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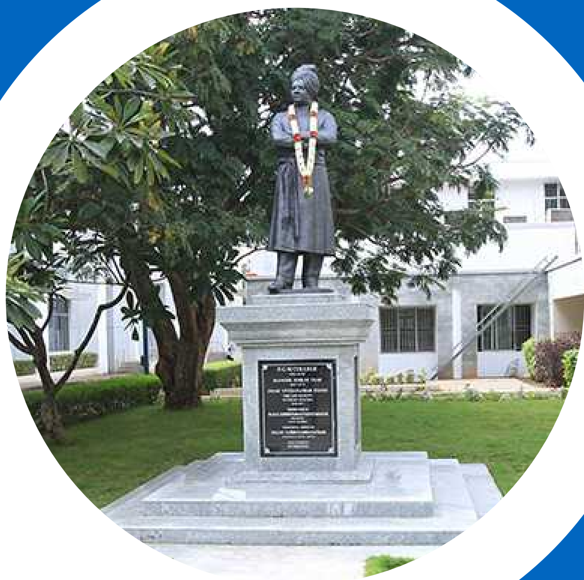

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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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## ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

## ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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# ON $Ng^*\alpha$ -NORMAL AND $Ng^*\alpha$ -REGULAR SPACES IN NANO TOPOLOGICAL SPACES

**V. Rajendran[1] – P. Sathishmohan[2] – M. Amsaveni[3] – M. Chitra[4]**



**ABSTRACT:** In this paper, we introduced a class of space called $Ng^*\alpha$ -normal and $Ng^*\alpha$ -regular spaces and analyzed few of its properties. We have observed some preservation theorem. Also we define $Ng^*\alpha$ -$T_0$ space, $Ng^*\alpha$ -$T_1$ space, and $Ng^*\alpha$ -$T_2$ space, and investigated their properties. Also we have obtain some of their basic results and give have an appropriate examples to understand the abstract concept clearly.

**Keywords:** $Ng^*\alpha$ -continuous functions, $Ng^*\alpha$ -normal space, $Ng^*\alpha$ -regular space, $Ng^*\alpha$ -$T_0$ space, $Ng^*\alpha$ -$T_1$ space, and $Ng^*\alpha$ -$T_2$ space.

## 1. INTRODUCTION

In 1971 Viglino[8] introduced generalized Normal space. Singal and Arya[5] introduced almost normal space and proved that a space is normal if and only if it is both a semi Normal and an almost Normal space. In 1987 Gongulay and Chandel R.S[2] generalized the usual notion of regularity and normality by replacing closed with g-closed set and obtained g -regularity and g -normality. Ganster et.al[3] studied semi g - regularity and semi g -normality. M. Bhuvaneswari and N. Nagaveni[1] introduced Nwg -normal and Nwg - regular in nano topological spaces and P. Sathishmohan et.al[7] Introduced Nano Pre -regular and strongly Nano Pre -regular spaces in nano topological spaces. Further P. Sathishmohan et.al[6] introduced and investigated the further properties of nano -$T_0$ space, nano semi -$T_0$ space, nano pre -$T_0$ space, nano -$T_1$ space, nano semi -$T_1$ space, nano pre -$T_1$ space, nano -$T_2$ space , nano semi -$T_2$ space, nano pre $T_2$ space and obtain some of its basic results. The main purpose of this paper is to bring up the idea about $Ng^*\alpha$ -$T_0$ space, $Ng^*\alpha$ -$T_1$ space, and $Ng^*\alpha$ -$T_2$ -space and obtain some of its basic results. Further we introduced $Ng^*\alpha$ -normal and $Ng^*\alpha$ -regular spaces and investigated some of their properties.

The structure of this manuscript as follows:

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In section 2, some basic definitions and results in nano topological spaces are recalled which are useful to prove the main results.

In section 3, we introduced $Ng^{*}\alpha$ -normal and $Ng^{*}\alpha$ -regular spaces and investigated some of their properties.

In section 4, we define and study the notions of $Ng^{*}\alpha$ -$T_0$ space in nano topological spaces and obtained some of its basic results.

In section 5, we define and study the notions of $Ng^{*}\alpha$ -$T_1$ space in nano topological spaces and some of the properties has been investigated.

