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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

International Conference on Emerging Trends in Science and Technology (ETIST 2021)

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Nano generalized α** closed sets in Nano Topological Spaces

Kalarani.M[1], Nithyakala.R[2], Santhi.R[3]

ABSTRACT: The objective of this paper is to introduce a new class of set called Nano α**- set (briefly Nα**- set) and a new closed set Nano generalized α**-closed set (Ngα**-closed set) in Nano topological spaces. The relation between Ngα**-closed set with other closed sets are discussed. Further the interior and closure of the closed set is defined and studied its properties.

Keywords: Nα**- set, Ngα**-closed set, Ngα**-interior, Ngα**-closure.

1. INTRODUCTION

Levine [6] introduced the concepts of generalized closed sets in Topological spaces. Lellis Thivagar [5] introduced nano topological space and also defined nano closed sets, nano interior and nano closure of a set. Bhuvaneswari [2] introduced the generalized closed sets in nano topological spaces. The present paper aims to introduce a new set nano α**-set and a new class of closed set called nano generalized α**- closed set in nano topological spaces is defined and investigate its relationship with other nano closed sets. In addition, the interior, closure of nano generalized α**-closed are defined and studied its properties.

2. PRELIMINARIES

Definition 2.1: [8] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. The pair (U,R) is said to be approximation space. Let X ⊆ U. Then

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$.

$L_R(X)=\bigcup_{x\in U}\{R(x): R(x)\subseteq X\}$, where R(x) denotes the equivalence class determined by $L_R(X)$.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$.

$U_R(X) = \bigcup_{x\in U} \{R(x): R(x) \cap X \neq \varphi\}$

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(iii) The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$ and $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2: [5] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{\varphi, U, L_R(X), U_R(X), B_R(X)\}$ where X ⊆ U. Then $\tau_R(X)$ satisfies the following axioms.

(i) *U and* $\varphi \in \tau_R(X)$.

(ii) The union of all the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Here $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X and (U, $\tau_R(X)$) as a nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. The complement of the nano open set is called nano closed sets.

Definition 2.3: [5] If (U, $\tau_R(X)$) is a nano topological space with respect to X where X ⊆ U and if A ⊆ U then (i) The nano interior of A is defined as the union of all nano open subsets contained in A and is denotedby $Nint(A)$. That is $Nint(A)$) is the largest nano open subset of A. (ii) The nano closure of A is defined as the intersection of all nano closed sets containing A and isdenoted by $Ncl(A)$. That is $Ncl(A)$is the smallest nano closed set containing A.

Definition 2.4: Let (U, $\tau_R(X)$) be a nano topological space. Then A ⊆ U is said to be a

(i) nano preopen set [5] if $A \subseteq Nint(Ncl(A))$

(ii) nano semi open set [5] if $A \subseteq Ncl(Nint(A))$

(iii) nano α-open set [5] if $A \subseteq Nint(Ncl(Nint(A)))$

(iv) nano regular open set [5] if $A = Nint(Ncl(A))$

(v) nano β-open set if $A \subseteq Ncl(Nint(Ncl(A)))$

The complements of the above sets are called as their respective closed sets.

Definition 2.5: Let (U, $\tau_R(X)$) be a nano topological space and $A \subseteq G$. Then A is called

(i) nano generalized closed (Ng-closed) set [2] if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open set in U.

(ii) nano generalized star closed (Ng*-closed) set [2] if $Ncl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano gopen set in U.

(iii) nano α- generalized closed (Nαg-closed) set [4] if $N\alpha cl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open set in U.

(iv) nano generalized α closed (Ngα-closed) set [4] if $N\alpha cl(A) \subseteq G$whenever $A \subseteq G$ and G is nano α open set in U.

(v) nano regular generalized closed (Nrg-closed) set [6] if $Nrcl(A) \subseteq G$whenever $A \subseteq G$ and G is nano regular open set in U.

