



### **VOLUME X**

ISBN No.: 978-81-953602-6-0

**Physical Science** 

## **NALLAMUTHU GOUNDER MAHALINGAM COLLEGE**

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

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**One day International Conference EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)** 27th October 2021

**Jointly Organized by** 

Department of Biological Science, Physical Science and Computational Science

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### Proceeding of the

One day International Conference on

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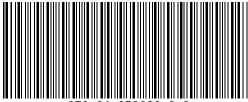
27<sup>th</sup> October 2021

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#### **ABOUT THE INSTITUTION**

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001: 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

#### ABOUT CONFERENCE

The International conference on "Emerging Trends in Science and Technology (ETIST-2021)" is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discust the innovative ideas and will promote to work in interdisciplinary mode.

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ISBN No.: 978-81-953602-6-0

Perfectly Regular Generalized Continuous Mappings in Bipolar Pythagorean Fuzzy Topological Spaces

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**ABSTRACT:** The main focus of this paper is to introduce the concept of Perfectly Regular Generalized Continuous Mappings in Bipolar Pythagorean Fuzzy Topological spaces. We further study some of the basic properties of Perfectly Regular Continuous mappings and interrelation with other existing Bipolar Pythagorean Fuzzy mappings in Bipolar Pythagorean Fuzzy Topological Spaces.

**KEYWORDS:** Bipolar Pythagorean Fuzzy sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagorean Fuzzy Regular Generalized Closed sets, Bipolar Pythagorean Fuzzy Perfectly Regular Generalized Continuous Mappings, Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Mappings.

#### 1. INTRODUCTION

The concept of Fuzzy sets was introduced by Zadeh [1] which shows many applications in the field of Science, Engineering, Management and Technology. The idea of Fuzzy sets deals with the data which are vague and uncertain. After that Atanassov [2] introduced the notion of intuitionistic fuzzy sets and Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. Yager [4] proposed another class of nonstandard fuzzy sets, called Pythagorean fuzzy sets and Murat Olgun, Mehmet Ünver, Seyhmus Yardimci[5] introduced the notion of Pythagorean fuzzy topological spaces. Zhang [6] introduced the extension of fuzzy set with bipolarity, called Bipolar value fuzzy sets. Bosc and Pivert [10] said that "Bipolarity" refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired or considered as acceptable. Negative statement corresponds to what is impossible, rejected or forbidden. Negative preferences to constraints, since they specify which values or objects have to be

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rejected, while positive preferences correspond to wishes, as they specify which objects are more desirable than others, without rejecting those that do not meet the wishes. In bipolar valued fuzzy set interval of membership value is [-1,1].

In this paper, we introduce, Bipolar Pythagorean Fuzzy Perfectly Regular Generalized Continuous Mappings (BPFpRG continuous mappings) and study some of the basic properties.

#### 1. PRELIMINARIES

**Definition 2.1:** Let X be the non-empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by  $P=\{(x,\mu_P(x),\nu_P(x)):x\in X\}$  where the functions  $\mu_P(x)\in[0,1]$  and  $\nu_P(x)\in[0,1]$  denote the degree of membership and degree of non-membership of each element  $x\in X$  to the set P, respectively, and  $0\leqslant \mu_P^2(x)+\nu_P^2(x)\leqslant 1$  for each  $x\in X$ . The degree of indeterminacy  $I_P=\sqrt{1-\mu_P^2(x)-\nu_P^2(x)}$  for each  $x\in X$ .

**Definition 2.2:** Let X be a non-empty set. A Bipolar Pythagorean Fuzzy Set  $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-): x \in X\}$  where  $\mu_A^+: X \to [0,1]$ ,  $\nu_A^+: X \to [0,1]$ ,  $\mu_A^-: X \to [-1,0]$ ,  $\nu_A^-: X \to [-1,0]$  are the mappings such that  $0 \le (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \le 1$  and  $-1 \le (\mu_A^-(x))^2 + (\nu_A^-(x))^2 \le 0$  where  $\mu_A^+(x)$  denote the positive membership degree,  $\nu_A^+(x)$  denote the positive non membership degree,  $\nu_A^-(x)$  denote the negative membership degree.

