VOLUME X
ISBN No.: 978-81-953602-6-0
Physical Science

Estd. 1957

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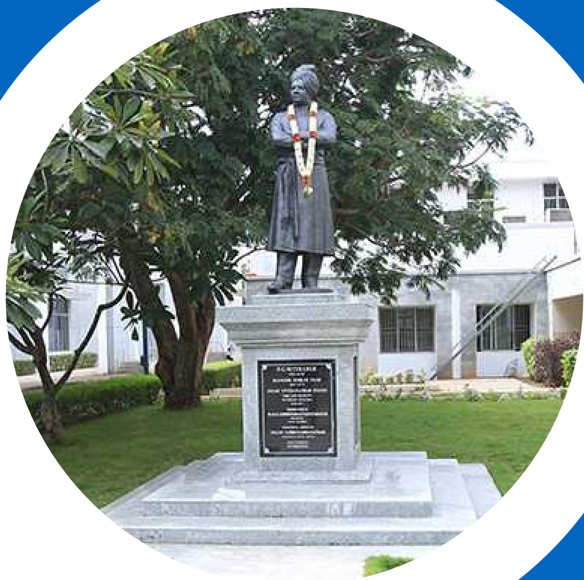

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EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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International Conference on Emerging Trends in Science and Technology (ETIST 2021) Jointly Organized by Department of Biological Science, Physical Science and Computational Science Nallamuthu Gounder Mahalingam College, Affiliated to Bharathiar University, Tamilnadu, India.
Published by NGMC - November 2021

$\delta_{\mathcal{I}}$-Semi-Connected and Compact Spaces in Ideal Topological Spaces

V. Inthumathi[1], **M. Maheswari**[2], **A. Anis Fathima**[3],

Abstract - In this paper, we introduce $\delta_{\mathcal{I}}$-semi-separated sets, $\delta_{\mathcal{I}}$-semi-connected and $\delta_{\mathcal{I}}$-semi-compact spaces also study some of its properties in topological spaces via ideals.

Keywords *$\delta_{\mathcal{I}}$-semi-separated sets, $\delta_{\mathcal{I}}$-semi-connected spaces, $\delta_{\mathcal{I}}$-semi-disconnected spaces and $\delta_{\mathcal{I}}$-semi-compact spaces.*
2010 Subject classification: *54A05*

1 Introduction

The notion of ideal in topological spaces was studied by Kuratowski [8] & Vaidyanathaswamy [12]. Applications to various fields in ideal topological spaces were investigated by Jankovic and Hamlett [7], Dontchev et al. [3], Mukherjee et al. [9], Arenas et al. [2], Navaneethakrishnan et al. [11], Nasef and Mahmoud [10], etc. In 2008, Ekici and Noiri [4] introduced the notion of connectedness in ideal topological spaces.

2 Preliminaries

Throughout this paper, $(X, \tau, \mathcal{I})$ and $(Y, \sigma, \mathcal{I})$ (or simply X and Y), always mean ideal topological spaces on which no separation axioms are assumed.

Definition 2.1. *[1] A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\delta_{\mathcal{I}}$-semi-open if $A \subseteq cl^*(int_\delta(A))$.*

The complement of $\delta_{\mathcal{I}}$-semi-open set is called $\delta_{\mathcal{I}}$-semi-closed set.

Definition 2.2. *[1] Let A be a subset of an ideal topological space $(X, \tau, \mathcal{I})$ and x be a point of X. Then*

1. *x is called a $\delta_{\mathcal{I}}$-semi-cluster point of A if $A \cap U \neq \emptyset$ for every $U \in \delta_{\mathcal{I}}SO(X)$,*
2. *the family of all $\delta_{\mathcal{I}}$-semi-cluster points of A is called $\delta_{\mathcal{I}}$-semi-closure of A and is denoted by $scl_{\delta_{\mathcal{I}}}(A)$.*

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Definition 2.3. *[5] A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ is said to be $\delta_{\mathcal{I}}$-semi-irresolute if inverse image of every $\delta_{\mathcal{I}}$-semi-open set in Y is $\delta_{\mathcal{I}}$-semi-open set in X.*

Definition 2.4. *[6] A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be contra $\delta_{\mathcal{I}}$-semi-continuous if $f^{-1}(V)$ is $\delta_{\mathcal{I}}$-semi-closed in X for each open set V of Y.*

3 $\delta_{\mathcal{I}}$-semi-separated

Definition 3.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Two non-empty subsets M and N are said to be $\delta_{\mathcal{I}}$-semi-separated if and only if $M \cap scl_{\delta_{\mathcal{I}}}(N) = \emptyset$ and $scl_{\delta_{\mathcal{I}}}(M) \cap N = \emptyset$.*
i.e.,$[M \cap scl_{\delta_{\mathcal{I}}}(N)] \cup [scl_{\delta_{\mathcal{I}}}(M) \cap N] = \emptyset$.