(vi) nano semi generalized closed (Nsg-closed) set [3] if $Nscl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano semi open set in U.

(vii) nano generalized semi closed (Ngs-closed) set [3] if $Nscl(A) \subseteq G$ whenever $A \subseteq G$ and G is nano open set in U.

(viii) nano c-set if A=G⋂F where G is nano open and F is nano α*- set.

(ix) nano α^*- set if $Nint(A) = Nint\ (Ncl(Nint(A)))$

(x) nano t-set (Nt-set) [3] if $Nint$(A)= $Nint\ (Ncl(A))$.

(xi) nano c*-set if A=G⋂F where G is nano g-open and F is nano α*- set.

(xii) nano c(s)-set if A=G⋂F where G is nano g-open and F is nano t-set.

3. NANO α**- SETS IN NANO TOPOLOGICAL SPACES

In this section, a new set Nano α** set is defined and studied its relations with other existing sets.

Definition.3.1: A subset A of a nano topological space (U, $\tau_R\ (X)$) is called a nano α**-set (Nα**-set) if $Nint(Ncl\ (A)) = Nint\ (Ncl(Nint(A)))$.

Example 3.2: Let $U = \{a, b, c, d\}$ with $U/_R = \{\{a\},\{c\},\{b,d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X)$={φ,U,$\{a\}$,$\{b,d\}$, $\{a,b,d\}$} is a nano topology with respect to X and the complement $\tau_R{}^c(X)$={φ,U,$\{c\}$,$\{a,c\}$,$\{b,c,d\}$}. Some of the nano sets for the topology are as follows.

(1) nano pre-open= {φ, U,$\{a\}$,$\{b\}$,$\{d\}$,$\{a,b\}$,$\{a,d\}$, $\{b,d\}$,$\{a,b,c\}$,$\{a,b,d\}$,$\{a,c,d\}$}

(2) nano semi-open = {φ,U,$\{a\}$,$\{a,c\}$,$\{b,d\}$,$\{a,b,d\}$,$\{b,c,d\}$}

(3) nano α-open = {φ,U,$\{a\}$,$\{b,d\}$,$\{a,b,d\}$}

(4) nano regular open = {φ,U,$\{a\}$,$\{b,d\}$}

(5) nano β-open= {φ,U,$\{a\}$,$\{b\}$,$\{d\}$,$\{a,b\}$,$\{a,c\}$,$\{a,d\}$,$\{b,c\}$,$\{b,d\}$,$\{c,d\}$, $\{a,b,c\}$,$\{a,b,d\}$,$\{a,c,d\}$,$\{b,c,d\}$}

(6) nano t-set={φ,U,$\{a\}$,$\{c\}$,$\{a,c\}$,$\{b,d\}$,$\{b,c,d\}$}

(7) nano c-set ={φ,U,$\{a\}$,$\{b\}$,$\{c\}$,$\{d\}$,$\{a,b\}$,$\{a,c\}$,$\{a,d\}$,$\{b,c\}$,$\{b,d\}$,$\{c,d\}$, $\{a,b,c\}$,$\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$}

(8) nano c*-set ={φ,U,$\{a\}$,$\{b\}$,$\{c\}$,$\{d\}$,$\{a,b\}$,$\{a,c\}$,$\{a,d\}$,$\{b,c\}$,$\{b,d\}$,$\{c,d\}$, $\{a,b,c\}$,$\{a,b,d\}$, $\{a,c,d\}$, $\{b,c,d\}$}

(9) nano c(s)-set={φ,U,$\{a\}$,$\{b\}$,$\{c\}$,$\{d\}$,$\{a,b\}$,$\{a,c\}$,$\{b,d\}$,$\{a,b,d\}$,$\{b,c,d\}$}

(10) Nα*-set = {φ,U,$\{a\}$,$\{b\}$,$\{c\}$,$\{d\}$,$\{a,b\}$,$\{a,c\}$,$\{a,d\}$, $\{b,c\}$,$\{b,d\}$,$\{c,d\}$, $\{a,b,c\}$,$\{a,c,d\}$,$\{b,c,d\}$}

(11) Nα**-closed set = {φ, U, $\{a\}$,$\{c\}$,$\{a,c\}$,$\{b, d\}$,$\{a,b, d\}$,$\{b,c,d\}$}

Theorem 3.3: The union of any two Nα**-set is Nα**-set.

