VOLUME X
ISBN No.: 978-81-953602-6-0
Physical Science

Estd. 1957

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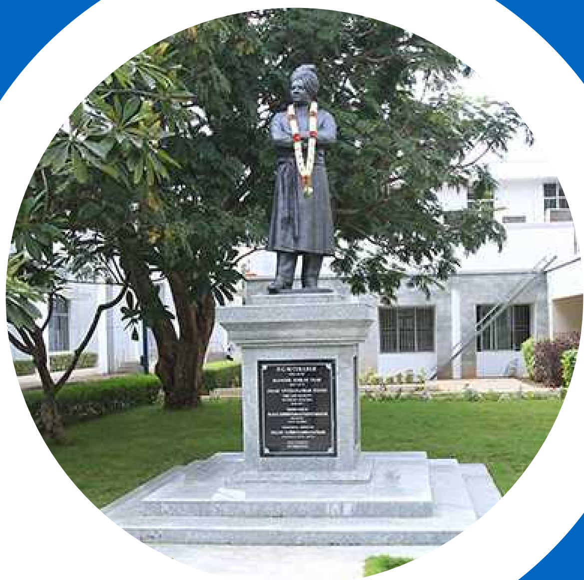

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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

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ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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Interval Valued Pythagorean Fuzzy Soft Sets and Their Properties

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ABSTRACT: The Pythagorean soft set theory is a mathematical model for dealing with uncertainty and vagueness in multi criteria decision making problems. In this paper, we introduce the concept of interval valued Pythagorean fuzzy soft set theory which is a combination of interval valued Pythagorean fuzzy set and soft set theory. Some operations such as complement, union, intersection, necessity and possibility operations are defined on the interval valued Pythagorean fuzzy soft set. Also some of their properties are studied.

Keywords: Soft sets, fuzzy soft sets, Pythagorean fuzzy sets, Pythagorean soft sets, Pythagorean fuzzy soft sets, interval valued Pythagorean fuzzy sets, interval valued Pythagorean fuzzy soft sets

1. INTRODUCTION

Soft set theory, a parameterization of objects has been introduced by Molodtsov [1] which helps in dealing with uncertainty. Later on Maji P.K., Biswas R., Roy A.R., [3] proposed the theory of fuzzy soft set. Atanassov [2] extended fuzzy soft sets to intuitionistic fuzzy soft sets. The characteristic that sum of membership and non membership degree in intuitionistic fuzzy soft sets are less than or equal to one is insufficient for multi criteria decision making problems. Hence Yager R.R., Abbasov A.M. [8] introduced the concept of Pythagorean fuzzy sets which satisfies the case. Since a crisp value cannot quantify the opinions made by experts in decision making, interval valued Pythagorean fuzzy set theory has been developed by Peng, X. and Yang, Y. [6], some operations, their properties were discussed in it.

In this paper, we propose interval valued Pythagorean fuzzy soft set theory which is a combination of interval valued Pythagorean fuzzy set and soft sets. Some basic operations were defined and its properties were discussed.

2. PRELIMINARIES

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In this section, we recall some basic definitions such as Pythagorean fuzzy set, Pythagorean fuzzy soft set and interval valued Pythagorean fuzzy set.

Definition 2.1[8]: Pythagorean fuzzy set, A is defined as a set of ordered pairs over a universal set X given by $A = \{< x, \mu_A(x), \vartheta_A(x) > | \ x \in X\}$, where $\mu_A, \vartheta_A : X \to [0,1]$ are the degrees of membership and nonmembership of the element $x \in$ X, respectively, with the condition that $(\mu_A(x))^2 + (\vartheta_A(x))^2 \leq 1$. Corresponding to its membership functions, the degree of indeterminacy is given by $\pi_A(x) = \sqrt{1 - (\mu_A(x))^2 - (\vartheta_A(x))^2}$.

Definition 2.2[4]: A pair (F,E) is called Pythagorean fuzzy soft set if a map F : E $\to$ P^U defined as $F_{u_i}(e_j) = \{(u_i, \zeta_j(u_i), \vartheta_j(u_i)) : u_i \in U$ where P^U is the Pythagorean fuzzy subset of U and ζ, ϑ satisfies $\zeta^2 + \vartheta^2 \leq 1$ for all $u_i \in$ U.

A pair (F,E) is termed as Pythagorean soft set and denote $F_{u_i}(e_j) = \{(u_i, \zeta_j(u_i), \vartheta_j(u_i))$ called as Pythagorean fuzzy soft number (PFSN) with $\zeta^2_{F(e_j)} + \vartheta^2_{F(e_j)} \leq 1$ for $\zeta^2_{F(e_j)}, \vartheta^2_{F(e_j)} \in [0,1]$.