**Definition 2.3:** Let  $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x) \rangle : x \in X \}$  and  $B = \{\langle x, \mu_B^+(x), \nu_B^+(x), \mu_B^-(x), \nu_B^-(x) \rangle : x \in X \}$  be two Bipolar Pythagorean Fuzzy sets over X. Then

- (i)  $A^c = \{(x, \nu_A^+(x), \mu_A^+(x), \nu_A^-(x), \mu_A^-(x)) : x \in X\},$
- (ii)  $A \cap B = \{\langle x, \min \{\mu_A^+(x), \mu_B^+(x)\}, \max \{\nu_A^+(x), \nu_B^+(x)\}, \max \{\mu_A^-(x), \mu_B^-(x)\}, \min \{\nu_A^-(x), \nu_B^-(x)\} \rangle : x \in X\}$
- (iii)  $A \cup B = \{(x, \max\{\mu_A^+(x), \mu_B^+(x)\}, \min\{\nu_A^+(x), \nu_B^+(x)\}, \min\{\mu_A^-(x), \mu_B^-(x)\}, \max\{\nu_A^-(x), \nu_B^-(x)\}\}: x \in X\}$
- (iv) A is a Bipolar Pythagorean subset of B and write  $A \subseteq B$  if

$$\langle \mu_A^+(x)\leqslant \mu_B^+(x), \nu_A^+(x)\geqslant \nu_B^+(x), \mu_A^-(x)\geqslant \mu_B^-(x), \nu_A^-(x)\leqslant \nu_B^-(x)\rangle \ for \ each \ x\in X$$

(v) 
$$0_X = \{(x, 0, 1, 0, -1) : x \in X\}$$
 and  $1_X = \{(x, 1, 0, -1, 0) : x \in X\}$ .

**Definition 2.4:** Bipolar Pythagorean Fuzzy Topological Spaces: Let  $X \neq \emptyset$  be a set and  $\tau_p$  be a family of Bipolar Pythagorean fuzzy subsets of X. If

$$T_1 \ 0_X, 1_X \in \tau_p.$$
 $T_2 \ \text{For any } P_1, P_2 \in \tau_p, \text{ we have } P_1 \cap P_2 \in \tau_p.$ 
 $T_3 \cup P_i \in \tau_p \text{ for an arbitrary family } \{P_i : i \in J\} \subseteq \tau_p.$ 

Then  $\tau_p$  is called Bipolar Pythagorean Fuzzy Topology on X and the pair  $(X, \tau_p)$  is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of  $\tau_p$  is called Bipolar Pythagorean fuzzy open set (BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set (BPFCS).

**Definition 2.5:** If BPFS 
$$A = \{(x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\}: x \in X\}$$
 in a BPTS  $(X, \tau_p)$  is said to be

- (a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if  $\mathcal{B}_{PF}int(\mathcal{B}_{PF}cl(A)) \subseteq A$ .
- (c) Bipolar Pythagorean Fuzzy Pre closed set (BPFPCS) if  $\mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(A)) \subseteq A$ .
- (e) Bipolar Pythagorean Fuzzy  $\alpha$  closed set (BPF $\alpha$ CS) if  $\mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(cl(A)) \subseteq A$ .

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- (i) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if  $A = \mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(A))$ .
- (k) A Bipolar Pythagorean Fuzzy set A of a BPFTS  $(X, \tau_p)$  is a Bipolar Pythagorean Fuzzy Generalized closed set (BPFGCS), if  $\mathcal{B}_{PF}cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is BPFOS in  $(X, \tau_p)$ .
- (1) A Bipolar Pythagorean Fuzzy set A of a BPFTS  $(X, \tau_p)$  is a Bipolar Pythagorean Fuzzy Generalized open set (BPFGCS), if  $A^c$  is a BPFGCS in  $(X, \tau_p)$ .
- (m) A Bipolar Pythagorean Fuzzy set A of a BPFTS  $(X, \tau_p)$  is a Bipolar Pythagorean Fuzzy Regular Generalized closed set (BPFGCS), if  $\mathcal{B}_{PF}cl(A) \subseteq U$  whenever  $A \subseteq U$  and U is BPFROS in  $(X, \tau_p)$ .

**Definition 2.6:** Let  $\mathcal{H}$  be a mapping from an BPFTS in  $(X, \tau_p)$  into a BPFTS  $(Y, \sigma_p)$ . Then  $\mathcal{H}$  is said to be Bipolar Pythagorean Fuzzy Continuous mapping if  $\mathcal{H}^{-1}(A) \in BPFO(X)$  for every  $A \in (Y, \sigma_p)$ .

