VOLUME X
ISBN No.: 978-81-953602-6-0
Physical Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,
Pollachi-642001

SUPPORTED BY

ABT Industries Ltd.,
Dairy Division

PROCEEDING

One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, Pollachi-642001.

Estd. 1957

Proceeding of the

One day International Conference on

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

ISBN No: 978-81-953602-6-0

978- 81- 953602- 6- 0

Nallamuthu Gounder Mahalingam College

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

www.ngmc.org

ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

EDITORIAL BOARD

International Conference on Emerging Trends in Science and Technology (ETIST 2021)
Jointly Organized by Department of Biological Science, Physical Science and Computational Science
Nallamuthu Gounder Mahalingam College, Affiliated to Bharathiar University, Tamilnadu, India.
Published by NGMC - November 2021

Perfectly Regular Generalized Continuous Mappings in Bipolar Pythagorean Fuzzy Topological Spaces

Vishalakshi.K[1], Maragathavalli.S[2], Santhi.R[2]
[1]Research Scholar, Government Arts College, Udumalpet, Tamilnadu, INDIA.
Email: vishalkapil2006@gmail.com
[2]Assistant Professor, Department of Mathematics, Government Arts College, Udumalpet, Tamilnadu, INDIA.
Email: smvalli@rediffmail.com
[3]Assistant Professor, Department of Mathematics, NGM College, Pollachi, Tamilnadu, INDIA.
Email: santhir2004@yahoo.co.in

ABSTRACT: The main focus of this paper is to introduce the concept of Perfectly Regular Generalized Continuous Mappings in Bipolar Pythagorean Fuzzy Topological spaces. We further study some of the basic properties of Perfectly Regular Continuous mappings and interrelation with other existing Bipolar Pythagorean Fuzzy mappings in Bipolar Pythagorean Fuzzy Topological Spaces.

KEYWORDS: Bipolar Pythagorean Fuzzy sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagorean Fuzzy Regular Generalized Closed sets, Bipolar Pythagorean Fuzzy Perfectly Regular Generalized Continuous Mappings, Bipolar Pythagorean Fuzzy Regular Generalized Irresolute Mappings.

1. INTRODUCTION

The concept of Fuzzy sets was introduced by Zadeh [1] which shows many applications in the field of Science, Engineering, Management and Technology. The idea of Fuzzy sets deals with the data which are vague and uncertain. After that Atanassov [2] introduced the notion of intuitionistic fuzzy sets and Coker [3] introduced the notion of intuitionistic fuzzy topological spaces. Yager [4] proposed another class of nonstandard fuzzy sets, called Pythagorean fuzzy sets and Murat Olgun, Mehmet Ünver, Seyhmus Yardimci[5] introduced the notion of Pythagorean fuzzy topological spaces. Zhang [6] introduced the extension of fuzzy set with bipolarity, called Bipolar value fuzzy sets. Bosc and Pivert [10] said that "Bipolarity" refers to the propensity of the human mind to reason and make decisions on the basis of positive and negative effects. Positive information states what is possible, satisfactory, permitted, desired or considered as acceptable. Negative statement corresponds to what is impossible, rejected or forbidden. Negative preferences to constraints, since they specify which values or objects have to be

Vishalakshi.K*[1], Research Scholar, Government Arts College,Udumalpet, Tamilnadu, India. vishalkapail2006@gmail.com
Maragathavalli.S*[2], Assistant Professor, Government Arts College,Udumalpet, Tamilnadu, India· smvalli@rediffmail.com
Santhi.R*[3], Assistant Professor, NGM College,Pollachi, Tamilnadu, India· santhir2004@yahoo.co.in

rejected, while positive preferences correspond to wishes, as they specify which objects are more desirable than others, without rejecting those that do not meet the wishes. In bipolar valued fuzzy set interval of membership value is [-1,1].

In this paper, we introduce, Bipolar Pythagorean Fuzzy Perfectly Regular Generalized Continuous Mappings (BPFpRG continuous mappings) and study some of the basic properties.

1. PRELIMINARIES

Definition 2.1: Let X be the non-empty universe of discourse. A Pythagorean fuzzy set(PFS) P in X is given by P=$\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non-membership of each element $x \in X$ to the set P, respectively, and $0 \leqslant \mu_P^2(x) + \nu_P^2(x) \leqslant 1$ for each $x \in X$. The degree of indeterminacy $I_P = \sqrt{1 - \mu_P^2(x) - \nu_P^2(x)}$ for each $x \in X$.