Definition 2.3[6]: Let X is the universe of discourse. An interval-valued Pythagorean fuzzy set (IVPFS) A defined in X is given as $A = \{\langle x, [\mu^L_A(x), \ \mu^U_A(x)], \ [\vartheta^L_A(x), \ \vartheta^U_A(x)]\rangle : x \in X\}$, where $0 \leq \mu^L_A(x) \leq \mu^U_A(x) \leq 1, \ 0 \leq \vartheta^L_A(x) \leq \vartheta^U_A(x) \leq 1$ and $(\mu^U_A(x))^2 + (\vartheta^U_A(x))^2 \leq 1$ for all $x \in X$. Similar to PFSs, corresponding to interval-valued membership values, its hesitation interval relative to A is given as $\pi_A(x) = \left[\sqrt{1 - (\mu^U_A(x))^2 - (\vartheta^U_A(x))^2}, \ \sqrt{1 - (\mu^L_A(x))^2 - (\vartheta^L_A(x))^2}\right]$.

If for every $x \in X, \mu_A(x) = \mu^L_A(x) = \mu^U_A(x), \ \vartheta_A(x) = \vartheta^L_A(x) = \vartheta^U_A(x)$, then IVPFS reduces to PFS. For an IVPFS A, the pair $\langle [\mu^L_A(x), \ \mu^U_A(x)], \ [\vartheta^L_A(x), \ \vartheta^U_A(x)]\rangle$ is called an interval-valued Pythagorean fuzzy number (IVPFN). For convenience, this pair is often denoted by $\alpha = \langle [a,b], [c,d]\rangle$, where [a, b] $\subseteq$ [0, 1], [c, d] $\subseteq$ [0, 1], and $b^2 + d^2 \leq 1$.

3. MAJOR SECTION

Definition 3.1: Let the parameter set be $\mathbb{E}$. Let $\mathcal{A} \subseteq \mathbb{E}$. A pair $(f, \ \mathcal{A})$ is called an interval-valued Pythagorean fuzzy soft set (IVPFSS) over $\mathbb{U}$, where $\mathbb{U}$ is the universe and f is a mapping given by $f : \mathcal{A} \to IP^{\mathbb{U}}$, where $IP^{\mathbb{U}}$ denotes the collection of all interval-valued Pythagorean fuzzy subsets of $\mathbb{U}$.

Definition 3.2: Let $\mathbb{U}$ be the universe and the parameter set be $\mathbb{E}$. Let $\mathcal{A}, \mathcal{B} \subset \mathbb{E}$, $(f, \ \mathcal{A})$ and $(g, \mathcal{B})$ be two IVPSS, then $(f, \ \mathcal{A})$ is an interval-valued Pythagorean fuzzy soft subset of $(g, \mathcal{B})$ if and only if

1. $\mathcal{A} \subseteq \mathcal{B}$,
2. $\forall \epsilon \in \mathcal{A}, f(\epsilon)$ is an interval-valued Pythagorean fuzzy soft subsets of $g(\epsilon)$. ie., $\forall x \in \mathbb{U}$ and $\epsilon \in \mathcal{A}, \ \mu^L_{f(\epsilon)}(x) \leq \mu^L_{g(\epsilon)}(x), \mu^U_{f(\epsilon)}(x) \leq \mu^U_{g(\epsilon)}(x), \ \vartheta^L_{f(\epsilon)}(x) \geq \vartheta^L_{g(\epsilon)}(x), \ \vartheta^U_{f(\epsilon)}(x) \geq \vartheta^U_{g(\epsilon)}(x)$ and is denoted by $(f, \ \mathcal{A}) \subseteq (g, \mathcal{B})$. Similarily, $(f, \ \mathcal{A})$ is said to be an interval-valued Pythagorean fuzzy soft superset of $(g, \mathcal{B})$, if $(g, \mathcal{B})$ is an interval-valued Pythagorean fuzzy soft subset of $(f, \ \mathcal{A})$ and is denoted by $(f, \ \mathcal{A}) \supseteq (g, \mathcal{B})$.

Definition 3.3: Let $(f,\ \mathcal{A})$ and $(g, \mathcal{B})$ be two IVPFSS over $\mathbb{U}$. $(f,\ \mathcal{A})$ and $(g, \mathcal{B})$ are said to be interval-valued Pythagorean fuzzy soft equal if and only if $(f,\ \mathcal{A})$ is an interval-valued Pythagorean fuzzy soft subset of $(g, \mathcal{B})$ and vice – versa.