Definition 3.2. *If $X = M \cup N$ such that M and N are non-empty $\delta_{\mathcal{I}}$-semi-separated sets in $(X, \tau, \mathcal{I})$ then M, N form a $\delta_{\mathcal{I}}$-semi-separation of X.*

Example 3.3. *Let $X=\{a, b, c, d\}$ with topology $\tau=\{\emptyset, \{a\}, \{b, d\}, \{a, b, d\}, X\}$ and $\mathcal{I}=\{\emptyset, \{a\}\}$. Consider $P=\{a\}$, $Q=\{b\}$ and $R=\{d\}$. Then the sets P and Q are $\delta_{\mathcal{I}}$-semi-separated but the sets Q and R are not $\delta_{\mathcal{I}}$-semi-separated.*

Definition 3.4. *A point $x \in X$ is said to be an $\delta_{\mathcal{I}}$-semi-adherent point of a subset A of an ideal topological space $(X, \tau, \mathcal{I})$ if every $\delta_{\mathcal{I}}$-semi-open set containing x, contains atleast one point of A.*

Remark 3.5. *Two $\delta_{\mathcal{I}}$-semi-separated sets are always disjoint. But two disjoint sets need not be $\delta_{\mathcal{I}}$-semi-separated. In Example 3.3, the sets Q and R are disjoint but not $\delta_{\mathcal{I}}$-semi-separated.*

Theorem 3.6. *Two sets are $\delta_{\mathcal{I}}$-semi-separated if and only if they are disjoint and neither of them contains $\delta_{\mathcal{I}}$-semi-cluster point of the other.*

Proof. Let A and B be $\delta_{\mathcal{I}}$-semi-separated. Now, $A \cap scl_{\delta_{\mathcal{I}}}(B) = \emptyset \Leftrightarrow A \cap (B \cup B_l) = \emptyset$, where the set B_l denotes the set of all $\delta_{\mathcal{I}}$-semi-cluster points of $B \Leftrightarrow A$ and B are disjoint and A contains no $\delta_{\mathcal{I}}$-semi-cluster point of B. Similaly, $scl_{\delta_{\mathcal{I}}}(A) \cap B = \emptyset$ if and only if A and B are disjoint and B contains no $\delta_{\mathcal{I}}$-semi-cluster point of A.

Theorem 3.7. *Subsets of $\delta_{\mathcal{I}}$-semi-separated sets are $\delta_{\mathcal{I}}$-semi-separated.*

Proof. Let C and D be subsets of two $\delta_{\mathcal{I}}$-semi-separated sets A and B respectively. Then $A \cap scl_{\delta_{\mathcal{I}}}(B) = \emptyset$ and $scl_{\delta_{\mathcal{I}}}(A) \cap B = \emptyset$. Then we have $C \cap scl_{\delta_{\mathcal{I}}}(D) \subseteq A \cap scl_{\delta_{\mathcal{I}}}(B) = \emptyset$ and $scl_{\delta_{\mathcal{I}}}(C) \cap D \subseteq scl_{\delta_{\mathcal{I}}}(A) \cap B = \emptyset$. Thus C and D are $\delta_{\mathcal{I}}$-semi-separated.

Theorem 3.8. *Two $\delta_{\mathcal{I}}$-semi-closed subsets of X are $\delta_{\mathcal{I}}$-semi-separated if and only if they are disjoint.*

Proof. By Remark 3.5 $\delta_{\mathcal{I}}$-semi-closed separated sets are disjoint.
Conversely, let A and B be two $\delta_{\mathcal{I}}$-semi-closed disjoint sets. Then we have $scl_{\delta_{\mathcal{I}}}(A) = A$, $scl_{\delta_{\mathcal{I}}}(B) = B$ and $A \cap B = \emptyset$. Consequently, $A \cap scl_{\delta_{\mathcal{I}}}(B) = \emptyset$ and $scl_{\delta_{\mathcal{I}}}(A) \cap B = \emptyset$. Hence A and B are $\delta_{\mathcal{I}}$-semi-separated.