**Definition 2.7:** A mapping  $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$  is said to be

- (i) BPF semi continuous mapping if  $\mathcal{H}^{-1}(A) \in BPFSO(X)$  for every  $A \in (Y, \sigma_p)$ .
- (ii)  $\text{BPF}\alpha$  continuous mapping if  $\mathcal{H}^{-1}(A) \in \text{BPF}\alpha O(X)$  for every  $A \in (Y, \sigma_p)$ .
- (iii) BPF Pre continuous mapping if  $\mathcal{H}^{-1}(A) \in BPFPO(X)$  for every  $A \in (Y, \sigma_p)$ .

**Definition 2.8:** A BPFTS  $(X, \tau_p)$  is said to be a  $BPFR_cT_{\frac{1}{2}}$  space (Bipolar Pythagorean Fuzzy Regular  $_cT_{\frac{1}{2}}$  space) if every BPFRGCS in  $(X, \tau_p)$  is a BPFCS in  $(X, \tau_p)$ .

**Definition 2.9:** A BPFTS  $(X, \tau_p)$  is said to be a  $BPFR_gT_{\frac{1}{2}}$  space (Bipolar Pythagorean Fuzzy Regular Generalized  ${}_gT_{\frac{1}{2}}$  space) if every BPFRGCS in  $(X, \tau_p)$  is a BPFGCS in  $(X, \tau_p)$ .

**Definition 2.10:** A BPFTS  $(X, \tau_p)$  is said to be a  $BPFR_{\alpha}T_{\frac{1}{2}}$  space (Bipolar Pythagorean Fuzzy Regular Generalized  ${}_{\alpha}T_{\frac{1}{2}}$  space) if every BPFRGCS in  $(X, \tau_p)$  is a BPF $\alpha$ CS in  $(X, \tau_p)$ .

# 2. BIPOLAR PYTHAGOREAN FUZZY PERFECTLY REGULAR GENERALIZED CONTINUOUS MAPPINGS (BPFpRG continuous mappings)

In this section we have introduced Bipolar Pythagorean Fuzzy Perfectly Regular Generalized continuous mappings and studied some of its properties.

**Definition 3.1:** A map  $\mathcal{H}: X, \tau_p) \to (Y, \sigma_p)$  is said to be BPF Perfectly Regular Generalized continuous mapping if  $\mathcal{H}^{-1}(A)$  is BPF clopen in  $(X, \tau_p)$  for every BPFRCS A in  $(Y, \sigma_p)$ .

**Theorem 3.2:** Let  $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$  be map. Then the following are equivalent.

- (a)  $\mathcal{H}$  is BPF perfectly Regular Generalized continuous.
- (b) The inverse image of BPFRGOS in  $(Y, \sigma_p)$  is BPF clopen in  $(X, \tau_p)$ .
- (c) The inverse image of BPFRGCS in  $(Y, \sigma_p)$  is BPF clopen in  $(X, \tau_p)$ .

**Proof:** (i)Let  $\mathcal{H}$  is BPF perfectly Regular Generalized continuous mapping. The inverse image of BPFRGOS in  $(Y, \sigma_v)$  is BPF clopen in  $(X, \tau_v)$ , from the definition.

(ii) (b) implies (c): Let G be any BPFRGCS in  $(Y, \sigma_p)$ . Then  $G^c$  is BPFRGOS in  $(Y, \sigma_p)$ . Hence by assumption  $\mathcal{H}^{-1}(G^c)$  is BPF clopen in  $(X, \tau_p)$ .

(iii) (c)implies (a): Let H be any BPFRGOS in  $(Y, \sigma_p)$ . Then  $H^c$  is BPFRGCS in  $(Y, \sigma_p)$ . As by (c)  $\mathcal{H}^{-1}(H^c)$  is BPF clopen in  $(X, \tau_p)$  which implies that  $\mathcal{H}^{-1}(H)$  is BPF clopen in  $(X, \tau_p)$ . Hence  $\mathcal{H}$  is BPF Perfectly Regular Generalized Continuous Mapping (BPFpRG continuous mapping).

