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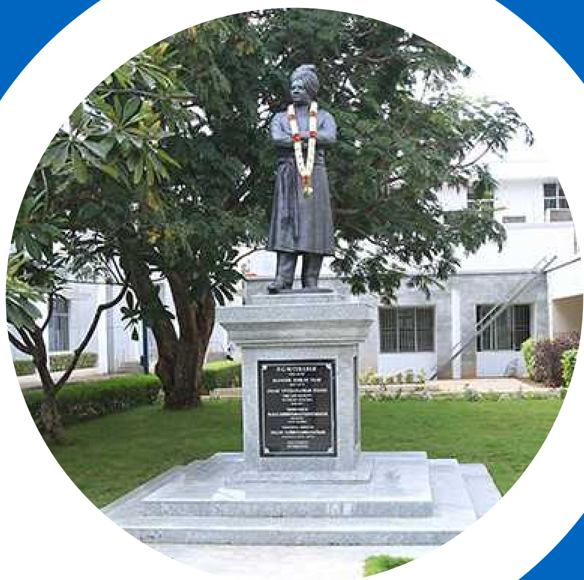

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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

Almost Regular α Generalized Continuous Mappings in Bipolar Pythagorean Fuzzy Topological Spaces

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ABSTRACT: In this paper, the concept of Almost Regular α Generalized Continuous Mappings was introduced and investigated some of their properties. Also, We have provided some characterization of Bipolar Pythagoren Fuzzy Almost Regular α Generalized Continuous Mappings.

KEYWORDS: Bipolar Pythagorean Fuzzy sets, Bipolar Pythagorean Fuzzy Topology, Bipolar Pythagoren Fuzzy Regular α Generalized Closed sets, Bipolar Pythagorean Fuzzy Regular α generalized continuous mappings,Bipolar Pythagorean Fuzzy Almost Regular α generalized continuous mappings.

1. INTRODUCTION

Atanassov [7] proposed an intuitionistic fuzzy set using the notion of fuzzy sets. On the other hand Coker [1] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. Yager [2] proposed another class of nonstandard fuzzy sets, called Pythagorean Fuzzy sets. Zhang [4] introduced the extension of fuzzy sets with Bipolarity, called Bipolar value fuzzy sets. In this paper we introduced the notion of Bipolar Pythagorean Fuzzy Almost Regular α Generalized Continuous Mappings and studied their behaviour and properties in Bipolar pythagorean fuzzy topological spaces. Also we obtained some interesting theorems.

2. PRELIMINARIES

Definition 2.1: Let X be a non-empty set. A Bipolar Pythagorean Fuzzy Set(BPFS in short) $A = \{(x, \mu_A^+, \mu_A^-, \nu_A^+, \nu_A^-): x \in X\}$ where $\mu_A^+: X \to [0,1]$, $\nu_A^+: X \to [0,1]$, $\mu_A^-: X \to [-1,0]$, $\nu_A^-: X \to [-1,0]$ are the mappings such that $0 \leqslant (\mu_A^+(x))^2 + (\nu_A^+(x))^2 \leqslant 1$ and $-1 \leqslant (\mu_A^-(x))^2 + (\nu_A^-(x))^2 \leqslant 0$ where $\mu_A^+(x)$ denote the positive membership degree, $\nu_A^+(x)$ denote the positive non membership degee, $\mu_A^-(x)$ denote the negative membership degree, $\nu_A^-(x)$ denote the negative non membership degree.

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Definition 2.2: Let $X \neq \emptyset$ be a set and τ_p be a family of Bipolar Pythagorean fuzzy subsets of X. If

(a) $0_X, 1_X \in \tau_p$.

(b) For any $P_1, P_2 \in \tau_p$, we have $P_1 \cap P_2 \in \tau_p$.

(c) $\cup P_i \in \tau_p$ for an arbitrary family $\{P_i : i \in J\} \subseteq \tau_p$.