Theorem 3.9. *Two $\delta_{\mathcal{I}}$-semi-open subsets of X are $\delta_{\mathcal{I}}$-semi-separated if and only if they are disjoint.*

Proof. By Remark 3.5 $\delta_{\mathcal{I}}$-semi-open separated sets are disjoint.
Conversely, let P and Q be two $\delta_{\mathcal{I}}$-semi-open disjoint sets. Suppose that $P \cap scl_{\delta_{\mathcal{I}}}(Q) \neq \emptyset$ and let $x \in P \cap scl_{\delta_{\mathcal{I}}}(Q)$. Then $x \in P$ and x is a $\delta_{\mathcal{I}}$-semi-adherent point of Q. Since P is a $\delta_{\mathcal{I}}$-semi-open set containing x and x is a $\delta_{\mathcal{I}}$-semi-adherent point of Q, therefore P must contain atleast one point of Q. Thus we have $P \cap Q \neq \emptyset$ which is a contradicton. Therefore $P \cap scl_{\delta_{\mathcal{I}}}(Q) = \emptyset$. Similarly, $scl_{\delta_{\mathcal{I}}}(P) \cap Q = \emptyset$. Hence P and Q are $\delta_{\mathcal{I}}$-semi-separated.

Theorem 3.10. *If the union of two $\delta_{\mathcal{I}}$-semi-separated sets is a $\delta_{\mathcal{I}}$-semi-closed set then the individual sets are $\delta_{\mathcal{I}}$-semi-closures of themselves.*

Proof. Let M and N be two $\delta_{\mathcal{I}}$-semi-separated sets such that $M \cup N$ is $\delta_{\mathcal{I}}$-semi-closed. Now, $M \cup N = scl_{\delta_{\mathcal{I}}}(M \cup N) \supseteq scl_{\delta_{\mathcal{I}}}(M) \cup scl_{\delta_{\mathcal{I}}}(B)$. Therefore $scl_{\delta_{\mathcal{I}}}(M) = scl_{\delta_{\mathcal{I}}}(M) \cap [scl_{\delta_{\mathcal{I}}}(M) \cup scl_{\delta_{\mathcal{I}}}(N)] \subseteq scl_{\delta_{\mathcal{I}}}(M) \cap [M \cup N] = M$. Thus we have $scl_{\delta_{\mathcal{I}}}(M) = M$. Similarly, $scl_{\delta_{\mathcal{I}}}(M) = M$.

Theorem 3.11. *If the union of two $\delta_{\mathcal{I}}$-semi-separated sets is δ-open, then the individual sets are $\delta_{\mathcal{I}}$-semi-open.*

Proof. Let M and N be two $\delta_{\mathcal{I}}$-semi-separated sets such that $M \cup N$ is δ-open. Therefore we have $M \cup N \cap [scl_{\delta_{\mathcal{I}}}(N)]^c$ is $\delta_{\mathcal{I}}$-semi-open and so $M \cup N \cap [scl_{\delta_{\mathcal{I}}}(N)]^c = M$. This implies M is $\delta_{\mathcal{I}}$-semi-open. Similarly, we can prove N is $\delta_{\mathcal{I}}$-semi-open.

4 $\delta_{\mathcal{I}}$-semi-connected

Definition 4.1. *A space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-connected if and only if X has no $\delta_{\mathcal{I}}$-semi-separation.*
If X is not $\delta_{\mathcal{I}}$-semi-connected then it is $\delta_{\mathcal{I}}$-semi-disconnected.

Definition 4.2. *A subset of $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-connected if it is $\delta_{\mathcal{I}}$-semi-connected as a subspace.*

Theorem 4.3. *An ideal topological space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-disconnected if and only if there exist a non-empty proper subset of X which is both $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed.*

Proof. Necessity: Let $(X, \tau, \mathcal{I})$ be $\delta_{\mathcal{I}}$-semi-disconnected. Then there exist non-empty $\delta_{\mathcal{I}}$-semi-separated subsets M and N of X such that $M \cup N = X$. Therefore $scl_{\delta_{\mathcal{I}}}(M) \cup N = X$, $M \cup scl_{\delta_{\mathcal{I}}}(N) = X$ and $M \cap N = \emptyset$. Thus we have $M = X - N$, $M = X - scl_{\delta_{\mathcal{I}}}(N)$ and $N = X - scl_{\delta_{\mathcal{I}}}(M)$. This shows that, M is non-empty proper subset of X which is both $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed.