**Proposition 3.3:** Every BPFpRG continuous mapping is a BPF continuous mapping but not conversely in general.

Example 3.4: Let X= {a,b} and Y={u,v} and  $T_1$ =(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)),  $T_2$ =(y, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)). Then  $\tau_p$ ={0<sub>p</sub>, T<sub>1</sub>, 1<sub>p</sub>} and  $\sigma_p$ ={0<sub>p</sub>, T<sub>2</sub>, 1<sub>p</sub>} be a BPFTs on X and Y respectively. Define a mapping  $\mathcal{H}$ : (X,  $\tau_p$ ) → (Y,  $\sigma_p$ ) by  $\mathcal{H}$ (a) = u and  $\mathcal{H}$ (b) = v. The BPFS A = ⟨y, (0.8, 0.6), (0.3, 0.5), (-0.9, -0.7), (-0.4, -0.5)) is a BPFCS in (Y,  $\sigma_p$ ). Then  $\mathcal{H}^{-1}$ (A) is a BPFCS in (X,  $\tau_p$ ). Therefore  $\mathcal{H}$  is a BPF continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFRGCS A in (Y,  $\sigma_p$ ),  $\mathcal{H}^{-1}$ (A) is not a BPF clopen in (X,  $\tau_p$ ), as  $\mathcal{B}_{PF}$  cl( $\mathcal{H}^{-1}$ (A)) =  $T_1^c = \mathcal{H}^{-1}$ (A) but  $\mathcal{B}_{PF}$ int( $\mathcal{H}^{-1}$ (A)) =  $T_1 \neq \mathcal{H}^{-1}$ (A).

**Proposition 3.5:** Every BPFpRG continuous mapping is a BPFG continuous mapping but not conversely in general. **Example 3.6:** Let  $X = \{a, b\}$ ,  $Y = \{u, v\}$  and  $T_1 = (x, (0.5, 0.4), (0.6, 0.5), (-0.4, -0.3), (-0.5, -0.4))$ ,  $T_2 = (y, (0.7, 0.8), (0.3, 0.3), (-0.8, -0.8), (-0.3, -0.4))$ . Then  $\tau_p = \{0_p, T_1, 1_p\}$  and  $\sigma_p = \{0_p, T_2, 1_p\}$  be a BPFTs on X and Y respectively. Define a mapping  $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$  by  $\mathcal{H}(a) = u$  and  $\mathcal{H}(b) = v$ . The BPFS  $A = \langle y, (0.2, 0.3), (0.8, 0.9), (-0.2, -0.2), (-0.9, -0.8))$  is a BPFCS in  $(Y, \sigma_p)$ . Then  $\mathcal{H}^{-1}(A)$  is a BPFCS in  $(X, \tau_p)$ . Therefore  $\mathcal{H}$  is a BPFG continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFRGCS A in  $(Y, \sigma_p)$ , and  $\mathcal{H}^{-1}(A)$  is not a BPF clopen in  $(X, \tau_p)$ , as  $\mathcal{B}_{PF}cl(\mathcal{H}^{-1}(A)) = T_1^c \neq \mathcal{H}^{-1}(A)$  but  $\mathcal{B}_{PF}int(\mathcal{H}^{-1}(A)) = 0_p \neq \mathcal{H}^{-1}(A)$ .

*Proposition 3.7:* Every BPFpRG continuous mapping is a BPFα continuous mapping but not conversely in general. *Example 3.8:* Let X={a,b}, Y={u,v} and  $T_1$ =(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)),  $T_2$ =(y, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)). Then  $\tau_p$ ={0<sub>p</sub>,  $T_1$ , 1<sub>p</sub>} and  $\sigma_p$ ={0<sub>p</sub>,  $T_2$ , 1<sub>p</sub>} be a BPFTs on X and Y respectively. Define a mapping  $\mathcal{H}$ : (X,  $\tau_p$ ) → (Y,  $\sigma_p$ ) by  $\mathcal{H}$ (X) = X0 and X1 and X2 and X3 and X4 respectively. Define a mapping X3 and X4 respectively. Define a mapping X4 and X5 and X6 and X7 and X8 and X9 by X1 and X1 and X2 and X3 and X4 respectively. Define a mapping X5 and X6 and X7 and X8 and X9 by X1 and X1 and X2 and X3 and X4 respectively. Define a mapping X5 and X6 and X7 and X8 and X9 by X1 and X1 and X2 and X3 and X4 respectively. Define a mapping X5 and X6 and X9 by X1 and X1 and X2 and X3 and X4 respectively. Define a mapping X5 and X6 and X9 by X1 and X1 and X2 and X3 and X4 respectively. Define a mapping X1 and X1 and X2 and X3 and X4 respectively. Define a mapping X5 and X4 and X4 respectively. Define a mapping X5 and X6 and X4 and X4 respectively. Define a mapping X5 and X6 and X7 and X8 and X9 and