Then τ_p is called Bipolar Pythagorean Fuzzy Topology(BPFT) on X and the pair (X, τ_p) is said to be Bipolar Pythagorean Fuzzy Topological space. Each member of τ_p is called Bipolar Pythagorean fuzzy open set (BPFOS). The complement of a Bipolar Pythagorean Fuzzy open set is called a Bipolar Pythagorean fuzzy Closed set (BPFCS).

Definition 2.3: Let (X, τ_p) be a BPFTS and $P=\{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ be a BPFS over X.

Then the Bipolar Pythagorean Fuzzy Interior, Bipolar Pythagorean Fuzzy Closure of P are defined by:

(i) BPF int(P) = $\cup\{G\ /G\ is\ a\ BPFOS\ in\ (X, \tau_p)\ and\ G\subseteq P\}$.

(ii) BPF cl(P) = $\cap\{K\ /K\ is\ a\ BPFCS\ in\ (X, \tau_p)\ and\ P\subseteq K\}$.

It is clear that

a. BPF int(P) is the biggest Bipolar Pythagorean Fuzzy Open set contained in P.

b. BPF cl(P) is the smallest Bipolar Pythagorean Fuzzy Closed set containing P.

Definition 2.4: If BPFS $A = \{\langle x, \mu_A^+(x), \nu_A^+(x), \mu_A^-(x), \nu_A^-(x)\rangle : x \in X\}$ in a BPTS (X, τ_p) is said to be

(a) Bipolar Pythagorean Fuzzy Semi closed set (BPFSCS) if $int(cl(A)) \subseteq A$

(b) Bipolar Pythagorean Fuzzy Pre-closed set(BPFPCS) if $cl(int(A)) \subseteq A$

(c) Bipolar Pythagorean Fuzzy α closed set (BPFαCS) if $cl(int(cl(A)) \subseteq A$

(d) Bipolar Pythagorean Fuzzy γ closed set (BPFγCS) if $A \subseteq int(cl(A)) \cup cl(int(A))$

(e) Bipolar Pythagorean Fuzzy regular closed set (BPFRCS) if $A = cl(int(A))$

(f) If BPF set A of a BPFTS (X, τ_p) is a Bipolar Pythagorean Fuzzy Regular Generalized closed set(BPFRGCS), if $cl(A) \subseteq U$whenever $A \subseteq U$ and U is BPFROS in X.

Definition 2.5: A Bipolar Pythagorean Fuzzy Set A of a Bipolar Pythagorean Fuzzy Topological Space (X, τ_p) is called Bipolar Pythagorean Fuzzy Regular α Generalized closed set (BPFRαGCS in short), if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is BPF regular open set in X.

Definition 2.6: A function $\phi : (X, \tau_p) \to (Y, \sigma_p)$ is called BPFRαG continuous mapping if the inverse image of every BPF closed set in Y is BPFRαG closed set in X.

Definition 2.7: A mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ is said to be

(i) BPF semi continuous mapping if $\phi^{-1}(A) \in BPFSO(X)$ for every $A \in (Y, \sigma_p)$.

(ii) BPFα continuous mapping if $\phi^{-1}(A) \in BPF\alpha O(X)$ for every $A \in (Y, \sigma_p)$.

(iii) BPF Pre continuous mapping if $\phi^{-1}(A) \in BPFPO(X)$ for every $A \in (Y, \sigma_p)$.

(iv) BPFγ continuous mapping if $\phi^{-1}(A) \in BPF\gamma O(X)$ for every $A \in (Y, \sigma_p)$.

3. BIPOLAR PYTHAGOREAN FUZZY ALMOST REGULAR α GENERALIZED CONTINUOUS MAPPINGS

In this section we introduced Almost Regular α Generalized continuous mappings in Bipolar Pythagorean Fuzzy Topological Spaces and investigated some of its properties.