Definition 2.2: Let X be a non-empty set. A Bipolar Pythagorean Fuzzy Set $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-) : x \in X\}$ where $\mu_A^+ : X \to [0,1]$, $\nu_A^+ : X \to [0,1]$, $\mu_A^- : X \to [-1,0]$, $\nu_A^- : X \to [-1,0]$ are the mappings such that $0 \leqslant (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leqslant 1$ and $-1 \leqslant (\mu_A^-(x))^2 + (\nu_A^-(x))^2 \leqslant 0$ where $\mu_A^+(x)$ denote the positive membership degree, $\nu_A^+(x)$ denote the positive non membership degree, $\mu_A^-(x)$ denote the negative membership degree, $\nu_A^-(x)$ denote the negative non membership degree.

Definition 2.3: Let $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B^+(x), \nu_B^+(x), \mu_B^-(x), \nu_B^-(x)\rangle : x \in X\}$ be two Bipolar Pythagorean Fuzzy sets over X. Then

(i) $A^c = \{\langle x, \nu_A^+(x), \mu_A^+(x), \nu_A^-(x), \mu_A^-(x)\rangle : x \in X\}$,

(ii) $A \cap B = \{\langle x, \min\{\mu_A^+(x), \mu_B^+(x)\}, \max\{\nu_A^+(x), \nu_B^+(x)\}, \max\{\mu_A^-(x), \mu_B^-(x)\}, \min\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$

(iii) $A \cup B = \{\langle x, \max\{\mu_A^+(x), \mu_B^+(x)\}, \min\{\nu_A^+(x), \nu_B^+(x)\}, \min\{\mu_A^-(x), \mu_B^-(x)\}, \max\{\nu_A^-(x), \nu_B^-(x)\}\rangle : x \in X\}$

(iv) A is a Bipolar Pythagorean subset of B and write $A \subseteq B$ if

$\langle \mu_A^+(x) \leqslant \mu_B^+(x), \nu_A^+(x) \geqslant \nu_B^+(x), \mu_A^-(x) \geqslant \mu_B^-(x), \nu_A^-(x) \leqslant \nu_B^-(x)\rangle$ *for each* $x \in X$

(v) $0_X = \{\langle x, 0,1,0,-1\rangle : x \in X\}$ and $1_X = \{\langle x, 1,0,-1,0\rangle : x \in X\}$.

***Definition 2.4*: Bipolar Pythagorean Fuzzy Topological Spaces:** Let $X \neq \emptyset$ be a set and τ_p be a family of Bipolar Pythagorean fuzzy subsets of X. If

T_1 $0_X, 1_X \in \tau_p$.

T_2 For any $P_1, P_2 \in \tau_p$, we have $P_1 \cap P_2 \in \tau_p$.

T_3 $\cup P_i \in \tau_p$ for an arbitrary family $\{P_i : i \in J\} \subseteq \tau_p$.

Then τ_p is called Bipolar Pythagorean Fuzzy Topology on X and the pair (X, τ_p) is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of τ_p is called Bipolar Pythagorean fuzzy open set (BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set (BPFCS).

Definition 2.5: If BPFS $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ in a BPTS (X, τ_p) is said to be

(a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if $\mathcal{B}_{PF}int(\mathcal{B}_{PF}cl(A)) \subseteq A$.

(c) Bipolar Pythagorean Fuzzy Pre closed set (BPFPCS) if $\mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(A)) \subseteq A$.

(e) Bipolar Pythagorean Fuzzy α closed set (BPFαCS) if $\mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(cl(A)) \subseteq A$.

(i) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if $A = \mathcal{B}_{PF}cl(\mathcal{B}_{PF}int(A))$.

(k) A Bipolar Pythagorean Fuzzy set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Generalized closed set (BPFGCS), if $\mathcal{B}_{PF}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPFOS in (X, τ_p).

(l) A Bipolar Pythagorean Fuzzy set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Generalized open set (BPFGCS), if A^c is a BPFGCS in (X, τ_p).

(m) A Bipolar Pythagorean Fuzzy set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Regular Generalized closed set (BPFGCS), if $\mathcal{B}_{PF}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPFROS in (X, τ_p).