Theorem 3.4: The intersection of any two Nα**-set is also a Nα**-set.

Remark 3.5: From the above example, the following implications are obtained.

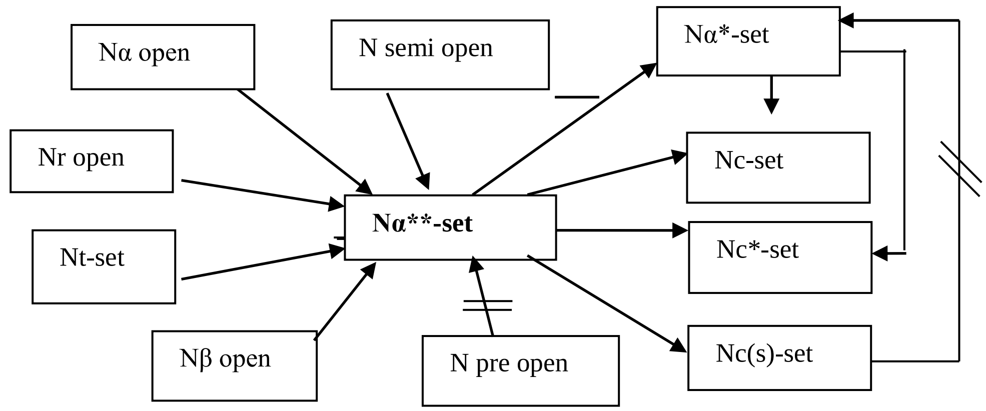

Figure.3.1

Remark.3.6: The sets nano β open and nano preopen sets are independent with Nα** set.

Theorem.3.7: Let (U, τ_R (X)) be a nano topological space, then every Nα** set is not a Nα* set.

Proof: The proof of the theorem follows in the above example.

4. NANO GENERALIZED α**- CLOSED SETS IN NANO TOPOLOGICAL SPACES

In this section, a new closed set nano generalized α**- closed (Ngα**-closed) set is defined and its relation with other nano closed sets are discussed.

Definition 4.1: A subset A of a nano topological space (U, $\tau_R(X)$) is called a nano generalized α**- closed set (Ngα**- closed set) if Ncl(A) $\subseteq G$ whenever A $\subseteq G$ and G is nano α**-set.

Example 4.2: Let $U = \{a, b, c, d\}$ with $U/_R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b,d\}$. Then τ_R (X)={φ, U, $\{b\}$,$\{c,d\}$, $\{b,c,d\}$} is a nano topology with respect to X and the complement is $\tau_R{}^c$ (X) = {φ, U,$\{a\}$, $\{a,b\}$,$\{a,c,d\}$}. The following are some nano closed sets for this nano topology.

(1) nano pre closed= {φ ,U,{a},{c},{d},{a,b},{a,c},{a,d}, {a,b,c},{a, b,d},{a,c,d}}

(2) nano semi-closed = {φ ,U,{a},{b},{a,b},{c,d}, {a,c,d}}

(3) nano α-closed = {φ ,U,{a},{a,b},{a,c,d}}

(4) nano regular closed = {φ ,U,{a,b},{a,c,d}}.