Definition 3.1: A mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ is said to be a Bipolar Pythagorean Fuzzy Almost Regular α Generalized Continuous (BPFaRαG continuous in short) mapping if $\phi^{-1}(\omega)$ is a BPFRαGCS in X for every BPFRCS ω in Y.

Example 3.2: Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where T_1= (x, (0.6, 0.5), (0.2, 0.2), (-0.6, -0.7), (-0.2, -0.1)), T_2 = (x, (0.3, 0.2), (0.7, 0.5), (-0.3, -0.2), (-0.7, -0.5)), T_3 = (y, (0.4, 0.4), (0.2, 0.2), (-0.4, -0.4), (-0.2, -0.1)) and T_4 = (y, (0.3, 0.2), (0.6, 0.6), (-0.3, -0.2), (-0.6, -0.6)). Define a mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here T_4^c = (y, (0.6, 0.6), (0.3, 0.2), (-0.6, -0.6), (-0.3, -0.2)) is BPFRCS in Y and $\phi^{-1}(T_4^c)$ = (x, (0.6, 0.6), (0.3, 0.2), (-0.6, -0.6), (-0.3, -0.2)) is BPFS in X. Then $\alpha cl(\phi^{-1}(T_4^c)) = T_2^c \subseteq 1_p$ as $\phi^{-1}(T_4^c) \subseteq 1_p$. Therefore, $\phi^{-1}(T_4^c)$ is BPFRαGCS in X. Thus ϕ is BPFaRαG continuous mapping in X.

Proposition 3.3: Every BPF continuous mapping and BPFα continuous mapping are BPFaRαG continuous mapping but not conversely.

Example 3.4: From Example 4.2, ϕ is BPFaRαG continuous mapping but not BPF continuous mapping, as $cl(\phi^{-1}(T_4^c)) = T_2^c \neq \phi^{-1}(T_4^c)$.

Example 3.5: From Example 4.2, ϕ is BPFaRαG continuous mapping but not BPF continuous mapping, as $cl(int(cl(\phi^{-1}(T_4^c))) = T_2^c \nsubseteq \phi^{-1}(T_4^c)$.

Proposition 3.6: Every BPFR continuous mapping is a BPFaRαG continuous mapping but not conversely.

Example 3.7: From Example 4.2, ϕ is BPFaRαG continuous mapping but not BPFR continuous mapping, as $int(cl(\phi^{-1}(T_4^c))) = T_2^c \neq \phi^{-1}(T_4^c)$.

Proposition 3.8: Every BPFG continuous mapping and BPFαG continuous mapping are BPFaRαG continuous mapping but not conversely.

Example 3.9: From Example 4.2, ϕ is BPFaRαG continuous mapping but not BPFG continuous mapping, as $cl(\phi^{-1}(T_4^c)) = T_2^c \nsubseteq U$.

Example 3.10: From Example 4.2, ϕ is BPFaRαG continuous mapping but not BPFαG continuous mapping, as $cl(int(cl(\phi^{-1}(T_4^c))) = T_2^c \nsubseteq U$.

Remark: 3.11: Every BPFP continuous mapping and BPFaRαG continuous mapping are independent to each other.

Example 3.12: Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where T_1= (x, (0.6, 0.5), (0.6, 0.4), (-0.6, -0.4), (-0.6, -0.3)), T_2 = (x, (0.2, 0.3), (0.7, 0.6), (-0.4, -0.3), (-0.6, -0.6)), T_3 = (y, (0.6, 0.6), (0.3, 0.3), (-0.6, -0.5), (-0.3, -0.2)) and T_4 = (y, (0.1, 0.1), (0.9, 0.9), (-0.1, -0.1), (-0.9, -0.9)). Define a mapping ϕ : $(X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here T_3^c = (y, (0.3, 0.3), (0.6, 0.6), (-0.3, -0.2), (-0.6, -0.5)) is BPFRCS in Y and T_1, T_2 are BPFROS in X. Now, $\phi^{-1}(T_3^c)$ = (x, (0.3, 0.3), (0.6, 0.6), (-0.3, -0.2), (-0.6, -0.5)) is a BPFPCS in X but $\phi^{-1}(T_3^c)$ is not a BPFRαGCS, as

$\overline{\alpha cl}(\phi^{-1}(T_3^c)) = T_1^c \nsubseteq T_1$. Therefore, ϕ is BPFP continuous mapping but not a BPFaRαG continuous mapping in X.