In section 6, we define and study the notions of $Ng^{*}\alpha$ -$T_2$ space in nano topological spaces and some of the properties has been investigated.

## 2. PRELIMINARIES

In this section, some basic definitions and results in nano topological spaces are given, which are useful to prove the main results.

**Definition:2.1** [4] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.
   That is, $L_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}\}$, where R(x) denotes the equivalence class determined by $x \in U$.
2. The Upper approximation of X with respect to R is the set of all objects, which can be certain classified as X with respect to R and it is denoted by $U_R(X)$.
   That is, $U_R(X) = \{\bigcup_{x\in U} \{R(x) : R(x)\cap X \neq \varphi\}\}$
3. The Boundary region of X with respect to R is the set of all objects which can be classified as neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
   That is, $B_R(X) = U_R(X) - L_R(X)$.

**Definition:2.2.** [4] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$

(ii) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U is called the nano topology on U with respect to X. We call $\{U, \tau_R(X)\}$ is called the nano topological space. Elements of the nano topology are known as nano open sets in U. Elements of $[\tau_R(X)]^c$ are called nano closed sets.

**Remark: 2.3.** [4] If $[\tau_R(X)]$ is the nano topology on U with respect to X. Then the set $B = \{U, \tau_R(X), B_R(X)\}$ is the basis for $[\tau_R(X)]$.

**Definition: 2.4.** [1] A nano topological space $(U, \tau_R(X))$ is said to be nano normal space if for any pair of disjoint nano closed sets A and B, there exists disjoint nano open sets M and N such that $A \subset M$ and $B \subset N$.

**Definition:2.5.** [1] A nano topological space $(U, \tau_R(X))$ is said to be nano regular space, if for each nano closed set F and each point $x \notin F$, there exists disjoint nano open sets G and H such that $x \in G$ and $F \subset H$.

**Definition: 2.6.** [1] The map $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is called

1. Nano continuous on U, if the inverse image of every nano -closed set in V is nano -closed in U.
2. Nano closed on U, if the image of every nano -closed set in U is nano -closed set in V.
3. $Ng^{*}\alpha$ -closed on U, if the image of every nano -closed set in U is $Ng^{*}\alpha$ -closed set in V.

**Definition:2.7.** [6] A space U is called nano $-T_0$ space for $x, y \in U$ and $x \neq y$, there exist a nano -open set G such that $x \in G$ and $y \notin G$.

**Definition:2.8.** [6] A space U is called nano pre $-T_0$ space for $x, y \in U$ and $x \neq y$, there exists a nano pre -open set G such that $x \in G$ and $y \notin G$.

**Definition:2.9.** [6] A space U is called $N\alpha$ $-T_0$ space for $x, y \in U$ and $x \neq y$, there exists a $N\alpha$ -open set G such that $x \in G$, $y \notin G$.

**Definition:2.10.**[6] A space U is called nano $-T_1$ space for $x, y \in U$ and $x \neq y$, there exists a nano -open sets G and H such that $x \in G$ and $y \notin G$ and $y \in H$, $x \notin H$.

**Definition:2.11.**[6] A space U is called nano pre $-T_1$ space for $x, y \in U$ and $x \neq y$, there exists a nano pre -open sets G and H such that $x \in G$ and $y \notin G$ and $y \in H$, $x \notin H$.

**Definition:2.12.**[6] A space U is called $N\alpha$ $-T_1$ space for $x, y \in U$ and $x \neq y$, there exists a $N\alpha$ -open sets G and H such that $x \in G$ and $y \notin G$ and $y \in H$, $x \notin H$.

**Definition:2.13.**[6] A space U is called nano $-T_2$ space for $x, y \in U$ and $x \neq y$, there exists a disjoint nano -open sets G and H such that $x \in G$ and $y \in H$.

**Definition: 2.14.** [6] A space U is called nano pre -$T_2$ space for x, y ∈ U and $x \neq y$, there exists a disjoint nano pre -open sets G and H such that x ∈ G and y ∈ H.

**Definition: 2.15.** [6] A space U is called Nα -$T_2$ space for x, y ∈ U and $x \neq y$, there exists a adjoint Nα -open sets G and H such that x ∈ G and y ∈ H.