Definition 3.4: Let $\mathbb{E} = \{e_1, e_2, e_3, \ldots. e_n\}$ be a parameter set. The Not set of $\mathbb{E}$ is denoted by $\rceil\mathbb{E}$, defined as $\rceil\mathbb{E} = \{\neg e_1, \neg e_2, \neg e_3, \ldots, \neg e_n\}$ where $\neg e_i$ is not e_i.

Definition 3.5: The complement of an IVPFSS $(f,\ \mathcal{A})$ is denoted by $(f,\ \mathcal{A})^C$, defined as $(f,\ \mathcal{A})^C = (f^c, \rceil\mathcal{A})$, where $f^c : \rceil\mathcal{A} \to \mathrm{IP}^{\mathbb{U}}$, where $\mathrm{IP}^{\mathbb{U}}$ denotes the collection of all interval-valued Pythagorean fuzzy subsets of $\mathbb{U}$ and the mapping is given by $f^c(\varepsilon) = \left(x, \vartheta_{f(\neg\varepsilon)}(x), \mu_{f(\neg\varepsilon)}(x)\right)\ \forall x \in \mathbb{U}$ and $\varepsilon \in \rceil\mathcal{A}$.

Definition 3.6: An IVPFSS $(f,\ \mathcal{A})$ over $\mathbb{U}$ is said to be a null IVPFSS denoted by $\emptyset$, if $\forall \varepsilon \in \mathcal{A}, \mu_{f(\varepsilon)}(x) = [0,0], \vartheta_{f(\varepsilon)}(x) = [1,1], x \in \mathbb{U}$.

Definition 3.7: An IVPFSS $(f,\ \mathcal{A})$ over $\mathbb{U}$ is said to be a absolute IVPFSS denoted by $\sum$, if $\forall \varepsilon \in \mathcal{A}, \mu_{f(\varepsilon)}(x) = [1,1], \vartheta_{f(\varepsilon)}(x) = [0,0], x \in \mathbb{U}$.

Definition 3.8: The union of two IVPFSS $(f,\ \mathcal{A})$ and $(g, \mathcal{B})$ over a universe $\mathbb{U}$ is an IVPFSS $(h,\ \mathcal{C})$ where $\mathcal{C} = \mathcal{A} \cup \mathcal{B}$ and $\forall \varepsilon \in \mathcal{C}$.

$$\mu_{h(\varepsilon)}(x) = \begin{cases} \mu_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A} - \mathcal{B},\ x \in \mathbb{U} \\ \mu_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B} - \mathcal{A},\ x \in \mathbb{U} \\ \left[\sup\left(\mu^L_{f(\varepsilon)}(x), \mu^L_{g(\varepsilon)}(x)\right), \sup\left(\mu^U_{f(\varepsilon)}(x), \mu^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

$$\vartheta_{h(\varepsilon)}(x) = \begin{cases} \vartheta_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A} - \mathcal{B},\ x \in \mathbb{U} \\ \vartheta_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B} - \mathcal{A},\ x \in \mathbb{U} \\ \left[\inf\left(\vartheta^L_{f(\varepsilon)}(x), \vartheta^L_{g(\varepsilon)}(x)\right), \inf\left(\vartheta^U_{f(\varepsilon)}(x), \vartheta^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases},$$

denoted by $(f,\ \mathcal{A}) \cup (g, \mathcal{B}) = (h,\ \mathcal{C})$.

Definition 3.9: The intersection of two IVPFSS $(f,\ \mathcal{A})$ and $(g, \mathcal{B})$ over a universe $\mathbb{U}$ is an IVFPSS $(h,\ \mathcal{C})$ where $\mathcal{C} = \mathcal{A} \cap \mathcal{B}$ and $\forall \varepsilon \in \mathcal{C}$.

$$\mu_{h(\varepsilon)}(x) = \begin{cases} \mu_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A} - \mathcal{B},\ x \in \mathbb{U} \\ \mu_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B} - \mathcal{A},\ x \in \mathbb{U} \\ \left[\inf\left(\mu^L_{f(\varepsilon)}(x), \mu^L_{g(\varepsilon)}(x)\right), \inf\left(\mu^U_{f(\varepsilon)}(x), \mu^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

$$\vartheta_{h(\varepsilon)}(x) = \begin{cases} \vartheta_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A} - \mathcal{B},\ x \in \mathbb{U} \\ \vartheta_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B} - \mathcal{A},\ x \in \mathbb{U} \\ \left[\sup\left(\vartheta^L_{f(\varepsilon)}(x), \vartheta^L_{g(\varepsilon)}(x)\right), \sup\left(\vartheta^U_{f(\varepsilon)}(x), \vartheta^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases},$$

denoted by $(f,\ \mathcal{A}) \cap (g, \mathcal{B}) = (h,\ \mathcal{C})$.

