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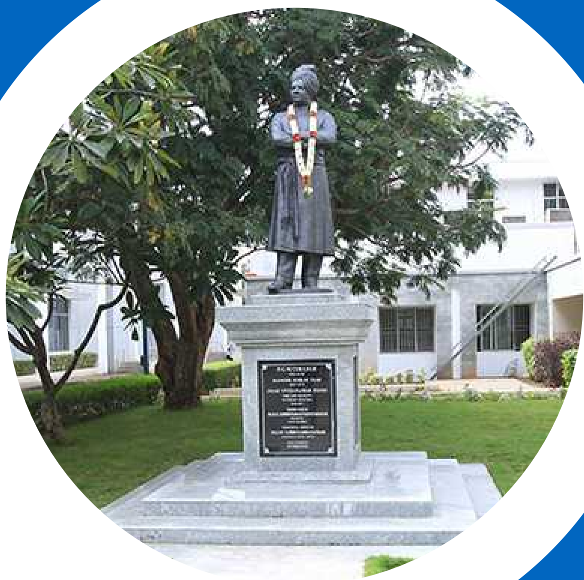

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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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## ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

## ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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# Regular Generalized Irresolute Continuous Mappings in Bipolar Pythagorean Fuzzy Topological Spaces

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**ABSTRACT:** The concept of this paper is to highlight the theory of Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Continuous Mappings in Bipolar Pythagorean Fuzzy Topological Spaces and study some of their basic properties and inter relationship with other existing Bipolar Pythagorean Fuzzy Continuous Mappings.
**KEYWORDS:** Bipolar Pythagorean Fuzzy sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagorean Fuzzy Regular Generalized Closed sets, Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Continuous Mappings.

## 1. INTRODUCTION

The concept of Fuzzy sets was introduced by Zadeh [1] which shows many applications in the field of Science, Engineering, Management and Technology. The idea of Fuzzy sets deals with the data which are vague and uncertain. After that Atanassov [2] introduced the notion of intuitionistic fuzzy sets and Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. Yager [4] proposed another class of nonstandard fuzzy sets, called Pythagorean fuzzy sets and Murat Olgun, Mehmet Ünver, Seyhmus Yardimci[5] introduced the notion of Pythagorean fuzzy topological spaces. Zhang [6] introduced the extension of fuzzy set with bipolarity, called Bipolar value fuzzy sets. Bosc and Pivert [11] said that "Bipolarity" refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired or considered as acceptable. Negative statement corresponds to what is impossible, rejected or forbidden, as they specify which objects are more desirable than others, without rejecting those that do not meet the wishes. In bipolar valued fuzzy set interval of membership value is [-1,1].

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In this paper, we introduce, Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Mappings and study some of the basic properties. In this paper, we introduce, Bipolar Pythagorean Fuzzy Perfectly Regular Generalized Continuous Mappings (BPF$p$RG continuous mappings) and study some of the basic properties.

## 2. PRELIMINARIES

***Definition 2.1:*** Let X be the non-empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by P=$\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non-membership of each element $x \in X$ to the set P, respectively, and $0 \leqslant \mu_P^2(x) + \nu_P^2(x) \leqslant 1$ for each $x \in X$. The degree of indeterminacy $I_P = \sqrt{1-\mu_P^2(x) - \nu_P^2(x)}$ for each $x \in X$.

***Definition 2.2:*** Let X be a non-empty set. A Bipolar Pythagorean Fuzzy Set $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-) : x \in X\}$ where$\mu_A^+ : X \to [0,1]$, $\nu_A^+ : X \to [0,1]$, $\mu_A^- : X \to [-1,0]$, $\nu_A^- : X \to [-1,0]$ are the mappings such that $0 \leqslant (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leqslant 1$ and $-1 \leqslant (\mu_A^-(x))^2 + (\nu_A^-(x))^2 \leqslant 0$ where $\mu_A^+(x)$ denote the positive membership degree, $\nu_A^+(x)$ denote the positive non membership degree, $\mu_A^-(x)$ denote the negative membership degree, $\nu_A^-(x)$ denote the negative non membership degree.