Definition 2.6: Let $\mathcal{H}$ be a mapping from an BPFTS in (X, τ_p) into a BPFTS (Y, σ_p). Then $\mathcal{H}$ is said to be Bipolar Pythagorean Fuzzy Continuous mapping if $\mathcal{H}^{-1}(A) \in BPFO(X)$ for every $A \in (Y, \sigma_p)$.

Definition 2.7: A mapping $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ is said to be

(i) BPF semi continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFSO(X) for every $A \in (Y, \sigma_p)$.

(ii) BPFα continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFαO(X) for every $A \in (Y, \sigma_p)$.

(iii) BPF Pre continuous mapping if $\mathcal{H}^{-1}(A) \in$ BPFPO(X) for every $A \in (Y, \sigma_p)$.

Definition 2.8: A BPFTS (X, τ_p) is said to be a $BPFR_cT_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular ${}_cT_{\frac{1}{2}}$ space) if every BPFRGCS in (X, τ_p) is a BPFCS in (X, τ_p).

Definition 2.9: A BPFTS (X, τ_p) is said to be a $BPFR_gT_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular Generalized ${}_gT_{\frac{1}{2}}$ space) if every BPFRGCS in (X, τ_p) is a BPFGCS in (X, τ_p).

Definition 2.10: A BPFTS (X, τ_p) is said to be a $BPFR_\alpha T_{\frac{1}{2}}$ space (Bipolar Pythagorean Fuzzy Regular Generalized ${}_\alpha T_{\frac{1}{2}}$ space) if every BPFRGCS in (X, τ_p) is a BPFαCS in (X, τ_p).

2. BIPOLAR PYTHAGOREAN FUZZY PERFECTLY REGULAR GENERALIZED CONTINUOUS MAPPINGS (BPFpRG continuous mappings)

In this section we have introduced Bipolar Pythagorean Fuzzy Perfectly Regular Generalized continuous mappings and studied some of its properties.

Definition 3.1: A map $\mathcal{H}: X, \tau_p) \to (Y, \sigma_p)$ is said to be BPF Perfectly Regular Generalized continuous mapping if $\mathcal{H}^{-1}(A)$ is BPF clopen in (X, τ_p) for every BPFRCS A in (Y, σ_p).

Theorem 3.2: Let $\mathcal{H}: (X, \tau_p) \to (Y, \sigma_p)$ be map. Then the following are equivalent.

(a) $\mathcal{H}$ is BPF perfectly Regular Generalized continuous.

(b) The inverse image of BPFRGOS in (Y, σ_p) is BPF clopen in (X, τ_p).

(c) The inverse image of BPFRGCS in (Y, σ_p) is BPF clopen in (X, τ_p).

Proof: (i)Let $\mathcal{H}$ is BPF perfectly Regular Generalized continuous mapping. The inverse image of BPFRGOS in (Y, σ_p) is BPF clopen in (X, τ_p), from the definition.

(ii) (b) implies (c): Let G be any BPFRGCS in (Y,σ_p). Then G^c is BPFRGOS in (Y,σ_p). Hence by assumption $\mathcal{H}^{-1}(G^c)$ is BPF clopen in (X,τ_p).

(iii) (c)implies (a): Let H be any BPFRGOS in (Y,σ_p). Then H^c is BPFRGCS in (Y,σ_p). As by (c) $\mathcal{H}^{-1}(H^c)$ is BPF clopen in (X,τ_p) which implies that $\mathcal{H}^{-1}(H)$ is BPF clopen in (X,τ_p). Hence $\mathcal{H}$ is BPF Perfectly Regular Generalized Continuous Mapping (BPFpRG continuous mapping).

Proposition 3.3: Every BPFpRG continuous mapping is a BPF continuous mapping but not conversely in general.