**Proposition 3.9:** Every BPFpRG continuous mapping is a BPFR continuous mapping but not conversely in general. **Example 3.10:** Let X={a,b}, Y={u,v} and  $T_1$ =(x, (0.2, 0.2), (0.8, 0.8), (-0.2, -0.1), (-0.8, -0.7)),  $T_2$ =(y, (0.6, 0.4), (0.7, 0.5), (-0.6, -0.4), (-0.7, -0.6)). Then  $\tau_p$ ={0 $_p$ ,  $T_1$ , 1 $_p$ } and  $\sigma_p$ ={0 $_p$ ,  $T_2$ , 1 $_p$ } be a BPFTs on X and Y respectively. Define a mapping  $\mathcal{H}$ : (X,  $\tau_p$ )  $\rightarrow$  (Y,  $\sigma_p$ ) by  $\mathcal{H}$ (A) = A1 and A2 and A3 be a BPFRCS A3 in (A3 in (A4 in (A5 in (A5 in (A5 in (A7 in (A7 in (A8 in (A8 in (A9 in (A9)) in (A9 in (A9)) in (A9 in (A9)) in (A9 in (A9)) in (A9) in (A

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BPFS continuous

BPFR continuous

BPFG continuous

BPFAG continuous

BPFRG continuous

BPFRG continuous

Figure 1: The relation between BPFpRG continuous mappings and other existing BPFs.

**Theorem 3.11:** A mapping  $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$  is a BPFpRG continuous mapping Iff the inverse image of each BPFRGOS is a BPF clopen in  $(X, \tau_p)$ .

**Proof:** Necessity: Let  $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$  be a  $(Y, \sigma_p)$  BPFpRG continuous mapping. Let A be a BPFRGOS in  $(Y, \sigma_p)$ . Since  $\mathcal{H}$  is a BPFpRG continuous mapping,  $\mathcal{H}^{-1}(A^c)$  is BPF clopen in  $(X, \tau_p)$ . As  $\mathcal{H}^{-1}(A^c) = (\mathcal{H}^{-1}(A))^c$ , we have  $\mathcal{H}^{-1}(A)$  is a BPF clopen in  $(X, \tau_p)$ .

Sufficiency: Let B be a BPFRGCS in  $(Y, \sigma_p)$ . Then  $B^c$  is a BPFRGOS in  $(Y, \sigma_p)$ . By hypothesis,  $(\mathcal{H}^{-1}(B^c))$  is BPF clopen in  $(X, \tau_p)$ , as  $\mathcal{H}^{-1}(B^c) = (\mathcal{H}^{-1}(B))^c$ . Therefore  $\mathcal{H}$  is a BPFpRG continuous mapping.

**Theorem 3.12:** Let  $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$  be a BPF continuous mapping and  $\mathcal{R}: (Y, \sigma_p) \to (Z, \zeta_p)$  is a BPFpRG continuous mapping, then  $\mathcal{R}o\mathcal{H}: (X, \tau_p) \to (Z, \zeta_p)$  is a BPFpRG continuous mapping.

**Proof:** Let A be a BPFRGCS in  $(Z, \zeta_p)$ . Since  $\mathcal{R}$  is a BPFpRG continuous mapping,  $\mathcal{R}^{-1}(A)$  is a BPF clopen in  $(Y, \sigma_p)$ . Since  $\mathcal{H}$  is a BPF continuous mapping, then  $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$  is a BPFCS in  $(X, \tau_p)$ , and also BPFOS in  $(X, \tau_p)$ . Hence  $\mathcal{R}o\mathcal{H}$  is a BPFpRG continuous mapping.

**Theorem 3.13:** The composition of two BPFpRG continuous mapping is a BPFpRG continuous mapping in general. **Proof:** Let  $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$  and  $\mathcal{R}: (Y, \sigma_p) \to (Z, \zeta_p)$  be any two BPFpRG continuous mappings. Let A be a BPFRGCS in  $(Z, \zeta_p)$ . By hypothesis,  $\mathcal{R}^{-1}(A)$  is BPF clopen in  $(Y, \sigma_p)$  and hence it is BPFCS in  $(Y, \sigma_p)$ . Since every BPFCS is BPFRGCS,  $\mathcal{R}^{-1}(A)$  is a BPFRGCS in  $(Y, \sigma_p)$  and  $\mathcal{H}$  is a BPFpRG continuous mapping,  $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A)) = (\mathcal{R}o\mathcal{H})^{-1}(A)$  is BPF clopen in  $(X, \tau_p)$ . Hence  $\mathcal{R}o\mathcal{H}$  is BPFRG continuous mapping.

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