## 3. Nano $g^{*}\alpha$ -normal and regular spaces

In this section we introduced a new space called Ng*α -normal and Ng*α -regular spaces and investigated some of their characteristics.

**Definition: 3.1.** A nano topological space (U, $\tau_R(X)$) is said to be Ng*α -normal space, if for every pair of disjoint nano closed sets A and B, there exists disjoint Ng*α -open sets M and N such that A ⊂ M and B ⊂ N.

**Definition: 3.2.** A nano topological space (U, $\tau_R(X)$) is said to be Ng*α -regular space, if for each nano -closed sets F and each point x ∉ F, there exists disjoint Ng*α -open sets G and H such that x ∈ G and F ⊂ H.

**Theorem: 3.3.** Every nano normal space is Ng*α -normal.

**Proof.** Let (U, $\tau_R(X)$) is nano normal space and A and B are two disjoint pair of nano -closed sets. Since (U, $\tau_R(X)$) is nano -normal there exists disjoint nano -open sets M and N such that A ⊂ M and B ⊂ N. Since every nano -open set is Ng*α -open. Therefore M and N are Ng*α -open sets. Hence (U, $\tau_R(X)$) is Ng*α -normal space.

The converse of the above theorem need not be true as seen from the following example. □

**Example: 3.4.** Let U = {a, b, c, d} with U/R = {{a, b}, {c}, {d}} and X = {a, b}. Then $\tau_R(X)$ = {U, ϕ, {a, b}}.

Then (U, $\tau_R(X)$) is Ng*α -normal space but not nano -normal space.

**Remark: 3.5.** Let U be the universe, X ⊆ U and if $U_R(X)$ = X and $L_R(X)$ = ϕ then (U, $\tau_R(X)$) is not Ng*α -normal.

**Remark: 3.6.** Let U be the universe, X ⊆ U and if $U_R(X)$ = X and $L_R(X)$ = U then (U, $\tau_R(X)$) is not Ng*α -normal.

**Theorem: 3.7.** If A nano topological space U is Ng*α -normal then for every pair of nano -open M and N whose union is U, there exist Ng*α -closed sets A and B such that A ⊂ M , B ⊂ N and A ∪ B = U.

**Proof.** Let M and N be a pair of nano open sets in a Ng*α -normal space U such that M ∪ N = U, then U − M, U − N are disjoint nano -closed sets. Since U is Ng*α -normal space, there exists two Ng*α -open sets $M_1$ and $N_1$ such that U − M ⊂ $M_1$ and U − N ⊂ $N_1$. Then A and B are Ng*α -closed sets such that

$A \subset M$, $B \subset N$ and $A \cup B = U$.

**Theorem: 3.8.** If $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is nano -continuous, injective, $Ng^{*}\alpha$ open function and U is nano normal space then V is $Ng^{*}\alpha$ -normal.

**Proof.** Let E and F be disjoint nano -closed set in V. Since f is nano -continuous bijective, $f^{-1}(E)$ and $f^{-1}(F)$ are disjoint nano -closed in U. Now U is nano -normal space, there exist disjoint nano -open sets G and H such that $f^{-1}(E) \subset G$ and $f^{-1}(F) \subset H$. That is $E \subset f(G)$ and $F \subset f(H)$. Since f is $Ng^{*}\alpha$ -open function, f(G), f (H) are $Ng^{*}\alpha$ -open sets in V and f is injective, $f(G) \cap f(H) = f(G \cap H) = f(\phi)$. Therefore V is $Ng^{*}\alpha$ -normal space.

**Remark: 3.9.** If $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is nano -continuous, injective, nano -open function and U is nano -normal space then V is $Ng^{*}\alpha$ -normal.

**Theorem: 3.10.** If $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is $Ng^{*}\alpha$ –continuous, nano -closed, injective and V is a nano normal space then U is $Ng^{*}\alpha$ -normal.