Example 3.13: Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where T_1= (x, (0.7, 0.5), (0.1, 0.1), (-0.7, -0.4), (-0.1, -0.1)), T_2 = (x, (0.1, 0.2), (0.5, 0.5), (-0.1, -0.2), (-0.4, -0.4)), T_3 = (y, (0.6, 0.7), (0.2, 0.1), (-0.6, -0.5), (-0.1, -0.2)) and T_4 = (y, (0.1, 0.1), (0.8, 0.8), (-0.1, -0.1), (-0.8, -0.7)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here T_4^c = (y, (0.8, 0.8), (0.1, 0.1), (-0.8, -0.7), (-0.1, -0.1)) is BPFRCS in Y and T_2 is BPFROS in X. Now, $\phi^{-1}(T_4^c)$ = (x, (0.8, 0.8), (0.1, 0.1), (-0.8, -0.7), (-0.1, -0.1)) is a BPFRαGCS in X but $\phi^{-1}(T_4^c)$ is not a BPFPCS, as $cl(int((\phi^{-1}(T_4^c))) = 1_p \nsubseteq \phi^{-1}(T_4^c)$. Therefore, ϕ is a BPFaRαG continuous mapping but not a BPFP continuous mapping in X.

Remark 3.14: Every BPFS continuous mapping and BPFaRαG continuous mapping are independent to each other.

Example 3.15: Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where T_1= (x, (0.5, 0.5), (0.3, 0.2), (-0.5, -0.4), (-0.3, -0.1)), T_2 = (x, (0.1, 0.2), (0.7, 0.6), (-0.1, -0.2), (-0.6, -0.6)), T_3 = (y, (0.5, 0.5), (0.3, 0.1), (-0.5, -0.4), (-0.3, -0.1)) and T_4 = (y, (0.3, 0.3), (0.8, 0.8), (-0.3, -0.2), (-0.5, -0.6)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here T_4^c = (y, (0.8, 0.8), (0.3, 0.3), (-0.5, -0.6), (-0.3, -0.2)) is BPFRCS in Y and T_1,T_2 are BPFROS in X. Now $\phi^{-1}(T_4^c)$ = (x, (0.8, 0.8), (0.3, 0.3), (-0.5, -0.6), (-0.3, -0.2)) is a BPFRαGCS in X but $\phi^{-1}(T_4^c)$ is not BPFSCS, as $int(cl((\phi^{-1}(T_4^c))) = 1_p \nsubseteq \phi^{-1}(T_4^c)$. Therefore, ϕ is BPFaRαG continuous mapping but not a BPFS continuous mapping in X.

Example 3.16: Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, whereT_1= (x, (0.6, 0.7), (0.2, 0.2), (-0.6, -0.5), (-0.3, -0.2)),T_2 = (x, (0.2, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.6, -0.5)), T_3 = (y, (0.2, 0.2), (0.6, 0.7), (-0.4, -0.3), (-0.6, -0.5)) and T_4 = (y, (0.5, 0.5), (0.4, 0.4), (-0.5, -0.5), (-0.5, -0.4)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here T_3^c = (y, (0.6, 0.7), (0.2, 0.2), (-0.6, -0.5), (-0.4, -0.3)) is BPFRCS in Y and T_1, T_2 are BPFROS in X. Now $\phi^{-1}(T_3^c)$ = (x, (0.6, 0.7), (0.2, 0.2), (-0.6, -0.5), (-0.4, -0.3)) is a BPFSCS in X but $\phi^{-1}(T_3^c)$ is not a BPFRαGCS, since $\alpha cl(\phi^{-1}(T_3^c))) = T_2^c \nsubseteq T_1$ as $\phi^{-1}(T_3^c) \subseteq T_1$. Therefore, ϕ is a BPFS continuous mapping but not a BPFaRαG continuous mapping in X.