Theorem 3.1: Let the parameter set be $\mathbb{E}$, $\mathcal{A} \subseteq \mathbb{E}$. If $\emptyset$ is a null IVPFSS, $\sum$ an absolute IVPFSS and $(f,\ \mathcal{A})$ and $(f,\ \mathbb{E})$ be two IVPFSS over $\mathbb{U}$, then

1. $(f,\ \mathcal{A}) \cup (f,\ \mathcal{A}) = (f,\ \mathcal{A})$
2. $(f,\ \mathcal{A}) \cap (f,\ \mathcal{A}) = (f,\ \mathcal{A})$
3. $(f,\ \mathbb{E}) \cup \emptyset = (f,\ \mathbb{E})$
4. $(f,\ \mathbb{E}) \cap \emptyset = \emptyset$

5. $(f,\ \mathbb{E}) \cup \sum = \sum$
6. $(f,\ \mathbb{E}) \cap \sum = (f,\ \mathbb{E})$

Proof: Theorem proof can be derived from the Definition 3.8 and Definition 3.9.

Theorem 3.2: If $(f,\ \mathcal{A})$ and $(g,\ \mathcal{B})$ be two IVPFSS over $\mathbb{U}$, then

1. $\left((f,\ \mathcal{A}) \cup (g,\ \mathcal{B})\right)^c = (f,\ \mathcal{A})^c \cap (g,\ \mathcal{B})^c$,
2. $\left((f,\ \mathcal{A}) \cap (g,\ \mathcal{B})\right)^c = (f,\ \mathcal{A})^c \cup (g,\ \mathcal{B})^c$

Proof: 1. By Definition 3.8,$(f,\ \mathcal{A}) \cup (g,\ \mathcal{B}) = (h,\ \mathcal{C})$, then $\left((f,\ \mathcal{A}) \cup (g,\ \mathcal{B})\right)^c = (h,\ \mathcal{C})^c = (h^C, \neg\mathcal{C})$, $h^C(\neg\varepsilon) = \left(x, \vartheta_{h(\varepsilon)}(x), \mu_{h(\varepsilon)}(x)\right)\ \forall x \in \mathbb{U}$ and $\neg\varepsilon \in \neg\mathcal{C} = \neg(\mathcal{A} \cup \mathcal{B}) = \neg\mathcal{A} \cap \neg\mathcal{B}$. Therefore,

$$\mu_{h^C(\neg\varepsilon)}(x) = \begin{cases} \vartheta_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A} - \mathcal{B}, \quad x \in \mathbb{U} \\ \vartheta_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B} - \mathcal{A}, \quad x \in \mathbb{U} \\ \left[\inf\left(\vartheta^L_{f(\varepsilon)}(x), \vartheta^L_{g(\varepsilon)}(x)\right), \inf\left(\vartheta^U_{f(\varepsilon)}(x), \vartheta^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

$$\vartheta_{h^C(\neg\varepsilon)}(x) = \begin{cases} \mu_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A} - \mathcal{B}, \quad x \in \mathbb{U} \\ \mu_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B} - \mathcal{A}, \quad x \in \mathbb{U} \\ \left[\sup\left(\mu^L_{f(\varepsilon)}(x), \mu^L_{g(\varepsilon)}(x)\right), \sup\left(\mu^U_{f(\varepsilon)}(x), \mu^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

Since $(f,\mathcal{A})^c = (f^C, \neg\mathcal{A})$and $(g,\mathcal{B})^C = (g^C, \neg\mathcal{B})$, then $(f,\mathcal{A})^c \cap (g,\mathcal{B})^c = (f^C, \neg\mathcal{A}) \cap (g^C, \neg\mathcal{B})$. Let $(f^C, \neg\mathcal{A}) \cap (g^C, \neg\mathcal{B}) = (i, \mathcal{D})$ where $\mathcal{D} = \neg\mathcal{C} = \neg(\mathcal{A} \cup \mathcal{B}) = \neg\mathcal{A} \cap \neg\mathcal{B}$ and $\neg\varepsilon \in \mathcal{D}$.

$$\mu_{i(\neg\varepsilon)}(x) = \begin{cases} \vartheta_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A} - \mathcal{B}, \quad x \in \mathbb{U} \\ \vartheta_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B} - \mathcal{A}, \quad x \in \mathbb{U} \\ \left[\inf\left(\vartheta^L_{f(\varepsilon)}(x), \vartheta^L_{g(\varepsilon)}(x)\right), \inf\left(\vartheta^U_{f(\varepsilon)}(x), \vartheta^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

$$\vartheta_{i(\neg\varepsilon)}(x) = \begin{cases} \mu_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A} - \mathcal{B}, \quad x \in \mathbb{U} \\ \mu_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B} - \mathcal{A}, \quad x \in \mathbb{U} \\ \left[\sup\left(\mu^L_{f(\varepsilon)}(x), \mu^L_{g(\varepsilon)}(x)\right), \sup\left(\mu^U_{f(\varepsilon)}(x), \mu^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