Example 3.4: Let X= {a,b} and Y={u,v} and T_1=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)), T_2=(y, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)). Then $\tau_p=\{0_p,T_1,1_p\}$ and $\sigma_p=\{0_p,T_2,1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ by $\mathcal{H}(a)=u$ and $\mathcal{H}(b)=v$. The BPFS A = ⟨y, (0.8, 0.6), (0.3, 0.5), (-0.9, -0.7), (-0.4, -0.5)) is a BPFCS in (Y,σ_p). Then $\mathcal{H}^{-1}(A)$ is a BPFCS in (X,τ_p). Therefore $\mathcal{H}$ is a BPF continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFRGCS A in (Y,σ_p), $\mathcal{H}^{-1}(A)$ is not a BPF clopen in (X,τ_p), as $\mathcal{B}_{\mathrm{PF}}\,cl(\mathcal{H}^{-1}(A))=T_1^c=\mathcal{H}^{-1}(A)$ but $\mathcal{B}_{\mathrm{PF}}int(\mathcal{H}^{-1}(A))=T_1\neq\mathcal{H}^{-1}(A)$.

Proposition 3.5: Every BPFpRG continuous mapping is a BPFG continuous mapping but not conversely in general.

Example 3.6*:* Let X= {a, b}, Y={u,v} and T_1=(x, (0.5, 0.4), (0.6, 0.5), (-0.4, -0.3), (-0.5, -0.4)), T_2=(y, (0.7, 0.8), (0.3, 0.3), (-0.8, -0.8), (-0.3, -0.4)). Then $\tau_p=\{0_p,T_1,1_p\}$ and $\sigma_p=\{0_p,T_2,1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ by $\mathcal{H}(a)=u$ and $\mathcal{H}(b)=v$. The BPFS A = ⟨y, (0.2, 0.3), (0.8, 0.9), (-0.2, -0.2), (-0.9, -0.8)) is a BPFCS in (Y,σ_p). Then $\mathcal{H}^{-1}(A)$ is a BPFCS in (X,τ_p). Therefore $\mathcal{H}$ is a BPFG continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFRGCS A in (Y,σ_p), and $\mathcal{H}^{-1}(A)$ is not a BPF clopen in (X,τ_p), as $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(A))=T_1^c\neq\mathcal{H}^{-1}(A)$ but $\mathcal{B}_{\mathrm{PF}}int(\mathcal{H}^{-1}(A))=0_p\neq\mathcal{H}^{-1}(A)$.

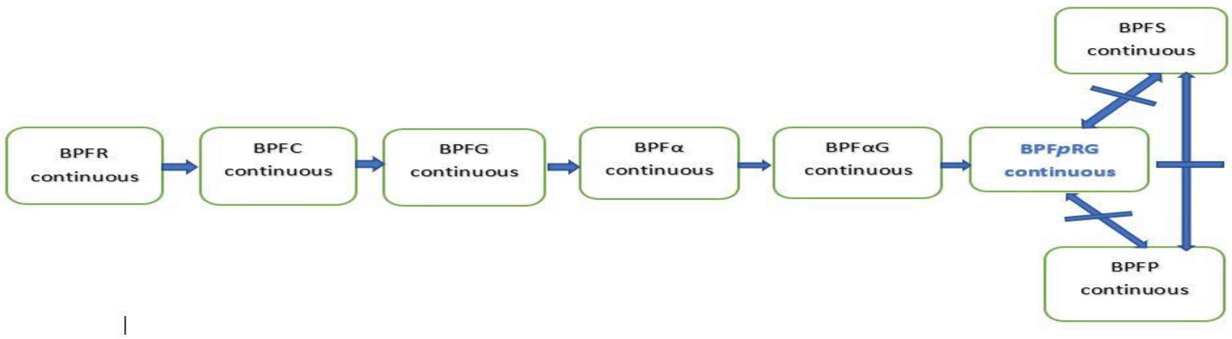
Proposition 3.7: Every BPFpRG continuous mapping is a BPFα continuous mapping but not conversely in general.

Example 3.8: Let X={a,b}, Y={u,v} and T_1=(x, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)), T_2=(y, (0.3, 0.5), (0.8, 0.6), (-0.4, -0.5), (-0.9, -0.7)). Then $\tau_p=\{0_p,T_1,1_p\}$ and $\sigma_p=\{0_p,T_2,1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ by $\mathcal{H}(a)=u$ and $\mathcal{H}(b)=v$. The BPFS A = ⟨y, (0.8, 0.6), (0.3, 0.5), (-0.9, -0.7), (-0.4, -0.5)) is a BPFαCS in (Y,σ_p). Then$\mathcal{H}^{-1}(A)$ is a BPFCS in (X,τ_p). Therefore $\mathcal{H}$ is a BPF continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFαCS A in (Y,σ_p), $\mathcal{H}^{-1}(A)$ is not a BPF clopen in (X,τ_p), as $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(A))=T_1^c=\mathcal{H}^{-1}(A)$ but $\mathcal{B}_{\mathrm{PF}}int(\mathcal{H}^{-1}(A))=T_1\neq\mathcal{H}^{-1}(A)$.