**Proof.** Let E and F be disjoint nano -closed sets in U. Since f is nano -continuous, bijective, f (E) and f(E) are disjoint nano -closed in V. Now V is nano -normal space, there exist disjoint nano -open sets G and H such that $f(E) \subset G$ and $f(F) \subset H$. That is $E \subset f^{-1}(G)$ and $F \subset f(H)$ since f is $Ng^{*}\alpha$ - function, continuous $f^{-1}(G)$, $f^{-1}(H)$ are $Ng^{*}\alpha$ -open sets in U. $f^{-1}(G) \cap f^{-1}(H) = \phi$. Therefore U is $Ng^{*}\alpha$ -normal space.

**Remark: 3.11.** If $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is nano -continuous, nano -closed, injective and V is nano -normal space then U is $Ng^{*}\alpha$ -normal.

**Theorem: 3.12.** Every nano -regular space is $Ng^{*}\alpha$ -regular space but not conversely.

**Proof.** Let F be a nano closed set and $x \notin F$ be a point of a nano regular space $(U, \tau_R(X))$. Since U is nano -regular space there exist two disjoint nano open sets G and H such that $x \in G$ and $F \subset H$. Since every nano open set is $Ng^{*}\alpha$ -open set, G and H are $Ng^{*}\alpha$ -open sets such that $x \in G$ and $F \subset H$. Hence $(U, \tau_R(X))$ is $Ng^{*}\alpha$ -regular space.

There converse of the above theorem need not be true as seen from the following example.

**Example: 3.13.** Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, d\}, \{c\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{U, \phi, \{b, d\}\}$. Then $(U, \tau_R(X))$ is $Ng^{*}\alpha$ -regular space but not nano regular space.

**Theorem 3.14.** Let $\tau_R(X)$ be a nano topology on U with respect to X. consider the following statements

1. U is nano Regular.
2. For each $x \in U$ and each $A \in \tau_R(X)$ with $x \in A$, there exists $B \in \tau_R(X)$ such that $x \in B \subseteq C_w(B) \subseteq A$. Then the implication (1) $\Rightarrow$ (2) holds if $Nint(A) \in \tau_R(X)$ for every nano closed A of U, then the statements are equivalent.

**Proof.** (1) $\Rightarrow$ (2)

Let $x \notin (U - A)$, where $A \in \tau_R(X)$. Then there exist disjoint G, $B \in \tau_R(X)$ such that $(U - A) \subseteq G$ and $x \in B$. Thus $B \subseteq U - G$ and hence $x \in B \subseteq Ncl(B) \subseteq Ncl(U - G) \subseteq A$.

(2) $\Rightarrow$ (1) Let F be a nano closed and $x \notin F$. Then $x \in U - F \in \tau_R(X)$ and hence there exists $B \in \tau_R(X)$ such that $x \in B \subseteq Ncl(B) \subseteq U - F$. Therefore $F \subseteq U - Ncl(B) = Nint(U - B) \in \tau_R(X)$.

**Theorem: 3.15.** If $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is nano -continuous, bijective, $Ng^*\alpha$ -open function and U is a nano - regular space then V is $Ng^*\alpha$ -regular.

**Proof.** Let F be nano -closed set in V and $y \notin F$. Let $y = f(x)$ for some $x \in U$. Since **f** is nano -continuous, $f^{-1}(F)$ is nano -closed in U such that $x \notin f^{-1}(F)$. Now U is nano -regular space, there exist disjoint nano -open sets G and H such that $x \in G$ and $f^{-1}(F) \subset H$. That is $y = f(x) \in f(G)$ and $F \in f(H)$. Since f is $Ng^*\alpha$ -open function, f (G) and f (H) are $Ng^*\alpha$ -open sets in V. $f(G) \cap f(H) = f(G \cap H) = \phi$. Therefore V is $Ng^*\alpha$ -regular space.

**Remark: 3.16.** If $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is nano -continuous, bijective, $Ng^*\alpha$ -open function and U is a $Ng^*\alpha$ -regular space then V is $Ng^*\alpha$ -regular.

**Theorem: 3.17.** If $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is nano -continuous, nano -closed, injective and V is a nano -regular space then U is $Ng^*\alpha$ -regular.

**Proof.** Let F be nano -closed set in U and $x \notin F$. Since f is nano -closed injective, f(F) is nano -closed set in V such that $f(x) \notin f(F)$. Now V is nano regular, there exist disjoint nano -open sets G and H such that $f(x) \subset G$ and $f(F) \subset H$. Thus $x \in f^{-1}(G)$ and $F \subset f^{-1}(H)$. Since f is $Ng^*\alpha$ -continuous function $f^{-1}(G)$ and $f^{-1}(H)$ are $Ng^*\alpha$ -open sets in U, $f(G) \cap f(H) = \phi$. Hence U is $Ng^*\alpha$ -regular.