***Definition 2.3*** Let $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B^+(x), \nu_B^+(x), \mu_B^-(x), \nu_B^-(x)\rangle : x \in X\}$ be two Bipolar Pythagorean Fuzzy sets over X. Then

(i) $A^c = \{\langle x, \nu_A^+(x), \mu_A^+(x), \nu_A^-(x), \mu_A^-(x)\rangle : x \in X\}$,

(ii) $A \cap B = \{\langle x, \min\{\mu_A^+(x), \mu_B^+(x)\}, \max\{\nu_A^+(x), \nu_B^+(x)\}, \max\{\mu_A^-(x), \mu_B^-(x)\}, \min\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$

(iii) $A \cup B = \{\langle x, \max\{\mu_A^+(x), \mu_B^+(x)\}, \min\{\nu_A^+(x), \nu_B^+(x)\}, \min\{\mu_A^-(x), \mu_B^-(x)\}, \max\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$

(iv) $A$ is a Bipolar Pythagorean subset of $B$ and write $A \subseteq B$ if

$\langle \mu_A^+(x) \leqslant \mu_B^+(x), \nu_A^+(x) \geqslant \nu_B^+(x), \mu_A^-(x) \geqslant \mu_B^-(x), \nu_A^-(x) \leqslant \nu_B^-(x)\rangle \ for\ each\ x \in X$

(v) $0_X = \{\langle x, 0,1,0,-1\rangle : x \in X\}$ and $1_X = \{\langle x, 1,0,-1,0\rangle : x \in X\}$.

***Definition 2.4: Bipolar Pythagorean Fuzzy Topological Spaces:*** Let $X \neq \emptyset$ be a set and $\tau_p$ be a family of Bipolar Pythagorean fuzzy subsets of X. If

$T_1$ $0_X, 1_X \in \tau_p$.

$T_2$ For any $P_1, P_2 \in \tau_p$, we have $P_1 \cap P_2 \in \tau_p$.

$T_3$ $\cup P_i \in \tau_p$ for an arbitrary family $\{P_i : i \in J\} \subseteq \tau_p$.

Then $\tau_p$ is called Bipolar Pythagorean Fuzzy Topology on X and the pair $(X, \tau_p)$ is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of $\tau_p$ is called Bipolar Pythagorean fuzzy open set (BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set (BPFCS).

***Definition 2.5:*** If BPFS $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ in a BPTS $(X, \tau_p)$ is said to be (a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if $\mathcal{B}_{PF}int(\mathcal{B}_{PF}cl(A)) \subseteq A$.

(c) Bipolar Pythagorean Fuzzy Preclosed set (BPFPCS) if $\mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(A)) \subseteq A$.

(e) Bipolar Pythagorean Fuzzy $\alpha$ closed set (BPF$\alpha$CS) if $\mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(cl(A)) \subseteq A$.

(i) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if $A = \mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(A))$.

(k) A Bipolar Pythagorean Fuzzy set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Generalized closed set (BPFGCS), if $\mathcal{B}_{\text{PF}}cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is BPFOS in $(X, \tau_p)$.

(l) A Bipolar Pythagorean Fuzzy set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Generalized open set (BPFGCS), if $A^c$ is a BPFGCS in $(X, \tau_p)$.

(m) A Bipolar Pythagorean Fuzzy set $A$ of a BPFTS $(X, \tau_p)$ is a Bipolar Pythagorean Fuzzy Regular Generalized closed set (BPFGCS), if $\mathcal{B}_{\text{PF}}cl(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is BPFROS in $(X, \tau_p)$.

***Definition 2.6:*** Let $\mathcal{H}$ be a mapping from an BPFTS in $(X, \tau_p)$ into a BPFTS $(Y, \sigma_p)$. Then $\mathcal{H}$ is said to be Bipolar Pythagorean Fuzzy Continuous mapping if $\mathcal{H}^{-1}(A) \in BPFO(X)$ for every $A \in (Y, \sigma_p)$.