Hence h^C and i are the same operators and $\left((f,\ \mathcal{A}) \cup (g,\ \mathcal{B})\right)^c = (f,\ \mathcal{A})^c \cap (g,\ \mathcal{B})^c$. Similarly we can prove 2.

Definition 3.10: The necessity operation on an IVPFSS $(f,\ \mathcal{A})$ is denoted by $\square(f,\ \mathcal{A}) = \left\{\left(x, \mu_{\square f(\varepsilon)}(x), \vartheta_{\square f(\varepsilon)}(x)\right) : x \in \mathbb{U}\ and\ \varepsilon \in \mathcal{A}\right\}$ where $\mu_{\square f(\varepsilon)}(x) = \left[\mu^L_{f(\varepsilon)}(x), \mu^U_{f(\varepsilon)}(x)\right]$ is the interval valued fuzzy membership degree that the object x holds on the parameter ε, $\vartheta_{\square f(\varepsilon)}(x) = \left[\sqrt{1 - \left(\mu^U_{f(\varepsilon)}(x)\right)^2}, \sqrt{1 - \left(\mu^L_{f(\varepsilon)}(x)\right)^2}\right]$is the interval valued fuzzy membership degree that the object x does not hold on the parameter ε and f is a mapping $f : \mathcal{A} \to IP^{\mathbb{U}}$ where $IP^{\mathbb{U}}$ is the interval-valued Pythagorean fuzzy subsets of $\mathbb{U}$.

Theorem: 3.3: Let $(f, \mathcal{A})$ and $(g, \mathcal{B})$ be two IVPFSS over $\mathbb{U}$, then the following properties hold:

1. $\square\left((f,\mathcal{A}) \cup (g,\mathcal{B})\right) = \square(f,\mathcal{A}) \cup \square(g,\mathcal{B})$
2. $\square\left((f,\mathcal{A}) \cap (g,\mathcal{B})\right) = \square(f,\mathcal{A}) \cap \square(g,\mathcal{B})$
3. $\square\square(f, \mathcal{A}) = \square(f, \mathcal{A})$.

Proof: 1. By Definition 3.8, we have $\square\left((f,\mathcal{A})\cup(g,\mathcal{B})\right) = \square(h,\mathcal{C})$ where $\mathcal{C} = \mathcal{A}\cup\mathcal{B}$ and $\forall\varepsilon\in\mathcal{C}$. By the Definition 3.10, we have

$$\mu_{\square h(\varepsilon)}(x) = \begin{cases} \mu_{f(\varepsilon)}(x) & if\ \varepsilon\in\mathcal{A}-\mathcal{B},\ x\in\mathbb{U} \\ \mu_{g(\varepsilon)}(x) & if\ \varepsilon\in\mathcal{B}-\mathcal{A},\ x\in\mathbb{U} \\ \left[\sup\left(\mu^L_{f(\varepsilon)}(x),\mu^L_{g(\varepsilon)}(x)\right),\sup\left(\mu^U_{f(\varepsilon)}(x),\mu^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon\in\mathcal{A}\cap\mathcal{B}, x\in\mathbb{U}\end{cases}$$

$$\vartheta_{\square h(\varepsilon)}(x) = \begin{cases} \left[\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{f(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon\in\mathcal{A}-\mathcal{B},\quad x\in\mathbb{U} \\ \left[\sqrt{1-\left(\mu^U_{g(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{g(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon\in\mathcal{B}-\mathcal{A},\quad x\in\mathbb{U} \\ \left[\sqrt{1-\sup\left(\mu^U_{f(\varepsilon)}(x),\mu^U_{g(\varepsilon)}(x)\right)},\sqrt{1-\sup\left(\mu^L_{f(\varepsilon)}(x),\mu^L_{g(\varepsilon)}(x)\right)}\right], & if\ \varepsilon\in\mathcal{A}\cap\mathcal{B}, x\in\mathbb{U}\end{cases}$$