**Remark: 3.18.** If $f : (U, \tau_R(X)) \longrightarrow (V, \tau_{R'}(Y))$ is nano -continuous, nano -closed, injective and V is a nano -regular space then U is $Ng^*\alpha$ -regular.

## 4. Nano $g^*\alpha$ -$T_0$ Spaces

In this section, we define and study the notions of $Ng^*\alpha$ -$T_0$ spaces, in nano topological spaces and obtain some of their basic results.

**Definition: 4.1.** A space U is called $Ng^*\alpha$ -$T_0$ spaces for $x, y \in U$ and $x \neq y$, there exists a $Ng^*\alpha$ -open set G such that $x \in G$ and $y \notin G$.

**Theorem:4.2.** Let $(U, \tau_R(X))$ be a nano topological space, then for every nano -$T_0$ space is $Ng^*\alpha$ -$T_0$ spaces.

**Proof**. Let U be nano $T_0$ space, x and y be two distinct points of U, as U is nano $T_0$ there exists nano open set G such that $x \in G$ and $y \notin G$, since every nano open set is $Ng^*\alpha$ -open and hence G is nano $Ng^*\alpha$ -open set such that $x \in G$ and $y \notin G$. Which implies U is $Ng^*\alpha$ -$T_0$ spaces.

The converse of the above theorem need not be true in general.

**Example 4.3.** Let U = {a, b, c, d} with U/R = {{a}, {b, c}, {d}} and X = {b, d}. Then $\tau_R(X)$ = {U, ϕ, {d}, {b, c}, {b, c, d}} be nano topology on U. We have

NPO(U, X) = {U, ϕ, {b}, {c}, {d}, {b, c}, {b, d}, {c, d}, {a, b, d}, {b, c, d}, {a, c, d}}

NαO(U, X) = {U, ϕ, {d}, {b, c}, {b, c, d}}

$Ng^*\alpha O$(U, X) = {U, ϕ, {b, c, d}, {c, d}, {b, d}, {b, c}, {d}, {c}, {b}}

Here x = {b, d} and y = {a} then it is $Ng^*\alpha$ -$T_0$ space but not nano -$T_0$ space.

**Theorem: 4.4.** Every $Ng^*\alpha$ -$T_0$ space is nano pre -$T_0$ space but not conversely.

**Proof.** Let U be $Ng^*\alpha$ -$T_0$ space, x and y be two distinct points of U, as U is $Ng^*\alpha$ -$T_0$ space there exists $Ng^*\alpha$ -open set G such that x ∈ G and y ∉ G, since every $Ng^*\alpha$ -$T_0$ -open set is nano pre-open and hence G is nano pre -open set such that x ∈ G and y ∉ G. Which implies U is nano pre -$T_0$ space.

The converse of the above theorem need not be true in general.

**Example: 4.5.** From the example (4.3), Let x = {a, c, d} and y = {a} then it is a nano pre -$T_0$ space but not $Ng^*\alpha$ -$T_0$ space.

**Theorem: 4.6.** Every Nα -$T_0$ space is $Ng^*\alpha$ -$T_0$ but not conversely.

**Proof**. Let U be Nα -$T_0$ space and x and y be two distinct points of U, as U is Nα -$T_0$ there exists Nα -open set G such that x ∈ G and y ∉ G, since every Nα -open set is $Ng^*\alpha$ -open and hence G is nano $Ng^*\alpha$ -open set such that x ∈ G and y ∉ G. Which implies U is Nα -$T_0$ space.

The converse of the above theorem need not be true in general.

**Example 4.7.** From the example(4.3), Let x = {c} and y = {a} then it is $Ng^*\alpha$ -$T_0$ space but not Nα -$T_0$ space.