Sufficiency: Let M be a non-empty proper subset of X which is both $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed. Then, M^c is a non-empty proper subset of X which is both $\delta_{\mathcal{I}}$-semi-closed and $\delta_{\mathcal{I}}$-semi-open. Thus $M \cap M^c = \emptyset$, $scl_{\delta_{\mathcal{I}}}(M) = M$ and $scl_{\delta_{\mathcal{I}}}(M^c) = M^c$ and therefore $scl_{\delta_{\mathcal{I}}}(M) \cap M^c = M \cap M^c = \emptyset$ and $M \cap scl_{\delta_{\mathcal{I}}}(M^c) = M \cap M^c = \emptyset$. Also $X = M \cup M^c$. Hence X is $\delta_{\mathcal{I}}$-semi-disconnected.

Theorem 4.4. *An ideal topological space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-disconnected if and only if X is the union of non-empty disjoint $\delta_{\mathcal{I}}$-semi-open sets.*

Proof. Necessity Let X be $\delta_{\mathcal{I}}$-semi-disconnected. Then there exist a non-empty proper subset M of X which is both $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed. Therefore M^c is a non-empty proper subset of X which is both $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed. This shows that $X = M \cup M^c$ and $M \cap M^c = \emptyset$. This implies that X is the union of two non-empty disjoint $\delta_{\mathcal{I}}$-semi-open sets.
Sufficiency , let X be the union of two non-empty disjoint $\delta_{\mathcal{I}}$-semi-open sets M and N. Then $N^c = M$.

Now N is $\delta_{\mathcal{I}}$-semi-open, it follows that M is $\delta_{\mathcal{I}}$-semi-closed. Since $N \neq \emptyset$, it implies that M is a non-empty proper subset of X which is both $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed. This shows that X is $\delta_{\mathcal{I}}$-semi-disconnected.

Theorem 4.5. *An ideal topological space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-connected if and only if X cannot be written as the union of non-empty disjoint $\delta_{\mathcal{I}}$-semi-open sets.*

Proof. Obvious.

Corollary 4.6. *A space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-connected (resp. $\delta_{\mathcal{I}}$-semi-disconnected) if and only if X cannot be written as (resp. can be written as) the union of non-empty disjoint $\delta_{\mathcal{I}}$-semi-closed sets.*

Theorem 4.7. *An ideal topological space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-connected if and only if the only subsets of X which is $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed are $\emptyset$ and X.*

Proof. Let F be a $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed subset of X. Then $X - F$ is both $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed. Since X is $\delta_{\mathcal{I}}$-semi-connected, X can not be expressed as union of two disjoint non empty $\delta_{\mathcal{I}}$-semi-open sets F and $X - F$, which implies $X - F$ is empty.
Conversely, suppose $X = U \cup V$ where U and V are disjoint non-empty $\delta_{\mathcal{I}}$-semi-open sets of X. Then U is both $\delta_{\mathcal{I}}$-semi-open and $\delta_{\mathcal{I}}$-semi-closed. Therefore by assumption, either $U = \emptyset$ or X, which contradicts the assumption that U and V are disjoint non-empty $\delta_{\mathcal{I}}$-semi-open subsets of X. Therefore X is $\delta_{\mathcal{I}}$-semi-connected.

Corollary 4.8. *If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is a $\delta_{\mathcal{I}}$-semi-irresolute surjective function and X is $\delta_{\mathcal{I}}$-semi-connected, then Y is δ-semi-connected.*

Theorem 4.9. *If the sets P and Q form a $\delta_{\mathcal{I}}$-semi-separation of $(X, \tau, \mathcal{I})$ and if $(Y, \sigma, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-connected subspace of X, then Y lies entirely within either P or Q.*

Proof. Since P and Q form a $\delta_{\mathcal{I}}$-semi-separation of X. If $P \cap Y$ and $Q \cap Y$ were both non-empty, they would form a $\delta_{\mathcal{I}}$-semi-separation of Y, which is a contradiction. Therefore one of them is empty. Hence Y must lie entirely in P or in Q.