Assume $\square(f,\mathcal{A}) = \left\{\left(x,\left[\mu^L_{f(\varepsilon)}(x),\mu^U_{f(\varepsilon)}(x)\right],\left[\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{f(\varepsilon)}(x)\right)^2}\right]\right) : x\in\mathbb{U}\ and\ \varepsilon\in\mathcal{A}\right\}$ and $\square(g,\mathcal{B}) = \left\{\left(x,\left[\mu^L_{g(\varepsilon)}(x),\mu^U_{g(\varepsilon)}(x)\right],\left[\sqrt{1-\left(\mu^U_{g(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{g(\varepsilon)}(x)\right)^2}\right]\right) : x\in\mathbb{U}\ and\ \varepsilon\in\mathcal{B}\right\}$. Hence $\square(f,\mathcal{A})\cup(g,\mathcal{B}) = (o,\mathcal{C})$ where $\mathcal{C} = \mathcal{A}\cup\mathcal{B}$ and $\varepsilon\in\mathcal{C}$.

$$\mu_{o(\varepsilon)}(x) = \begin{cases} \mu_{f(\varepsilon)}(x) & if\ \varepsilon\in\mathcal{A}-\mathcal{B},\ x\in\mathbb{U} \\ \mu_{g(\varepsilon)}(x) & if\ \varepsilon\in\mathcal{B}-\mathcal{A},\ x\in\mathbb{U} \\ \left[\sup\left(\mu^L_{f(\varepsilon)}(x),\mu^L_{g(\varepsilon)}(x)\right),\sup\left(\mu^U_{f(\varepsilon)}(x),\mu^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon\in\mathcal{A}\cap\mathcal{B}, x\in\mathbb{U}\end{cases}$$

$$\vartheta_{o(\varepsilon)}(x) = \begin{cases} \left[\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{f(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon\in\mathcal{A}-\mathcal{B},\quad x\in\mathbb{U} \\ \left[\sqrt{1-\left(\mu^U_{g(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{g(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon\in\mathcal{B}-\mathcal{A},\quad x\in\mathbb{U} \\ \left[\left(\inf\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^U_{g(\varepsilon)}(x)\right)^2}\right),\inf\left(\sqrt{1-\left(\mu^L_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{g(\varepsilon)}(x)\right)^2}\right)\right], & if\ \varepsilon\in\mathcal{A}\cap\mathcal{B}, x\in\mathbb{U}\end{cases}$$

$$= \begin{cases} \left[\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{f(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon\in\mathcal{A}-\mathcal{B},\quad x\in\mathbb{U} \\ \left[\sqrt{1-\left(\mu^U_{g(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^L_{g(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon\in\mathcal{B}-\mathcal{A},\quad x\in\mathbb{U} \\ \left[\sqrt{1-\sup\left(\mu^U_{f(\varepsilon)}(x),\mu^U_{g(\varepsilon)}(x)\right)},\sqrt{1-\sup\left(\mu^L_{f(\varepsilon)}(x),\mu^L_{g(\varepsilon)}(x)\right)}\right], & if\ \varepsilon\in\mathcal{A}\cap\mathcal{B}, x\in\mathbb{U}\end{cases}$$

Hence $\square(h,\mathcal{C}) = (o,\mathcal{C})$. Thus we have proved 1. Similarly we can prove 2 and 3.

Definition 3.11: The possibility operation on an IVPFSS $(f,\mathcal{A})$ is denoted by $\Diamond(f,\mathcal{A}) = \left\{\left(x,\mu_{\Diamond f(\varepsilon)}(x),\vartheta_{\Diamond f(\varepsilon)}(x)\right) : x\in\mathbb{U}\ and\ \varepsilon\in\mathcal{A}\right\}$ where $\vartheta_{\Diamond f(\varepsilon)}(x) = \left[\vartheta^L_{f(\varepsilon)}(x),\vartheta^U_{f(\varepsilon)}(x)\right]$ is the interval valued fuzzy membership degree that the object x does not hold on the parameter ε, $\mu_{\Diamond f(\varepsilon)}(x) = \left[\sqrt{1-\left(\vartheta^U_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\vartheta^L_{F(\varepsilon)}(x)\right)^2}\right]$ is the interval valued fuzzy membership degree that the object x hold on the parameter ε and f is a mapping $f:\mathcal{A}\to IP^{\mathbb{U}}$, where $IP^{\mathbb{U}}$ is the interval-valued Pythagorean fuzzy subsets of $\mathbb{U}$.