***Definition 2.7:*** A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be

(i) BPF semi continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFSO(X) for every $A \in (Y, \sigma_p)$.

(ii) BPF$\alpha$ continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPF$\alpha$O(X) for every $A \in (Y, \sigma_p)$.

(iii) BPF Pre continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFPO(X) for every $A \in (Y, \sigma_p)$.

***Definition 2.8:*** A BPFTS $(X, \tau_p)$ is said to be a $BPFR_cT_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular $_cT_{\frac{1}{2}}$ space) if every BPFRGCS in $(X, \tau_p)$ is a BPFCS in $(X, \tau_p)$.

***Definition 2.9:*** A BPFTS $(X, \tau_p)$ is said to be a $BPFR_gT_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular Generalized $_gT_{\frac{1}{2}}$ space) if every BPFRGCS in $(X, \tau_p)$ is a BPFGCS in $(X, \tau_p)$.

***Definition 2.10:*** A BPFTS $(X, \tau_p)$ is said to be a $BPFR_\alpha T_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular Generalized $_\alpha T_{\frac{1}{2}}$ space) if every BPFRGCS in $(X, \tau_p)$ is a BPF$\alpha$CS in $(X, \tau_p)$.

***Definition 2.12:*** A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy irresolute mapping (BPF irresolute mapping) if $\mathcal{H}^{-1}(A) \in$ BPFCS(X) for every BPFCS $A$ in $(Y, \sigma_p)$.

***Definition 2.13:*** A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Generalized irresolute mapping (BPFG irresolute mapping) if $\mathcal{H}^{-1}(A) \in$ BPFGCS(X) for every BPFGCS $A$ in $(Y, \sigma_p)$.

## 3. BIPOLAR PYTHAGOREAN FUZZY REGULAR GENERALIZED IRRESOLUTE MAPPINGS

In this section we have introduced Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Mappings and study some of its properties.

***Definition 3.1:*** A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be Bipolar Pythagorean Fuzzy Regular Generalized irresolute mapping (BPFRG irresolute) if $\mathcal{H}^{-1}(A)$ is a BPFRGCS in $(X, \tau_p)$ for every BPFRGCS $A$ in $(Y, \sigma_p)$.

***Example 3.2:*** Let X={a, b} and Y={u, v} and $T_1$=(x, (0.6, 0.7), (0.6, 0.6), (-0.5, -0.6), (-0.5, -0.5 )), $T_2$=(y, (0.5, 0.4), (0.5, 0.4), (-0.6, -0.5), (-0.6, -0.5)). Then $\tau_p$=$\{0_p, T_1, 1_p\}$ and $\sigma_p$=$\{0_p, T_2, 1_p\}$ be a BPFTs on $X$ and $Y$ respectively. Define a mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ by $\mathcal{H}(a) = u$ and $\mathcal{H}(b) = v$. The BPFS $A$ = (y, (0.2, 0), (0.6, 0.7), (-0.3, -0.1), (-0.7, -0.6)) is said to be BPFRG irresolute mapping (BPFRG irresolute), since $\mathcal{H}^{-1}(A)$ = (x, (0.2, 0), (0.6, 0.7), (-0.3, -0.1), (-0.7, -0.6)) is a BPFRGCS in $(X, \tau_p)$ for every BPFRGCS $A$ in $(Y, \sigma_p)$.

***Theorem 3.3:*** If $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is a BPFRG irresolute, then $\mathcal{H}$ is BPFRG continuous mapping but not conversely.

***Proof:*** Let $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFG irresolute mapping in $(X, \tau_p)$. Let $A$ be any BPFCS in $(Y, \sigma_p)$, Since every BPFCS is a BPFRGCS, $A$ is a BPFRGCS in $(Y, \sigma_p)$. By hypothesis, $\mathcal{H}^{-1}(A)$ is a BPFRGCS in $(X, \tau_p)$. Hence $\mathcal{H}$ is a BPFRG continuous mapping.