**Theorem 4.8.** If P ∈ $Ng^*\alpha O$(X) and Q ∈ $Ng^*\alpha O$(X) then Q ∈ $Ng^*\alpha O$(X)

**Theorem: 4.9.** A space U is $Ng^*\alpha$ -$T_0$ iff for each x ∈ U, there exists a $Ng^*\alpha$ -open set X of U containing x such that the subspace X is $Ng^*\alpha$ -$T_0$.

**Proof.** If U is nano $Ng^*\alpha$ –$T_0$, take U as X. Then X is a $Ng^*\alpha$ -open set containing x such that the subspace X is $Ng^*\alpha$ -$T_0$ , for every x ∈ U.

Next suppose that $x_1$, $x_2$ be any two distinct points of U. By hypothesis, there exists Xj ∈ $Ng^*\alpha$ O(U ) such that xj ∈ Xj and the subspace Xj is $Ng^*\alpha$ -$T_0$, for j = 1, 2. If $x_2$ ∈ $X_1$ then the proof is completed. If $x_2$ ∉ $X_1$ then as $X_1$ is $Ng^*\alpha$ -$T_0$, there exists $W_1$ ∈ $Ng^*\alpha O(X_1)$ such that $x_1$ ∈ $w_1$ and $x_2$ ∉ $W_1$ or there exists

$W_2 \in Ng^*\alpha O(X_1)$ such that $x_2 \in W_2$ and $x_1 \in W_2$. Since $X_1 \in Ng^*\alpha\ O(U)$, it follows from theorem(4.8), $Wj \in Ng^*\alpha O(U)$ for j = 1, 2. This means that the space U is $Ng^*\alpha$ -$T_0$.

## 5. Nano $g^*\alpha$ -$T_1$ Spaces

In this section, we define and study the notions of $Ng^*\alpha$ - $T_1$ spaces, in nano topological spaces and obtain some of their basic results.

**Definition: 5.1.** A space U is called $Ng^*\alpha$ -$T_1$ space for $x, y \in U$ and $x \neq y$, there exists a $Ng^*\alpha$ -open set G and H such that $x \in G$ and $y \notin G$. and $y \in H$, $x \notin H$.

**Theorem: 5.2.** Every nano-$T_1$ space is $Ng^*\alpha$ -$T_1$ spaces.

**Proof.** Let U be nano-$T_1$ space and let $x \neq y$ in U. Then there exists distinct nano open sets G and H such that $x \in G$ and $y \in H$. Since every nano open set is $Ng^*\alpha$ -open set. Hence G and H are distinct $Ng^*\alpha$ -open sets such that $x \in G$ and $y \in H$.

The converse of above theorem is need not be true in general.

**Example 5.3.** Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{b, d\}$.

Then $\tau_R(X) = \{U, \phi, \{d\}, \{b, c\}, \{b, c, d\}\}$ be nano topology on U. We have

$NPO(U, X) = \{U, \phi, \{b\}, \{c\}, \{d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$.

$N\alpha O(U, X) = \{U, \phi, \{d\}, \{b, c\}, \{b, c, d\}\}$.

$Ng^*\alpha O(U, X) = \{U, \phi, \{b, c, d\}, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}\}$.

Let $V = \{a, b, c, d\}$ with $V/R_2 = \{\{b\}, \{c\}, \{a, d\}\}$ and $Y = \{b, d\}$.

Then $\tau_R(Y) = \{V, \phi, \{b\}, \{a, d\}, \{a, b, d\}\}$ be a nano topology on V, we have

$NPO(V, Y) = \{V, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

$N\alpha O(V, Y) = \{V, \phi, \{b\}, \{a, d\}, \{a, b, d\}\}$.

$Ng^*\alpha O(V, Y) = \{V, \phi, \{a, b, d\}, \{b, d\}, \{a, d\}, \{a, b\}, \{d\}, \{a\}, \{b\}\}$.

Here $x = \{c\}$ and $y = \{a\}$ then it is $Ng^*\alpha$ -$T_1$ space but not nano $T_1$ space.

**Theorem 5.4.** Every $N\alpha$ -$T_1$ space is $Ng^*\alpha$ -$T_1$ space.

**Proof.** Let U be $N\alpha$ -$T_1$ space and let $x \neq y$ in U. Then there exists distinct nano open sets G and H such that $x \in G$ and $y \in H$. Since every nano open set is $Ng^*\alpha$ -open set. Here G and H are distinct $Ng^*\alpha$ -open sets such that $x \in G$ and $y \in H$.