Theorem 3.4: Let $(f, \mathcal{A})$ and $(g, \mathcal{B})$ be two IVPFSS over $\mathbb{U}$, then the following properties hold:

1. $\lozenge\left((f, \mathcal{A}) \cup (g, \mathcal{B})\right) = \lozenge (f, \mathcal{A}) \cup \lozenge (g, \mathcal{B})$
2. $\lozenge\left((f, \mathcal{A}) \cap (g, \mathcal{B})\right) = \lozenge (f, \mathcal{A}) \cap \lozenge (g, \mathcal{B})$
3. $\lozenge\lozenge (f, \mathcal{A}) = \lozenge (f, \mathcal{A})$

Proof: 1. By Definition 3.8, we have $\lozenge\left((f, \mathcal{A}) \cup (g, \mathcal{B})\right) = \lozenge (h, \mathcal{C})$. By Definition 3.11,

$$\mu_{\lozenge h(\varepsilon)}(x) = \begin{cases} \left[\sqrt{1-\left(\vartheta^U_{f(\varepsilon)}(x)\right)^2}, \sqrt{1-\left(\vartheta^L_{f(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon \in \mathcal{A}-\mathcal{B}, \quad x \in \mathbb{U} \\ \left[\sqrt{1-\left(\vartheta^U_{g(\varepsilon)}(x)\right)^2}, \sqrt{1-\left(\vartheta^L_{g(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon \in \mathcal{B}-\mathcal{A}, \quad x \in \mathbb{U} \\ \left[\sqrt{1-\inf\left(\vartheta^U_{f(\varepsilon)}(x), \vartheta^U_{g(\varepsilon)}(x)\right)}, \sqrt{1-\inf\left(\vartheta^L_{f(\varepsilon)}(x), \vartheta^L_{g(\varepsilon)}(x)\right)}\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

$$\vartheta_{\lozenge h(\varepsilon)}(x) = \begin{cases} \vartheta_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A}-\mathcal{B}, \quad x \in \mathbb{U} \\ \vartheta_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B}-\mathcal{A}, \quad x \in \mathbb{U} \\ \left[\inf\left(\vartheta^L_{f(\varepsilon)}(x), \vartheta^L_{g(\varepsilon)}(x)\right), \inf\left(\vartheta^U_{f(\varepsilon)}(x), \vartheta^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

Assume $\lozenge (f, \mathcal{A})$ and $\lozenge (g, \mathcal{B})$ by Definition 3.11 and by Definiton 3.8, we have $\lozenge (f, \mathcal{A}) \cup \lozenge (g, \mathcal{B}) = (o, \mathcal{C})$, where $\mathcal{C} = \mathcal{A} \cup \mathcal{B}$ and $\varepsilon \in \mathcal{C}$.

$$\mu_{o(\varepsilon)}(x) = \begin{cases} \left[\sqrt{1-\left(\vartheta^U_{f(\varepsilon)}(x)\right)^2}, \sqrt{1-\left(\vartheta^L_{f(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon \in \mathcal{A}-\mathcal{B}, \quad x \in \mathbb{U} \\ \left[\sqrt{1-\left(\vartheta^U_{g(\varepsilon)}(x)\right)^2}, \sqrt{1-\left(v^L_{g(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon \in \mathcal{B}-\mathcal{A}, \quad x \in \mathbb{U} \\ \left[\left(\sup\sqrt{1-\left(\vartheta^U_{f(\varepsilon)}(x)\right)^2}, \sqrt{1-\left(\vartheta^U_{g(\varepsilon)}(x)\right)^2}\right), \sup\left(\sqrt{1-\left(\vartheta^U_{f}(x)\right)^2}, \sqrt{1-\left(\vartheta^L_{g(\varepsilon)}(x)\right)^2}\right)\right], & \\ \quad if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

$$= \begin{cases} \left[\sqrt{1-\left(\vartheta^U_{f(\varepsilon)}(x)\right)^2}, \sqrt{1-\left(\vartheta^L_{f(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon \in \mathcal{A}-\mathcal{B}, \quad x \in \mathbb{U} \\ \left[\sqrt{1-\left(\vartheta^U_{g(\varepsilon)}(x)\right)^2}, \sqrt{1-\left(\vartheta^L_{g(\varepsilon)}(x)\right)^2}\right] & if\ \varepsilon \in \mathcal{B}-\mathcal{A}, \quad x \in \mathbb{U} \\ \left[\sqrt{1-\inf\left(\vartheta^U_{f(\varepsilon)}(x), \vartheta^U_{g(\varepsilon)}(x)\right)}, \sqrt{1-\inf\left(\vartheta^L_{f(\varepsilon)}(x), \vartheta^L_{g(\varepsilon)}(x)\right)}\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

$$\vartheta_{o(\varepsilon)}(x) = \begin{cases} \vartheta_{f(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{A}-\mathcal{B}, \quad x \in \mathbb{U} \\ \vartheta_{g(\varepsilon)}(x) & if\ \varepsilon \in \mathcal{B}-\mathcal{A}, \quad x \in \mathbb{U} \\ \left[\inf\left(\vartheta^L_{f(\varepsilon)}(x), \vartheta^L_{g(\varepsilon)}(x)\right), \inf\left(\vartheta^U_{f(\varepsilon)}(x), \vartheta^U_{g(\varepsilon)}(x)\right)\right], & if\ \varepsilon \in \mathcal{A} \cap \mathcal{B}, x \in \mathbb{U} \end{cases}$$

Hence $\lozenge$ $(h, \mathcal{C})$ and $(o, \mathcal{C})$ are same IVPFSS. Hence (i).Similarly we can prove 2 and 3.