(5) nano β-closed= {φ ,U,{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d}, {a,b,c},{a,b,d},{a,c,d}}

(6) Ng-closed = {φ,U,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

(7) Ng*-closed ={φ,U,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

(8) Nrg-closed={φ ,U,{a},{a,b},{a,c},{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}}

(9) Nsg-closed={φ ,U,{a},{b},{c},{d},{a,b},{a,c},{a,d}, {c,d},{a,b,c},{a,b,d},{a,c,d}}

(10) Ngs-closed={φ ,U,{a},{b},{c},{d},{a,b},{a,c},{a,d}, {c,d},{a,b,c},{a,b,d},{a,c,d}}

(11) Ngα-closed = {φ ,U,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

(12) Nαg-closed = {φ, U,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}

(13) Ngα**-set = {φ, U,{a},{a,b},{a,c},{a, d},{a,b,c}{a, b,d},{a,c,d}}

Theorem 4.3: The union of any two Ngα**-closed set is Ngα**-closed.

Theorem 4.4: The intersection of any two Ngα**-closed set is also a Ngα**-closed.

Remark 4.5: From the above example, the following implications are obtained.

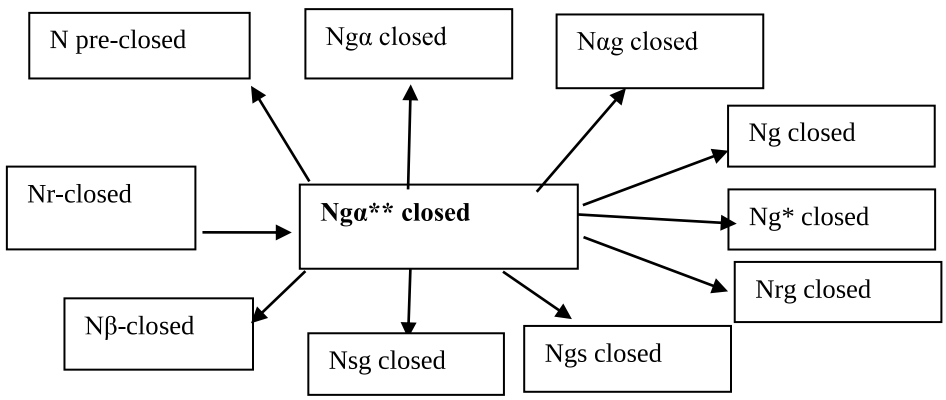

Figure 4.1

5. NANO gα**-INTERIOR AND NANO gα**-CLOSURE IN NANO TOPOLOGICAL SPACES

In this section, Nano gα**interior, Nano gα**closure is introduced and studied its characterizations.

Definition 5.1: Let (U, τ_R (X)) be a nano topological space and A ⊆G, then nano gα**- interior is defined as Ngα**int(A)=U{B/Bis a Ngα**-open, B⊂A}.

Definition 5.2: Let A be a subset of (U, τ_R (X)). A point x∈U is called Ngα**- interior point of A if A contains a Ngα**-open set containing x.

Definition 5.3: Let (U, τ_R (X)) be a nano topological space and A ⊆G, then nano gα**-closure is defined as Ngα**cl(A)=⋂{B/B is a Ngα**-closed, A⊂B}.

Theorem 5.4: A subset A of (U, τ_R (X)) is Ngα**-open if and only if Ngα**-int(A) = A.

Proof: Let A be Ngα**open in U then Ngα**-int(A) = A. Conversely let Ngα**-int(A) = A. Then by definition Ngα**-int (A) is a nano generalized α**open set, so A is a nano generalized α** open. Hence the proof.

Theorem 5.5: Let A and B be any two subsets of (U, τ_R(X)). Then

(i) Ngα**-int(U) = U and Ngα**-int(φ) = φ.

(ii) Ngα**-int(A)⊂A.

(iii) if B is any Ngα** open set contained in A, then B⊂Ngα**-int(A).

(iv) if A⊂B, then Ngα**-int(A)⊂Ngα**-int (B).

(v) Ngα**-int(Ngα**-int(A))=Ngα**-int (A).

Proof :

(i) Since U is a Ngα**open set, by definition Ngα**-int (U) = union of all Ngα** open sets that are contained in A = $U \cup$ {all Ngα** open sets} = U. That is Ngα**-int(U) = U. Here φ is the only Ngα** open set contained in φ, So Ngα**-int(φ) = φ.