Proposition 3.9: Every BPFpRG continuous mapping is a BPFR continuous mapping but not conversely in general.

Example 3.10: Let X={a,b}, Y={u,v} and T_1=(x, (0.2, 0.2), (0.8, 0.8), (-0.2, -0.1), (-0.8, -0.7)), T_2=(y, (0.6, 0.4), (0.7, 0.5), (-0.6, -0.4), (-0.7, -0.6)). Then $\tau_p=\{0_p,T_1,1_p\}$ and $\sigma_p=\{0_p,T_2,1_p\}$ be a BPFTs on X and Y respectively. Define a mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ by $\mathcal{H}(a)=u$ and $\mathcal{H}(b)=v$. Let us consider the BPFRCS A in (Y,σ_p). Then the BPFS A= (y, (0.7, 0.5), (0.6, 0.4), (-0.7, -0.6), (-0.6, -0.4)) Then $\mathcal{H}^{-1}(A)$ is a BPFRCS in (X,τ_p). Therefore $\mathcal{H}$ is a BPFRG continuous mapping, but not a BPFpRG continuous mapping. Since for a BPFRGCS A in (Y,σ_p), $\mathcal{H}^{-1}(A)$ is not a BPF clopen in (X,τ_p), as $\mathcal{B}_{\mathrm{PF}}cl(\mathcal{H}^{-1}(A))=T_1^c\neq\mathcal{H}^{-1}(A)$ but $\mathcal{B}_{\mathrm{PF}}int(\mathcal{H}^{-1}(A))=T_1\neq\mathcal{H}^{-1}(A)$.

Figure1: The relation between BPF*p*RG continuous mappings and other existing BPFs.



Theorem 3.11: A mapping $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ is a BPFpRG continuous mapping Iff the inverse image of each BPFRGOS is a BPF clopen in (X,τ_p).

Proof: Necessity: Let $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ be a (Y,σ_p) BPFpRG continuous mapping. Let A be a BPFRGOS in (Y,σ_p). Since $\mathcal{H}$ is a BPFpRG continuous mapping, $\mathcal{H}^{-1}(A^c)$ is BPF clopen in (X,τ_p). As $\mathcal{H}^{-1}(A^c)=(\mathcal{H}^{-1}(A))^c$, we have $\mathcal{H}^{-1}(A)$ is a BPF clopen in (X,τ_p).

Sufficiency: Let B be a BPFRGCS in (Y,σ_p). Then B^c is a BPFRGOS in (Y,σ_p). By hypothesis, $(\mathcal{H}^{-1}(B^c))$ is BPF clopen in (X,τ_p), as $\mathcal{H}^{-1}(B^c)=(\mathcal{H}^{-1}(B))^c$. Therefore $\mathcal{H}$ is a BPFpRG continuous mapping.

Theorem 3.12: Let $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ be a BPF continuous mapping and $\mathcal{R}:(Y,\sigma_p)\to(Z,\zeta_p)$ is a BPFpRG continuous mapping, then $\mathcal{R}o\mathcal{H}:(X,\tau_p)\to(Z,\zeta_p)$ is a BPFpRG continuous mapping.

Proof: Let A be a BPFRGCS in (Z,ζ_p). Since $\mathcal{R}$ is a BPFpRG continuous mapping, $\mathcal{R}^{-1}(A)$ is a BPF clopen in (Y,σ_p). Since $\mathcal{H}$ is a BPF continuous mapping, then $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))$ is a BPFCS in (X,τ_p), and also BPFOS in (X,τ_p). Hence $\mathcal{R}o\mathcal{H}$ is a BPFpRG continuous mapping.

Theorem 3.13: The composition of two BPFpRG continuous mapping is a BPFpRG continuous mapping in general.

