\overline{t + R}$, $(t + R) \cap K \neq \emptyset$. Let w be an element of $t + R$ and K. Then $-t + u \in s\overline{R}$. Hence $u \in t + s\overline{R}$.

(ii) Let u$\in s(t+R)^{\circ}$. Then $-t+u \in R^{\circ}$. Since S is an irresolute topological simple ring and R is semi-open neighbourhood in S , then there is semi-open sets J of –t and K of u such that $J+K \subseteq R \Rightarrow -t+K \subseteq R \Rightarrow$ K$\subseteq t+R$ .Hence $u \in s(t+R)^{\circ}$. Let $u \in s(t+R)^{\circ}$. Then $u = t+R$ for some $\in R$. We obtained semi-open set J and K in S containing t and r such that $+K \subseteq R \Rightarrow u \in t + sR^{\circ}$.

Theorem 2.14: Let S be an irresolute topological simple ring and $R \subseteq S$. For each $u \in S$, then

(i) $us\bar{R} = s\overline{uR}$ (ii) $usR^{\circ} = s(uR)^{\circ}$

Proof: (i)Let $g \in$ us$\bar{R}$. Then g = uh for some $h \in s\bar{R}$. Let L be semi-open neighbourhood J and K of u and h in S such that $JK \subseteq L$. Now h $\in s\bar{R} \Rightarrow R \cap K \neq \emptyset$. Therefore $t \in R \cap K \Rightarrow$ ut $\in$ (uR) $\cap$ JK $\subseteq$ (uR) $\cap$ $L \Rightarrow g \in s\overline{uR}$. Hence us$\bar{R} \subseteq$ $s\overline{uR}$. Let g $\in$ $s\overline{uR}$ and let L be an semi-open neighbourhood in S containing h = u^{-1}g. There exist a semi-open neighbourhood J and K of u^{-1} and g such that $JK \subseteq L$. By assumption $g \in s\overline{uR} \Rightarrow (uR) \cap K \neq \emptyset \Rightarrow t \in (uR) \cap K \Rightarrow u^{-1}t \in R \cap (JK) \subseteq R \cap L$. Thus $R \cap L \neq \emptyset \Rightarrow u^{-1}g \in sR \Rightarrow g \in$ us$\bar{R}$.
(ii) Let $g \in$ us$R^{\circ} \Rightarrow u^{-1}g \in sR^{\circ}$. Let R be a semi-open neighbourhood of s. By definition of an irresolute topological simple ring, there exist an semi-open J of u^{-1} and K of g such that $JK \subseteq R \Rightarrow u^{-1}K \subseteq R \Rightarrow K \subseteq$ u$R \Rightarrow g \in s$(uR)$^{\circ}$. Let $g \in s$(uR)$^{\circ} \Rightarrow g$ = ur for some r $\in R$ and R be a semi-open nighbourhood of S. By definition of S, we obtain semi-open set J of u and K of r such that $JK \subseteq$ uR. But g = ur $\in$ usR°.

Theorem 2.15: Let S be an irresolute topological simple ring. Then (i) $L_s: S \to S$ is defined by $L_s(t) = s+t$ and $R_s: S \to S$ is defined by $R_s(t) = t+s$ are irresolute (ii) $L_s: S \to S$ is defined by L_s(t) = st and $R_s: S \to S$ is defined by $R_s(t) = ts$ are irresolute.

Theorem 2.16: Let (S, τ_s) be an iorresolute topological simple ring and (T, τ_s) be topological simple ring . If $\varphi: S \to T$ is homomorphism and semi-homeomorphism, then T is also an irresolute topological simple ring.

Proof (i) Let g and h be any two point in T. Let $L \subseteq T$ be a semi-open neighbourhood of $g-h$. Let $u = \varphi^{-1}(g)$ and $v = \varphi^{-1}(h)$. Since φ is semi-homeomorphism, the $\varphi^{-1}(L)$ is semi-open neighbourhood of $u-v$ and S is an irresolute topological simple ring , there exist semi-pen neighbourhood J and K of u and v respectively with $J-K \subseteq \varphi^{-1}(L)$. Since φ is pre-semi-open, the set $G = \varphi(J)$ and $H = \varphi(K)$ are semi-open neighbourhood of g and h. Then $G-H = \varphi(J) - \varphi(K)$ = $\varphi(J-K) \subseteq L$.Analogously we can prove $\varphi(JK^{-1}) \subseteq L$. Hence T is an irresolute topological simple.

Theorem 2.17: Let S and T be an irresolute topological simple ring and $\varphi: S \to T$ be a homomorphism. If φ is irresolute at i_s of S, then φ is irresolute on S.

Theorem 2.18: Every open ideal T of an irresolute topological simple ring S is also an irresolute topological simple ring.

Theorem 2.19: A non-empty ideal T of an irresolute topological simple ring S is semi-open ⇔its semi-interior is non-empty.

Proof: Let $t \in sT^{\circ}$, then there is semi-open set J such that $t \in J \subseteq T \Rightarrow t+J \subseteq J$. Choose every $u \in T, u+J = u-t+t+J \subseteq u-t+T = T$. Since $u+J$ is semi-open, $T = \cup\{u+J/u \in T\}$ is semi-open.

Theorem 2.20: Every open ideal of an irresolute topological space S is semi-closed in S.

Definition 2.21: Let S be an irresolute topological simple ring. Then a subset $T \subseteq R$ is called symmetric if $T = -T$ and T^{-1} = T.

Theorem 2.22: If S is an irresolute topological simple ring of symmetric semi-open neighbourhood J of the identity i. Then $R = \bigcup_{n=1}^{\infty} J^{\alpha}$ is a semi-open and semi-closed ideal of S.

Proof: Let $\in J^{\alpha}$, $v \in J^{\beta} \in$ R, then $u * v \in J^{\alpha+\beta} \subseteq R$. If u$\in R$ and $u \in J^{\alpha}$, then $u^{-1} \in {J^{-1}}^{\alpha} = J^{\alpha} \subseteq R$. Hence R is an ideal of S. Since S is an irresolute topological simple ring, J^{α} is semi-open for each $\alpha \in N$. Thus R is semi-open in S. By theorem 3.20, L is semi-closed.

Corollary 2.23: Let S be an irresolute topological simple ring and Δ_i be collection of all semi-open neighbourhoods of i. Then (i) for every $J \in \Delta_i$, there exist $K \in \Delta_i$ such that $-K, K^{-1} \subseteq J$ (ii)for every $J \in \Delta_i$ and $s \in J$, there exist $K \in \Delta_i$ such that $s + K, K + s$, sK and Ks $\subseteq J$ (iii) for every $J \in \Delta_i$ there exist $K \in J$ such that $K - K, KK^{-1} \subseteq J$.

Theorem 2.24: Let S be an irresolute topological simple ring and $R \subseteq S$. Then $s\bar{R} = \cap \{R + J / J \in \Delta_i\}$

Proof: Let $u \in s\bar{R}$ and J be a semi-open neighbourhood of i. By corollary 3.23, there exist $K \in \Delta_i$ such that $-K \subseteq J$. Since $u \in s\bar{R}$, $(s + K) \cap R \neq \emptyset$ for some $k \in K$ and $r \in R$ such that $r = s + K \Rightarrow s = r - k \in R - K \subseteq R + J$. $s\bar{R} \subseteq \cap \{R + J / J \in \Delta_i\}$, obviously $\cap \{R + J / J \in \Delta_i\} \subseteq s\bar{R}$.

CONCLUSION

In this paper, we developed irresolute topological simple ring which is one of generalization of topological simple ring. The concept is further elaborated with properties and theorems.

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