Theorem 3.5: Let $(f, \mathcal{A})$ be an IVFPSS over $\mathbb{U}$, then we have the following properties

1. $\square(f, \mathcal{A}) \subset (f, \mathcal{A}) \subset \lozenge (f, \mathcal{A})$;
2. $\lozenge \square(f, \mathcal{A}) = \square(f, \mathcal{A})$;
3. $\square \lozenge (f, \mathcal{A}) = \lozenge (f, \mathcal{A})$

Proof: 1. Suppose that $(f,\mathcal{A}) = \{(x,[\mu^L_{f(\varepsilon)}(x),\mu^U_{f(\varepsilon)}(x)],[\vartheta^L_{f(\varepsilon)}(x),\vartheta^U_{f(\varepsilon)}(x)]) : x \in \mathbb{U}\ and\ \varepsilon \in \mathcal{A}\}$. Hence by Definition 3.10 and Definition 3.11, define $\square(f,\mathcal{A})\ and\ \lozenge(f,\mathcal{A})$. Since$\left(\left(\mu^U_{f(\varepsilon)}(x)\right)^2\right) + \left(\left(\vartheta^U_{f(\varepsilon)}(x)\right)^2\right) \le 1$, then $\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2} \ge \vartheta^U_{f(\varepsilon)}(x) \ge \vartheta^L_{f(\varepsilon)}$ and $\sqrt{1-\left(\mu^L_{f(\varepsilon)}(x)\right)^2} \ge \sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2} \ge \vartheta^U_{f(\varepsilon)}(x)$. Since $\mu^L_{f(\varepsilon)}(x) \ge \mu^L_{f(\varepsilon)}(x)$ and $\mu^U_{f(\varepsilon)}(x) \ge \mu^U_{f(\varepsilon)}(x)$. Hence $\square(f,\mathcal{A}) \subset (f,\mathcal{A})$. Now, $\sqrt{1-\left(\vartheta^U_{f(\varepsilon)}(x)\right)^2} \ge \mu^U_{f(\varepsilon)}(x) \ge \mu^L_{f(\varepsilon)}$ and $\sqrt{1-\left(\vartheta^L_{f(\varepsilon)}(x)\right)^2} \ge \sqrt{1-\left(\vartheta^U_{f(\varepsilon)}(x)\right)^2} \ge \mu^U_{f(\varepsilon)}(x)$. Since $\vartheta^L_{f(\varepsilon)}(x) \ge \vartheta^L_{f(\varepsilon)}(x)$ and $\vartheta^U_{f(\varepsilon)}(x) \ge \vartheta^U_{f(\varepsilon)}(x)$., hence $(f,\mathcal{A}) \subset \lozenge(f,\mathcal{A})$.

Assume definition 3.10. Then

$$\lozenge\square(f,\mathcal{A}) = \left\{\left(\begin{matrix} x,\left[\sqrt{1-\left(\sqrt{1-\left(\mu^L_{f(\varepsilon)}(x)\right)^2}\right)},\ \sqrt{1-\left(\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2}\right)}\right], \\ \left[\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2}\right]\end{matrix}\right) : x \in \mathbb{U}\ and\ \varepsilon \in \mathcal{A}\right\}$$

$$= \left\{\left(x,\ [\mu^L_{f(\varepsilon)}(x),\mu^U_{f(\varepsilon)}(x)],\left[\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2},\sqrt{1-\left(\mu^U_{f(\varepsilon)}(x)\right)^2}\right]\right) : x \in\ \mathbb{U}\ and\ \varepsilon \in \mathcal{A}\right\}$$

$$= \square(f,\mathcal{A}).$$

Similarly we can prove 2 as 3.

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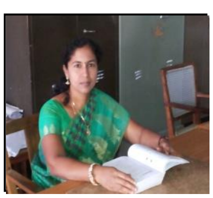
Biography



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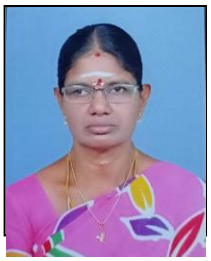

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