***Example 3.4:*** Let X={a,b}, Y={u,v} and $T_1$=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7 )), $T_2$=(y, (0.4, 0.6), (0.5, 0.6), (-0.5, -0.7), (-0.6, -0.7)). Then $\tau_p$=$\{0_p, T_1, 1_p\}$ and $\sigma_p$=$\{0_p, T_2, 1_p\}$ be a BPFTs on $X$ and $Y$ respectively. Define a mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ by $\mathcal{H}(a) = u$ and $\mathcal{H}(b) = v$. The BPFS $A$ = (y, (0.5, 0.6), (0.4, 0.6), (-0.6, -0.7), (-0.5, -0.7)) is not BPFRGCS in $(Y, \sigma_p)$, since $\mathcal{B}_{\text{PF}}cl(A) = T_2^c \nsubseteq U$, whenever $A \subseteq U$ but $\mathcal{H}^{-1}(A)$ is BPFRGCS in $(X, \tau_p)$, as $\mathcal{B}_{\text{PF}}cl(\mathcal{H}^{-1}(A)) = T_1^c \subseteq 1_p$, whenever $\mathcal{H}^{-1}(A) \subseteq 1_p$. Therefore $\mathcal{H}$ is not BPFRG irresolute mapping.

***Theorem 3.5:*** If $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ and $\mathcal{R}: (Y, \sigma_p) \to (Z, \zeta_p)$ are two BPFRG irresolute mappings, then $\mathcal{R}o\mathcal{H}: (X, \tau_p) \to (Z, \zeta_p)$ is a BPFRG irresolute mapping.

***Proof:*** Let $A$ be BPFRGCS in $(Z, \zeta_p)$. Then by hypothesis, $\mathcal{R}^{-1}(A)$ is a BPFRGCS in $(Y, \sigma_p)$. Since $\mathcal{R}$ is a BPFRG irresolute mapping, $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$ is a BPFRGCS in $(X, \tau_p)$. then, $(\mathcal{R}o\mathcal{H})^{-1}(A)$ is a BPFRGCS in $(X, \tau_p)$. Therefore, $\mathcal{R}o\mathcal{H}$ is a BPFRG irresolute mapping.

***Example 3.6:*** Let X={a,b}, Y={u,v} and Z={p,q} and $T_1$=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7 )), $T_2$=(y, (0.4, 0.6), (0.5, 0.6), (-0.5, -0.7), (-0.6, -0.7)), $T_3$=(z, (0.5, 0.7), (0.5, 0.7), (-0.6, -0.8), (-0.7, -0.8)). Then $\tau_p$=$\{0_p, T_1, 1_p\}$, $\sigma_p$=$\{0_p, T_2, 1_p\}$ and $\zeta_p$=$\{0_p, T_3, 1_p\}$ be a BPFTs on $X$, $Y$ and $Z$ respectively. Define the mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ by $\mathcal{H}(a) = u$ and $\mathcal{H}(b) = v$ and $\mathcal{R}: (Y, \sigma_p) \to (Z, \zeta_p)$ by $\mathcal{R}(u) = p$ and $\mathcal{H}(v) = q$. The BPFS $A$ = (z, (0.4, 0.3), (0.5, 0.7), (-0.5, -0.8), (-0.7, -0.8)) is a BPFRGCS in $(Z, \zeta_p)$, since $\mathcal{B}_{\text{PF}}cl(A) = T_3^c \subseteq T_3$, whenever $A \subseteq \{T_3, 1_p\}$ and $\mathcal{R}^{-1}(A)$ is a BPFRGCS in $(Y, \sigma_p)$, since $\mathcal{B}_{\text{PF}}cl(\mathcal{R}^{-1}(A)) = 1_p \subseteq U$, whenever $\mathcal{R}^{-1}(A) \subseteq U$ and $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$ is a BPFRGCS in $(X, \tau_p)$, since $\mathcal{B}_{\text{PF}}cl\left(\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))\right) = 1_p \subseteq U$, whenever $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A)) \subseteq U$. Therefore $\mathcal{R}o\mathcal{H}$ is BPFRG irresolute mapping.