(ii) Let $x \in$ Ngα**-int (A) $\Longrightarrow x$ is an interior point of A. $\Longrightarrow x \in A$. Therefore Ngα**-int(A) $\subset A$.

(iii) Let B be any Ngα** open set such that $B \subset A$. If $x \in B$, then x is an Ngα** interior point of A, since $B \subset A$. $\Longrightarrow x \in$ Ngα**-int (A). Hence $B \subset$ Ngα**-int(A).

(iv) Let A and B be subsets of U with $A \subset B$. Let $x \in$ Ngα**-int (A). Then x is a Ngα**interior point of A and A is a Ngα** neighbourhood of x. Since $A \subset B$, B is also a Ngα** neighbourhood of x.
$\Longrightarrow x \in$ Ngα**-int (B). Hence Ngα**-int (A) $\subset$ Ngα**-int (B).

(v) Let A be any subset of U. By definition of Ngα**-interior, Ngα**-int(A) is Ngα** open and hence
Ngα**-int (Ngα**-int(A)) = Ngα**-int (A).

Theorem 5.6: If A and B are two subsets of $(U, \tau_R(X))$, then

(i) Ngα**-int(A)⋃Ngα**-int(B) ⊂ Ngα**-int($A \cup B$).

(ii) Ngα**-int(A)⋂Ngα**-int(B)=Ngα**-int($A \cap B$).

Theorem 5.7: If A is a subset of $(U, \tau_R(X))$, then N-int(A)⊂Nα-int(A)⊂ Ngα**-int(A).

Proof : Let A be a subset of U. Let $x \in$ N int (A) $\Longrightarrow x \in \cup\{B : B$ is Nano-open and $B \subset A\}$. $\Longrightarrow$ there exists a nano open set B such that $x \in B \subset A$. Every open set is a nano α open $\Longrightarrow$ there exists a Nα-open set B such that $x \in B \subset A$. Hence $x \in \cup\{B : B$ is Nano α-open and $B \subset A\}$. We know every nano α-open set is Ngα**-open in U.
$\Longrightarrow x \in \cup\{B$:$B$ is Ngα**-open and $B \subset A\}$. $\Longrightarrow x \in$ Ngα**-int(A). Thus N-int(A)⊂Nα-int(A)⊂ Ngα**-int(A).

Theorem 5.8: Let A and B are any two subsets of $(U, \tau_R(X))$. Then

(i) Ngα**-cl(U)=U and Ngα**-cl(φ)=φ.

(ii) A⊂Ngα**-cl(A).

(iii) if B is any Ngα**-closed set containing A, then Ngα**-cl(A)⊂B.

(iv) If $A \subset B$ then Ngα**-cl(A)⊂Ngα**-cl(B).

(v) Ngα**-cl(Ngα**-cl(A))=Ngα**-cl(A).

Theorem 5.9: If A is a Ngα**-closed subset of $(U, \tau_R (X))$, then Ngα**-cl(A) = A.

Theorem 5.10: If A and B are two subsets of $(U, \tau_R(X))$, then

(i) Ngα**-cl(A)⋃Ngα**-cl(B)=Ngα**-cl($A \cup B$).

(ii) Ngα**-cl(A)⋂Ngα**-cl(B)=Ngα**-cl($A \cap B$).

Theorem 5.11: If A is a subset of $(U, \tau_R(X))$, then

(i) [Ngα**-int(A)]^c=[Ngα**-cl(A^c)].

(ii) Ngα**-int(A)=[Ngα**-cl(A^c)]^c.

6. CONCLUSION

In conclusion, a new set nano α**-set is introduced, using this set nano generalized α**- closed (Ngα**- closed) set in Nano Topological Spaces is defined. Its relation with other closed sets are investigated. Moreover, the interior and closure of Ngα**- closed sets are defined and examined its properties.

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