The converse of above theorem is need not be true in general.

**Example 5.5.** From the example 5.3, Let $x = \{c\}$ and $y = \{a\}$ then it is $Ng^*\alpha$ -T1 space but not $N\alpha$ -$T_1$ space.

**Theorem 5.6.** Every $Ng^*\alpha$ -$T_1$ space is nano pre -$T_1$ space.

**Proof.** Let U be $Ng^*\alpha$ -$T_1$ space and let $x \neq y$ in U. Then there exists distinct nano- open sets G and H such that $x \in G$ and $y \in H$. Since every nano open set is nano -open set. Hence G and H are distinct pre-open sets such that $x \in G$ and $y \in H$.

The converse of above theorem is need not be true in general.

**Example: 5.7.** From the example 5.3, Let x = {a, b, c} and y = {a, c, d} then it is clear that x ∈ G and y ∉ G. Then we can say that it is $Ng^*\alpha$ -$T_0$ space.

**Theorem: 5.8.** Let $(U, \tau_R(X))$ be a nano topological space, then for each $Ng^*\alpha$ -$T_1$ space is $Ng^*\alpha$ -$T_0$ space.

**Proof**. Let U be $Ng^*\alpha$ -$T_1$ space and x and y be two distinct points of U, as U is $Ng^*\alpha$ -$T_1$ space there exists $Ng^*\alpha$ -open set G such that x ∈ G and y ∉ G, since every nano open set is $Ng^*\alpha$-open and hence G is $Ng^*\alpha$-open set such that x ∈ G and y ∉ G ⇒ U is $Ng^*\alpha$ -$T_0$.

**Example: 5.9.** From the example 5.3, Let x = {b, c, d} and y = {a, d} then it is clear that x ∈ G and y ∉ G. Then we can say that it is $Ng^*\alpha$ -$T_0$ space.

**Lemma 5.10.** Union of $Ng^*\alpha$ -open sets is $Ng^*\alpha$ -open.

**Theorem: 5.11.** A space U is $Ng^*\alpha$ -T1 space iff for any point x ∈ U, the singleton set {x} is $Ng^*\alpha$ -closed set.

**Proof.** Let every singleton set {x} ∈ U of U be $Ng^*\alpha$ -closed. Therefore U − {x} is $Ng^*\alpha$ -open. Now we show that U is $Ng^*\alpha$ -$T_1$ space. Let x, y ∈ U with $x \neq y$. Then {x} and {y} are $Ng^*\alpha$ -closed sets. Therefore U − {x} is a $Ng^*\alpha$ -open set containing y but not x and U − {y} is a $Ng^*\alpha$ -open set containing x but not y. Thus U is $Ng^*\alpha$ -$T_1$ space.

Conversely, let U be a $Ng^*\alpha$ -T1 space. Assume that x ∈ U be an arbitrary point. Now, we show that {x} is $Ng^*\alpha$ -closed or U − {x} is nano $Ng^*\alpha$ -open. Let z ∈ ∪ − {x} then clearly $z \neq x$. Now, U is $Ng^*\alpha$ -$T_1$ and z is a point different from x so there exists a $Ng^*\alpha$ -open set GZ such that $z \in G_z$ but $x \notin G_z$. Hence $z \in G_z \subset U - \{x\}$. Therefore $U - \{x\} = \cup\{G_z / z \neq x\}$. So U − {x} being the union of $Ng^*\alpha$-open sets is $Ng^*\alpha$-open. Hence {x} is $Ng^*\alpha$- closed set.

**Theorem 5.12.** A space U is $Ng^*\alpha$ -$T_1$ space iff for each point x ∈ U, there exists a $Ng^*\alpha$-open set X of U containing x such that the subspace X is $Ng^*\alpha$ -$T_1$.

**Proof.** Proof is similar to theorem 4.9.

## 6. Nano g*α - $T_2$ Spaces

In this section, we define and study the notions of Ng*α -$T_2$ spaces, in nano topological spaces and obtained some of their basic results.

**Definition: 6.1.** A space U is called Ng*α - $T_2$ spaces for x, y ∈ U and $x \neq y$ there exists disjoint Ng*α -open set G and H such that x ∈ G and y ∈ H.