Theorem 3.17: A mapping $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a BPFaRαG continuous mapping if and only if the inverse image of each BPFROS in Y is a BPFRαGOS in X.

Proof: Necessity: Let ω be a BPFROS in Y. This implies ω^c is a BPFRCS in Y. Since ϕ is a BPFaRαG continuous mapping, $\phi^{-1}(\omega^c)$ is a BPFRαGCS in X. Since $\phi^{-1}(\omega^c) = (\phi^{-1}(\omega))^c$, $\phi^{-1}(\omega)$ is a BPFRαGOS in X.

Sufficiency: Let ω be a BPFRCS in Y. This implies ω^c is a BPFROS in Y. By hypothesis, $\phi^{-1}(\omega^c)$ is a BPFRαGOS in X. Since $\phi^{-1}(\omega^c) = (\phi^{-1}(\omega))^c$, $\phi^{-1}(\omega)$ is a BPFRαGCS in X. Thus ϕ is a BPFaRαG continuous mapping.

Theorem 3.18: Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping where $\phi^{-1}(\delta)$ is a BPFRCS in X for every BPFCS in Y. Then ϕ is a BPFaRαG continuous mapping but not conversely.

Proof: Let δ be a BPFRCS in Y. Since every BPFRCS is a BPFCS, δ is a BPFCS in Y. Then $\phi^{-1}(\delta)$ is a BPFRCS in X. Since every BPFRCS is a BPFRαGCS, $\phi^{-1}(\delta)$ is a BPFRαGCS in X. Hence ϕ is a BPFaRαG continuous mapping.

Example 3.19: Let X={a,b} and Y={u,v}. Then $\tau_p = \{0_p, T_1, T_2, 1_p\}$ and $\sigma_p = \{0_p, T_3, T_4, 1_p\}$ are BPFTs on X and Y respectively, where T_1= (x, (0.7, 0.7), (0.3, 0.2), (-0.7, -0.5), (-0.3, -0.2)),T_2 = (x, (0.2, 0.2), (0.8, 0.8), (-0.2, -0.2), (-0.7, -0.6)), T_3 = (y, (0.7, 0.7), (0.3, 0.2), (-0.7, -0.6), (-0.2, -0.2)) and T_4 = (y, (0.2, 0.2), (0.9, 0.9), (-0.1, -0.1), (-0.8, -0.7)). Define a mapping $\phi : (X, \tau_p) \to (Y, \sigma_p)$ by $\phi(a) = u$ and $\phi(b) = v$. Here T_3^c = (y, (0.3, 0.2), (0.7, 0.7), (-0.2, -0.1), (-0.7, -0.6)) is BPFRCS in Y and $\phi^{-1}(T_3^c)$ = (x, (0.3, 0.2), (0.7, 0.7), (-0.2, -0.1), (-0.7, -0.6)) is a BPFS in X. Now $\phi^{-1}(T_3^c) \subseteq T_1$ where T_1 is BPFROS in X and $\alpha cl(\phi^{-1}(T_3^c)) = T_1^c \subseteq T_1$. Therefore $\phi^{-1}(T_3^c)$ is a BPFRαGCS in X but not a BPFRCS in X, since T_3^c is BPFCS in Y but $cl(int(\phi^{-1}(T_3^c))) = T_1^c$. Thus ϕ is a BPFaRαG continuous mapping but not the mapping in Theorem 4.14.

Theorem 3.20: Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping. If $\phi^{-1}(\alpha int(\delta)) \subseteq \alpha int(\phi^{-1}(\delta))$ for every BPFS δ in Y, then ϕ is a BPFaRαG continuous mapping.