Theorem 4.10. *A contra $\delta_{\mathcal{I}}$-semi-continuous image of a $\delta_{\mathcal{I}}$-semi-connected space is connected.*

Proof. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a contra $\delta_{\mathcal{I}}$-semi-continuous function of a $\delta_{\mathcal{I}}$-semi-connected space $(X, \tau, \mathcal{I})$ onto a topological space (Y, σ). Suppose Y is disconnected. Let A and B form a separation of Y. Then A and B are clopen and $Y = A \cup B$ where $A \cap B = \emptyset$. Since f is contra $\delta_{\mathcal{I}}$-semi-continuous, $X = f^{-1}(Y) = f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty $\delta_{\mathcal{I}}$-semi-open sets in X. Also $f^{-1}(A) \cap f^{-1}(B) = \emptyset$. Hence X is not $\delta_{\mathcal{I}}$-semi-connected. This is a contradiction. Therefore Y is connected.

Theorem 4.11. *If A is $\delta_{\mathcal{I}}$-semi-connected and $A \subseteq B \subseteq scl_{\delta_{\mathcal{I}}}(A)$, then B is $\delta_{\mathcal{I}}$-semi-connected.*

Proof. Let A be $\delta_{\mathcal{I}}$-semi-connected and let $A \subseteq B \subseteq scl_{\delta_{\mathcal{I}}}(A)$. Suppose that B is not $\delta_{\mathcal{I}}$-semi-connected, then C and D form a $\delta_{\mathcal{I}}$-semi-seperation of B. By Theorem 4.9, A must lie entirely in C or in D. Suppose that $A \subseteq C$ implies $scl_{\delta_{\mathcal{I}}}(A) \cap D \subseteq scl_{\delta_{\mathcal{I}}}(C) \cap D = \emptyset$. Also, $D \subseteq B \subseteq scl_{\delta_{\mathcal{I}}}(A)$ implies $scl_{\delta_{\mathcal{I}}}(A) \cap D = D$. This shows that $D = \emptyset$, which is a contradiction. Similarly, we will have a contradiction for $A \subseteq D$. Therefore B is $\delta_{\mathcal{I}}$-semi-connected.

Corollary 4.12. *The $\delta_{\mathcal{I}}$-semi-closure of a $\delta_{\mathcal{I}}$-semi-connected set is $\delta_{\mathcal{I}}$-semi-connected.*

Theorem 4.13. *The union of any family of $\delta_{\mathcal{I}}$-semi-connected sets having a non-empty intersection is $\delta_{\mathcal{I}}$-semi-connected.*

Proof. Let $\{E_\alpha\}$ be any family of $\delta_{\mathcal{I}}$-semi-connected sets such that $\bigcap_\alpha E_\alpha \neq \emptyset$. Let $E = \bigcup_\alpha E_\alpha$. Suppose that E is not $\delta_{\mathcal{I}}$-semi-connected, then A and B constitute a $\delta_{\mathcal{I}}$-semi-seperation of E. Since $\bigcap_\alpha E_\alpha \neq \emptyset$, let $x \in \bigcap_\alpha E_\alpha$. Then x belongs to each E_α and so $x \in E$. Consequently, $x \in A$ or $x \in B$. Suppose that $x \in A$, $E_\alpha \cap A \neq \emptyset$ for every α. From Theorem 4.9, $E_\alpha \subseteq A$ or $E_\alpha \subseteq B$. Since A and B are disjoint and $E_\alpha \cap A \neq \emptyset$ for every α. we must have $E_\alpha \subseteq A$ for each α. Consequently, $\bigcup_\alpha E_\alpha \subseteq A$ or $E \subseteq A$. This shows that $B = \emptyset$, which is a contradiction. Hence E is $\delta_{\mathcal{I}}$-semi-connected.

Corollary 4.14. *Let $\{E_\alpha | \alpha \in \Lambda\}$ be a family of $\delta_{\mathcal{I}}$-semi-connected subsets of $(X, \tau, \mathcal{I})$ such that one of the members of this family intersects every other member. Then $\bigcup\{E_\alpha | \alpha \in \Lambda\}$ is $\delta_{\mathcal{I}}$-semi-connected.*