Proof: Let $\mathcal{H}:(X,\tau_p)\to(Y,\sigma_p)$ and $\mathcal{R}:(Y,\sigma_p)\to(Z,\zeta_p)$ be any two BPFpRG continuous mappings. Let A be a BPFRGCS in (Z,ζ_p). By hypothesis, $\mathcal{R}^{-1}(A)$ is BPF clopen in (Y,σ_p) and hence it is BPFCS in (Y,σ_p). Since every BPFCS is BPFRGCS, $\mathcal{R}^{-1}(A)$ is a BPFRGCS in (Y,σ_p) and $\mathcal{H}$ is a BPFpRG continuous mapping, $\mathcal{H}^{-1}(\mathcal{R}^{-1}(A))=(\mathcal{R}o\mathcal{H})^{-1}(A)$ is BPF clopen in (X,τ_p). Hence $\mathcal{R}o\mathcal{H}$ is BPFRG continuous mapping.

REFERENCES

[1] L. A. Zadeh, Fuzzy sets, Information and Control, vol. 8(1965), no. 3, pp. 338–353,. https://doi.org/10.1016/S0019- 9958(65)90241-X.

[2] K.T. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets Systems 20(1986), 87-96.

[3] Coker.D, An introduction to intuitionistic topological spaces, Busefal, (2000), 8151-56.

[4] Yager R., Pythagorean fuzzy subsets in Proceedings of the joint IFSA world congress NAFIPS annual meeting, (2013), 57-61.

[5] J.Chen,S.Li,S.Ma and X.Wang, m-Polar Fuzzy Sets: An Extension of Bipolar Fuzzy Sets, The ScientificWorldJournal (2014) See also http://dx.doi.org/10.1155/2014/416530.

[6] Zhang, W.R., Bipolar fuzzy sets, In Proceedings of the 1998 IEEE International Conference on Fuzzy Systems, 4-9 May., Anchorage, AK, USA, (1998)

[7] Santhi, R., and Sakthivel. K., Intuitionistic fuzzy generalized semi continuous mappings, Advances in Theoretical and Applied Mathematics, (2009) 5 ,73-82.

[8] Rajarajeswari, P. and Krishna Moorthy, R, Intuitionistic fuzzy weekly generalized continuous mappings, Far East Journal of Mathematical Sciences, 66(2012), 153-170

[9] Santhi, R., and Jayanthi, D, Intuitionistic Fuzzy Generalized Semi Pre-Continuous Mapping, Int. J. Contemp. Math. Science, Vol. 5(2010), no.30, pp.1455 – 1469.

[10] P. Bosc, O. Pivert, On a fuzzy bipolar relational algebra, Information-Sciences, 219 (2013),1-16.

[11] Santhi, R., and Sakthivel, K., Intuitionistic fuzzy almost alpha generalized continuous mapping, Advances in Fuzzy Mathematics, Vol.2(2010), 209-219.

[12] Kim, J. H., P. K. Lim, J. G. Lee, and K. Hur, Bipolar Fuzzy Intuitionistic topological spaces, Infinite Study, (2018).

[13] Vishalakshi. K., and Maragathavalli.S., Bipolar Pythagorean Fuzzy Regular Generalized closed sets, IJSRED, 4(2021), 55-62.

BIOGRAPHY

K. Vishalakshi is a Research Scholar in the Department of Mathematics Government Arts College, Udumalpet at Bharathiar University, India since June 2019. She worked as an Assistant Professor and has 9 years of experience in an Engineering College. Her research interests include Topology, Bipolar Pythagorean Fuzzy Topological spaces, etc. She has been qualified in TNSET-2016. She Published two articles. Email ID: vishalkapil2006@gmail.com

Dr. S. Maragathavalli received her Ph.D., degree in 2011 from Bharathiar University, Tamilnadu, India. She is working as an Assistant Professor in the Department of Mathematics, Government Arts College, Udumalpet (Bharathiar University), India. Her research interests include General Topology, Intuitionistic Fuzzy Topological spaces, Bipolar Pythagorean Fuzzy Topological spaces, etc. She Published more than 50 articles in reputed Journals and conferences. Email ID: smvalli@gmail.com

Dr.R.Santhi working as an Assistant Professor, Department of Mathematics in Nallamuthu Gounder Mahalingam College, Pollachi, Tamilnadu, India. She has 19 years of teaching experience and rich Research experience. She has guided 22 M.Phil. and 3 Ph.D. students. She has published more than 39 research articles in National and International journals. Her Research area is Topology and Operation Research. Email ID: santhir2004@yahoo.co.in