Proof: Let δ be a BPFROS in Y. By hypothesis, $\phi^{-1}(\alpha int(\delta)) \subseteq \alpha int(\phi^{-1}(\delta))$. Since δ is a BPFROS, it is a BPFαOS in Y. Therefore $\alpha\ int(\delta) = \delta$. Hence $\phi^{-1}(\delta) = \phi^{-1}(\alpha int(\delta)) \subseteq \alpha int(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\delta)$. Therefore $\phi^{-1}(\delta) = \alpha int(\phi^{-1}(\delta))$. This implies $\phi^{-1}(\delta)$ is a BPFαOS in X and hence $\phi^{-1}(\delta)$ is a BPFRαGOS in X. Thus ϕ is a BPFaRαG continuous mapping.

Remark 3.21: The converse of the above theorem 3.21 is true if δ is a BPFROS in Y and X is a BPFR$\alpha T_{1/2}$ space.

Theorem 3.22: Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping. If $\alpha cl(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\alpha cl(\delta))$ for every BPFS δ in Y, then ϕ is a BPFaRαG continuous mapping.

Proof: Let δ be a BPFRCS in Y. By hypothesis, $\alpha cl(\phi^{-1}(\delta)) \subseteq \phi^{-1}(\alpha cl(\delta))$. Since δ is a BPFRCS, it is a BPFαCS in Y. Therefore $\alpha\ cl(\delta) = \delta$. Hence $\phi^{-1}(\delta) = \phi^{-1}(\alpha cl(\delta)) \supseteq \alpha cl(\phi^{-1}(\delta)) \supseteq \phi^{-1}(\delta)$. Therefore $\phi^{-1}(\delta) = \alpha cl(\phi^{-1}(\delta))$. This implies $\phi^{-1}(\delta)$ is a BPFαCS in X and hence $\phi^{-1}(\delta)$ is a BPFRαGCS in X. Thus ϕ is a BPFaRαG continuous mapping.

Remark 3.23: The converse of the above theorem 3.23 is true if δ is a BPFRCS in Y and X is a BPFR$\alpha T_{1/2}$ space.

Theorem 3.24: Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping where X is a BPFR$\alpha T_{1/2}$ space. If ϕ is a BPFaRαG continuous mapping, then $cl(int(cl(\phi^{-1}(\delta)))) \subseteq \phi^{-1}(\alpha cl(\delta))$ for every BPFRCS δ in Y.

Proof: Let δ be a BPFRCS in Y. By hypothesis, $\phi^{-1}(\delta)$ is a BPFRαGCS in X. Since X is a BPFR$\alpha T_{1/2}$ space, $\phi^{-1}(\delta)$ is a BPFαCS in X. This implies $\alpha cl(\phi^{-1}(\delta)) = \phi^{-1}(\delta)$. Now $cl(int(cl(\phi^{-1}(\delta)))) \subseteq \phi^{-1}(\delta) \cup cl(int(cl(\phi^{-1}(\delta)))) \subseteq \alpha cl(\phi^{-1}(\delta)) = \phi^{-1}(\delta) = \phi^{-1}(\alpha cl(\delta))$, as every BPFRCS is a BPFαCS. Hence $cl(int(cl(\phi^{-1}(\delta)))) \subseteq \phi^{-1}(\alpha cl(\delta))$.

Theorem 3.25: Let $\phi: (X, \tau_p) \to (Y, \sigma_p)$ be a mapping where X is a BPFR$\alpha T_{1/2}$ space. If ϕ is a BPFaRαG continuous mapping, then $\phi^{-1}(\alpha int(\delta)) \subseteq int(cl(int(\phi^{-1}(\delta))))$ for every BPFROS B in Y.

Proof: This theorem can be easily proved by taking complement in Theorem 3.25.

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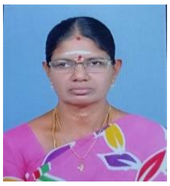

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