Proof. Let E_{α_o} be a member of the given family such that $E_{\alpha_o} \cap E_\alpha \neq \emptyset$ for every $\alpha \in \Lambda$. Then By Theorem 4.13, $C_\alpha = E_{\alpha_o} \cup E_\alpha$ is $\delta_{\mathcal{I}}$-semi-connected for each α. Now, $\bigcup\{C_\alpha | \alpha \in \Lambda\} = \bigcup\{E_{\alpha_o} \cup E_\alpha | \alpha \in \Lambda\} = E_{\alpha_o} \cup (\bigcup\{E_\alpha | \alpha \in \Lambda\}) = \bigcup\{E_\alpha | \alpha \in \Lambda\}$ and $\bigcap\{C_\alpha | \alpha \in \Lambda\} = \bigcap\{E_{\alpha_o} \cup E_\alpha | \alpha \in \Lambda\} = E_{\alpha_o} \cup (\bigcap\{E_\alpha | \alpha \in \Lambda\}) \neq \emptyset$. Thus $\bigcup\{C_\alpha | \alpha \in \Lambda\}$ is the union of $\delta_{\mathcal{I}}$-semi-connected sets having a non-empty intersection is $\delta_{\mathcal{I}}$-semi-connected. Therefore $\bigcup\{E_\alpha | \alpha \in \Lambda\}$ is $\delta_{\mathcal{I}}$-semi-connected.

5 $\delta_{\mathcal{I}}$-semi-Compact

Definition 5.1. *A collection $\{A_\alpha | \alpha \in \Lambda\}$ of $\delta_{\mathcal{I}}$-semi-open sets in an ideal topological space $(X, \tau, \mathcal{I})$ is called $\delta_{\mathcal{I}}$-semi-open cover of a subset B of X if $B \subseteq \bigcup\{A_\alpha | \alpha \in \Lambda\}$ holds.*

Definition 5.2. *An ideal topological space $(X, \tau, \mathcal{I})$ is said to be $\delta_{\mathcal{I}}$-semi-compact if every $\delta_{\mathcal{I}}$-semi-open cover of X has a finite subcover.*

Definition 5.3. *A subset B of an ideal topological space $(X, \tau, \mathcal{I})$ is called $\delta_{\mathcal{I}}$-semi-compact relative to X if for every collection $\{A_\alpha | \alpha \in \Lambda\}$ of $\delta_{\mathcal{I}}$-semi-open subsets of X such that $B \subseteq \bigcup\{A_\alpha | \alpha \in \Lambda\}$, there exists a finite subset Λ_o of Λ such that $B \subseteq \bigcup\{A_\alpha | \alpha \in \Lambda_o\}$*

Proposition 5.4. *A $\delta_{\mathcal{I}}$-semi-closed subset of a $\delta_{\mathcal{I}}$-semi-compact space $(X, \tau, \mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-compact relative to $(X, \tau, \mathcal{I})$.*

Proof. Let A be any $\delta_{\mathcal{I}}$-semi-closed subset of an ideal topological space $(X, \tau, \mathcal{I})$. Then A^c is $\delta_{\mathcal{I}}$-semi-open in $(X, \tau, \mathcal{I})$. Let $S = \{A_i | i \in \Lambda\}$ be a $\delta_{\mathcal{I}}$-semi-open cover of A. Then $S^*{=}S \cup A^c$ is a $\delta_{\mathcal{I}}$-semi-open cover of X. That is $X = (\bigcup_{i \in \Lambda} A_i) \cup A^c$. By assumption, X is $\delta_{\mathcal{I}}$-semi-compact and hence S^* is reducible to a finite subcover of X say $X = A_{i_1} \cup A_{i_2} \cup .... \cup A_{i_n} \cup A^c$ where $A_{i_k} \in S^*$. But A and A^c are disjoint. Hence $A \subseteq A_{i_1} \cup A_{i_2} \cup .... \cup A_{i_n} \in S$. Thus $\delta_{\mathcal{I}}$-semi-open cover S of A contains a finite subcover. Hence A is $\delta_{\mathcal{I}}$-semi-compact relative to X.

Proposition 5.5. *If a map $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is $\delta_{\mathcal{I}}$-semi-irresolute and a subset B of X is $\delta_{\mathcal{I}}$-semi-compact relative to X, then $f(B)$ is δ-semi-compact relative to Y.*

Proof. Let $\{A_\alpha | \alpha \in \Lambda\}$ be a collection of δ-semi-open sets in Y such that $f(B) \subseteq \bigcup_{\alpha \in \Lambda} A_\alpha$. Then $B \subseteq \bigcup_\alpha f^{-1}(A_\alpha)$, where $\{f^{-1}(A_\alpha) \mid \alpha \in \Lambda\}$ is $\delta_{\mathcal{I}}$-semi-open set in X. Since B is $\delta_{\mathcal{I}}$-semi-compact relative to X, there exists finite subcollection $\{f^{-1}(A_1), f^{-1}(A_2), ..., f^{-1}(A_n)\}$ such that $B \subseteq \bigcup_{\alpha=1}^{n} f^{-1}(A_\alpha)$. That is $f(B) \subseteq \bigcup_{\alpha=1}^{n} A_\alpha$. Hence $f(B)$ is δ-semi-compact relative to Y.