***Theorem 3.7:*** If $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ and $\mathcal{R}: (Y, \sigma_p) \to (Z, \zeta_p)$ are two BPFRG irresolute mappings, then $\mathcal{R}o\mathcal{H}: (X, \tau_p) \to (Z, \zeta_p)$ is a BPFRG continuous mapping.

***Proof:*** Let $A$ be BPFCS in $(Z, \zeta_p)$. Then by hypothesis, $\mathcal{R}^{-1}(A)$ is a BPFRGCS in $(Y, \sigma_p)$. Since $\mathcal{H}$ is a BPFRG irresolute mapping, $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$ is a BPFRGCS in $(X, \tau_p)$. then, $(\mathcal{R}o\mathcal{H})^{-1}(A)$ is a BPFRGCS in $(X, \tau_p)$. Therefore, $\mathcal{R}o\mathcal{H}$ is a BPFRG continuous mapping.

***Theorem 3.8:*** If $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is a BPFRG irresolute mapping in a $BPFR_cT_{\frac{1}{2}}$ space in $(X, \tau_p)$, Then $\mathcal{H}$ is a BPF continuous mapping.

***Proof:*** Let $A$ be a BPFCS in $(Y, \sigma_p)$. Then $A$ is a BPFRG irresolute mapping in $(Y, \sigma_p)$. Since $\mathcal{H}$ is a BPFRG irresolute, $\mathcal{H}^{-1}(A)$ is a BPFRGCS in $(X, \tau_p)$. Since X is a $BPFR_cT_{\frac{1}{2}}$ space, $\mathcal{H}^{-1}(A)$ is a BPFCS in $(X, \tau_p)$. Hence $\mathcal{H}$ is a BPF continuous mapping.

***Theorem 3.9:*** If $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ is a BPFRG irresolute mapping in a $BPFR_{\mathcal{R}}T_{\frac{1}{2}}$ space in $(X,\tau_p)$, Then $\mathcal{H}$ is a BPFG irresolute mapping.

***Proof:*** Let $A$ be a BPFGCS in $(Y,\sigma_p)$. Then $A$ is a BPFRGCS in $(Y,\sigma_p)$. Therefore $\mathcal{H}^{-1}(A)$ is a BPFRGCS in $(x,\tau_p)$, by hypothesis. Since$(X,\tau_p)$ is a $BPFR_{\mathcal{R}}T_{\frac{1}{2}}$ space, $\mathcal{H}^{-1}(A)$ is a BPFGCS in $(Y,\sigma_p)$. Hence $\mathcal{H}$ is a BPFG irresolute mapping.

***Theorem 3.10:*** Let $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ be a mapping from a BPFTS $(X,\tau_p)$ into a BPFTS $(Y,\sigma_p)$. Then the following conditions are equivalent if $(X,\tau_p)$ and $(Y,\sigma_p)$ are $BPFR_cT_{\frac{1}{2}}$ spaces:

(i) $\mathcal{H}$ is a BPFRG irresolute mapping.

(ii) $\mathcal{H}^{-1}(B)$ is a BPFRGOS in $(X,\tau_p)$ for each BPFRGOS in $(Y,\sigma_p)$.

(iii) $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B)\subseteq\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$ for each BPFS $B$ of $(Y,\sigma_p)$.

***Proof:*** (i)⇒(ii): Obviously true.

(ii)⇒(iii): Let $B$ be any BPFS in $(Y,\sigma_p)$. Clearly $B\subseteq\mathcal{B}_{\mathrm{PF}}cl(B)$. Then $\mathcal{H}^{-1}(B)\subseteq\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$. Since $cl(B)$ is a BPFCS in $(Y,\sigma_p)$, $\mathcal{B}_{\mathrm{PF}}cl(B)$ is a BPFRGCS in $(Y,\sigma_p)$. Therefore, $\mathcal{H}^{-1}(c\mathcal{B}_{\mathrm{PF}}l(B))$ is a BPFRGCS in $(X,\tau_p)$, by hypothesis. Since $(X,\tau_p)$ is a $BPFR_cT_{\frac{1}{2}}$space, $\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$ is a BPFCS in $(X,\tau_p)$. Hence $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B)\subseteq c\mathcal{B}_{\mathrm{PF}}l(\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B)))=\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$.That is $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B))\subseteq\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))$.