**Theorem: 6.2.** Every nano -$T_2$ space is Ng*α -$T_2$ space.

**Proof.** Let U be nano -$T_2$ space and let $x \neq y$ in U. Then there exists disjoint nano open sets G and H such that x ∈ G and y ∈ H. Since every nano open set is Ng*α -open set. Here G and H are disjoint Ng*α -open sets such that x ∈ G and y ∈ H.

The converse of above theorem is need not be true in general.

**Example 6.3.** Let U = {a, b, c, d} with U/R = {{a, b}, {c}, {d}} and X = {a, b}.

Then $\tau_R(X)$ = {U, ϕ, {a, b}} be nano topology on U. We have

NαO(U, X) = {U, ϕ, {a, b}, {a, b, c}, {a, b, d}}.

Ng*αO(U, X) = {U, ϕ, {a, b, d}, {a, b, c}, {a, b}, {a}, {b}}.

Let V = {a, b, c} with $V/R_2$ = {{a}, {b, c}} and Y = {a, c}.

Then $\tau_R(Y)$ = {V, ϕ, {a}, {b, c}} be a nano topology on V, we have

NαO(V, Y ) = {V, ϕ, {a}, {b, c}}.

Ng*αO(V, Y ) = {V, ϕ, {a}, {b, c}}.

Here x = {a} and y = {b, c} then it is Ng*α -$T_2$ space but not nano $T_2$ space.

**Theorem: 6.4.** Every Nα -$T_2$ space is Ng*α -$T_2$ space.

**Proof.** Let U be Nα -$T_2$ space and let $x \neq y$ in U. Then there exists disjoint nano open sets G and H such that x ∈ G and y ∈ H. As every nano open set is Ng*α -open set G and H are disjoint Ng*α -open sets such that x ∈ G and y ∈ H.

The converse of above theorem is need not be true in general.

**Example 6.5.** From the example 6.3, Let x = {a} and y = {b, c} then it is clear that x ∈ G and y ∈ H. Then we can say that it is Ng*α -$T_2$ space but not Nα -$T_2$ space.

**Theorem:6.6.** Let (U, $\tau_R(X)$) be a nano topological space, then for each Ng*α -$T_2$ space is Ng*α -$T_0$ space.

**Proof.** Let U be $Ng^*\alpha$ -$T_2$ space and x and y be two distinct points of U, as U is $Ng^*\alpha$ -$T_2$ space there exists $Ng^*\alpha$ -open set G such that $x \in G$ and $y \notin G$, since every nano open set is $Ng^*\alpha$-open and hence G is $Ng^*\alpha$-open set such that $x \in G$ and $y \notin G$ which implies U is $Ng^*\alpha$ -$T_0$. □

**Example: 6.7.** From the example (6.3), Let $x = \{a, b\}$ and $y = \{b, c\}$ then it is clear that it is $Ng^*\alpha$ -$T_0$ space but not $Ng^*\alpha$ -$T_2$ space.

**Lemma: 6.8.** Every $N\alpha$ -open subspace of a $Ng^*\alpha$ -$T_2$ space is $Ng^*\alpha$ -$T_2$ space.

**Proof.** Let X be a $N\alpha$ -open subspace of a $Ng^*\alpha$ -$T_2$ space. Let x and y be any two distinct points of X. Since U is $Ng^*\alpha$ -$T_2$ space and $x \subset U$, there exists two disjoint $Ng^*\alpha$- open sets G and H in U such that $x \in G$ and $y \in H$. Let $A = G \cap X$ and $B = H \cap X$. Then A and B are nano $g^*\alpha$ open sets in X containing x and y respectively. Also, $A \cap B = (G \cap X) \cap (H \cap X) = \phi$. Hence X is $Ng^*\alpha$ -$T_2$ space.

## CONCLUSION

In the above work we have look into the characterization of $Ng^*\alpha$ -Normal space, $Ng^*\alpha$ -Regular space, $Ng^*\alpha$ -$T_0$ -space, $Ng^*\alpha$ -$T_1$ -space, $Ng^*\alpha$ -$T_2$ -space and obtain some relationship between the existing sets.

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