Proposition 5.6. *Every finite union of $\delta_{\mathcal{I}}$-semi-compact sets is $\delta_{\mathcal{I}}$-semi-compact.*

Proof. Let U and V be any $\delta_{\mathcal{I}}$-semi-compact subsets of $(X,\tau,\mathcal{I})$. Let F be a $\delta_{\mathcal{I}}$-semi-open cover of $U\cup V$. Then F will also be a $\delta_{\mathcal{I}}$-semi-open cover of both U and V. By assumption, there exists a finite subcollection of F of $\delta_{\mathcal{I}}$-semi-open sets, say $\{U_1,U_2,...,U_n\}$ and $\{V_1,V_2,...,V_n\}$ covering U and V respectively. Then the collection $\{U_1,U_2,...,U_n,V_1,V_2,...,V_n\}$ is a finite collection of $\delta_{\mathcal{I}}$-semi-open sets covering $U\cup V$. By induction, every finite union of $\delta_{\mathcal{I}}$-semi-compact sets is $\delta_{\mathcal{I}}$-semi-compact.

Proposition 5.7. *Let A be a $\delta_{\mathcal{I}}$-semi-compact subset of a space $(X,\tau,\mathcal{I})$ and B be a $\delta_{\mathcal{I}}$-semi-closed subset of X. Then $A\cap B$ is $\delta_{\mathcal{I}}$-semi-compact in X.*

Proof. Let $\{G_\alpha\}$ be a $\delta_{\mathcal{I}}$-semi-open cover of $A\cap B$. Since B is $\delta_{\mathcal{I}}$-semi-closed, $\{G_\alpha,B^c\}$ is $\delta_{\mathcal{I}}$-semi-open. Then $\{G_\alpha,B^c\}$ is a $\delta_{\mathcal{I}}$-semi-open cover of A. By assumption A is $\delta_{\mathcal{I}}$-semi-compact, there exists a finite subcollection, say, $\{G_k,B^c\}$. Then $\{G_k\}$ is a finite $\delta_{\mathcal{I}}$-semi-open subcover of $A\cap B$. Thus $A\cap B$ is $\delta_{\mathcal{I}}$-semi-compact in X.

Theorem 5.8. *An ideal topological space $(X,\tau,\mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-compact if and only if every family of $\delta_{\mathcal{I}}$-semi-closed subsets of X having finite intersection property has a non-empty intersection.*

Proof. Suppose $(X,\tau,\mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-compact. Let $\{A_\alpha|\alpha\in\Lambda\}$ be a family of $\delta_{\mathcal{I}}$-semi-closed sets with finite intersection property. Suppose $\bigcap_{\alpha\in\Lambda}\{A_\alpha\}=\emptyset$. Then $[\bigcap_{\alpha\in\Lambda}\{A_\alpha\}]^c=X$. This implies $\bigcup_{\alpha\in\Lambda}\{A_\alpha^c\}$ $=X$. Thus the cover $\{A_\alpha^c|\alpha\in\Lambda\}$ is a $\delta_{\mathcal{I}}$-semi-open cover of $(X,\tau,\mathcal{I})$. Then by assumption, the $\delta_{\mathcal{I}}$-semi-open cover $\{A_\alpha^c|\alpha\in\Lambda\}$ has a finite subcover, say $\{A_\alpha^c|\alpha=1,2,...n\}$. This implies $X=\bigcup_{\alpha=1}^n\{A_\alpha^c\}=$ $[\bigcap_{\alpha=1}^n\{A_\alpha\}]^c$ and so $\emptyset=\bigcap_{\alpha=1}^n\{A_\alpha\}$. This contradicts the assumption. Hence $\bigcap_{\alpha\in\Lambda}\{A_\alpha\}\neq\emptyset$.