(iii)⇒(i): Let $B$ be a BPFRGCS in $(Y,\sigma_p)$. Since $(Y,\sigma_p)$ is a $BPFR_cT_{\frac{1}{2}}$space, $B$ is a BPFCS in $(Y,\sigma_p)$ and $\mathcal{B}_{\mathrm{PF}}cl(B)=B$. Hence $\mathcal{H}^{-1}(B)=\mathcal{H}^{-1}(\mathcal{B}_{\mathrm{PF}}cl(B))\supseteq\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B))$. Therefore, $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(B))=\mathcal{H}^{-1}(B)$. This implies $\mathcal{H}^{-1}(B)$ is a BPFCS in $(X,\tau_p)$ and hence it is a BPFRGCS in $(X,\tau_p)$. Thus $\mathcal{H}$ is a BPFRG irresolute mapping.

## CONCLUSION

We defined and studied a new concept of Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Mappings. The relationship between BPFPRG continuous mappings and other BPFs were proved. In the future, we intend to extent our research work in the applications of these mappings in decision making problems.

### REFERENCES

[1] L. A. Zadeh, Fuzzy sets, Information and Control, vol. 8 (1965), no. 3, pp.338–353,. https://doi.org/10.1016/S0019- 9958(65)90241-X.

[2] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Systems 20(1986), 87-96.

[3] Coker.D, An introduction to intuitionistic topological spaces, Busefal, (2000), 8151-56.

[4] Yager R., Pythagorean fuzzy subsets in Proceedings of the joint IFSA world congress NAFIPS annual meeting, (2013),57-61.

[5] J. Chen,S.Li,S.Ma and X.Wang, m-Polar Fuzzy Sets:An Extension of Bipolar Fuzzy Sets, The ScientificWorldJournal(2014). http://dx.doi.org/10.1155/2014/416530.

[6] Zhang, W.R., Bipolar fuzzy sets, In Proceedings of the 1998 IEEE International Conference on Fuzzy Systems, 4-9 May., Anchorage, AK, USA. (1998)

[7] Raja Rajeswari, P. and Krishna Moorthy, R., Intuitionistic fuzzy weekly generalized continuous mappings, Far East Journal of Mathematical Sciences, 66 (2012), 153-170.

[8] Santi, R., and Sakthivel. K., Intuitionistic fuzzy generalized semi continuous mappings, Advances in Theoretical and Applied Mathematics, 5 (2009) ,73-82.

[9] Saranya, M., and Jayanthi, D., On Intuitionistic Fuzzy β Generalized Open Sets, International Journal of Engineering Sciences and Management Research, vol. 3(2016), 48 - 53.

[10] C.L. Chang, Fuzzy Topological Spaces, J.Math.Anal.Appl.,24(1968),182-190.

[11] P. Bosc, O. Pivert, On a fuzzy bipolar relational algebra, Information-Sciences, (2013), 219,1-16.

[12] K.Sakthivel, Intuitionistic Fuzzy Alpha Generalized Continuous Mappings and Intuitionistic Fuzzy Alpha Generalized Irresolute Mappings, Applied Mathematical Sciences 4(2010), (1831-1842).

[13] Tapi U. D, Thakur S. S. and Rathore G.P. S, Fuzzy Sg-Irresolute mappings, Stud.Cert.Stii.Ser. Mat. niv. Bacau, 9(1999): 203-210.

[14] Kim, J. H., P. K. Lim, J. G. Lee, and K. Hur, Bipolar Fuzzy Intuitionistic topological spaces, Infinite Study (2018).

[15] Vishalakshi. K.., and Maragathavalli.S., Bipolar Pythagorean Fuzzy Regular Generalized closed sets, IJSRED, 4(2021), 55-62.

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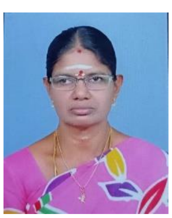

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