Conversely, suppose $(X,\tau,\mathcal{I})$ is not $\delta_{\mathcal{I}}$-semi-compact. Then there exists a $\delta_{\mathcal{I}}$-semi-open cover of $(X,\tau,\mathcal{I})$ say $\{G_\alpha|\alpha\in\Lambda\}$ having no finite subcover. This implies for any finite subfamily $\{G_\alpha|\alpha=1,2,...,n\}$ of $\{G_\alpha|\alpha\in\Lambda\}$ we have $\bigcup_{\alpha=1}^n G_\alpha\neq X$. Now, $\emptyset\neq[\bigcup_{\alpha=1}^n\{G_\alpha\}]^c=[\bigcap_{\alpha=1}^n\{G_\alpha^c\}]$. Then the family $\{G_\alpha^c|\alpha\in\Lambda\}$ of $\delta_{\mathcal{I}}$-semi-closed sets has a finite intersection property. Also by assumption $\bigcap_{\alpha\in\Lambda}\{G_\alpha^c\}\neq\emptyset$ and so $\bigcup_\alpha G_\alpha\neq X$. This implies $\{G_\alpha|\alpha\in\Lambda\}$ is not a $\delta_{\mathcal{I}}$-semi-cover of $(X,\tau,\mathcal{I})$. This contradicts the fact that $\{G_\alpha|\alpha\in\Lambda\}$ is a $\delta_{\mathcal{I}}$-semi-cover for $(X,\tau,\mathcal{I})$. Therefore $\delta_{\mathcal{I}}$-semi-open cover $\{G_\alpha|\alpha\in\Lambda\}$ of X has a finite subcover $\{G_\alpha|\alpha=1,2,...,n\}$. Hence $(X,\tau,\mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-compact.

Corollary 5.9. *An ideal topological space $(X,\tau,\mathcal{I})$ is $\delta_{\mathcal{I}}$-semi-compact if and only if every family of $\delta_{\mathcal{I}}$-semi-closed sets of X with empty intersection has a finite sub-family with empty intersection.*

Proposition 5.10. *The image of a $\delta_{\mathcal{I}}$-semi-compact space under $\delta_{\mathcal{I}}$-semi-irresolute surjective function is δ-semi-compact.*

Proof. Let $f:(X,\tau,\mathcal{I})\to(Y,\sigma,\mathcal{I})$ is a $\delta_{\mathcal{I}}$-semi-irresolute function from $\delta_{\mathcal{I}}$-semi-compact space $(X,\tau,\mathcal{I})$ onto an ideal topological space $(Y,\sigma,\mathcal{I})$. Let $\{A_\alpha|\alpha\in\Lambda\}$ be a δ-semi-open cover of Y. Then $\{f^{-1}(A_\alpha)|\alpha\in\Lambda\}$ is a $\delta_{\mathcal{I}}$-semi-open cover of X, since f is $\delta_{\mathcal{I}}$-semi-irresolute. As X is $\delta_{\mathcal{I}}$-semi-compact, $\delta_{\mathcal{I}}$-semi-open cover $\{f^{-1}(A_\alpha)|\alpha\in\Lambda\}$ of X has a finite subcover, say, $\{f^{-1}(A_\alpha)|\alpha=1,2,...,n\}$. Therefore $X=\bigcup_{\alpha=1}^n\{f^{-1}(A_\alpha)|\alpha=1,2,...,n\}$. Then $f(X)=\bigcup_{\alpha=1}^n\{A_\alpha|\alpha=1,2,...,n\}$, that is $Y=\bigcup_{\alpha=1}^n\{A_\alpha|\alpha=1,2,...,n\}$. Thus $\{A_1,A_2,...A_n\}$ is a finite subcover for Y. Hence Y is δ-semi-compact.

Definition 5.11. *An ideal toplogical space $(X,\tau,\mathcal{I})$ is called locally $\delta_{\mathcal{I}}$-semi-compact if every point in X has atleast one $\delta_{\mathcal{I}}$-semi-neighborhood whose closure is $\delta_{\mathcal{I}}$-semi-compact.*

Proposition 5.12. *Every $\delta_{\mathcal{I}}$-semi-compact space is locally $\delta_{\mathcal{I}}$-semi-compact.*

Proof. Let $(X,\tau,\mathcal{I})$ be a $\delta_{\mathcal{I}}$-semi-compact space. Let $x\in X$. Then X is a $\delta_{\mathcal{I}}$-semi-neighborhood of x such that $cl(X)=X$ is $\delta_{\mathcal{I}}$-semi-compact. Hence X is locally $\delta_{\mathcal{I}}$-semi-compact.

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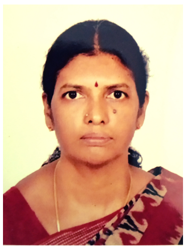
BIOGRAPHY



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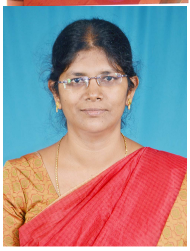

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