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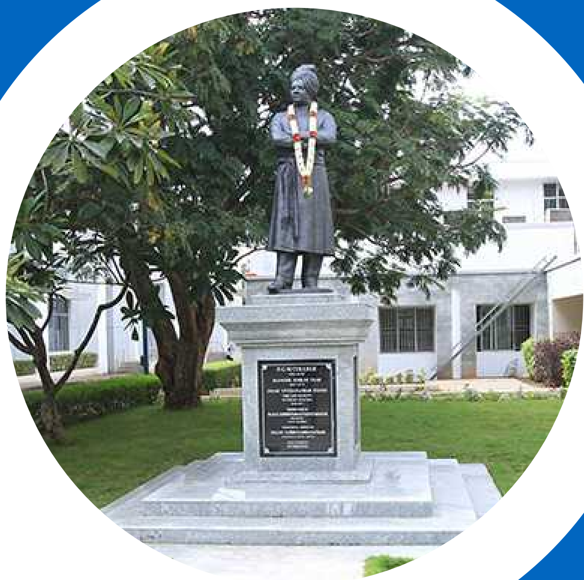

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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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Nonoscillatory properties of certain nonlinear difference equations with generalized difference

M. Raju*[1] – S. Kaleeswari[2] – N. Punitha[3]

ABSTRACT: A New criteria is obtained for an asymptotic behavior of fixed solutions of certain nonlinear delay difference equation with generalized difference of the type $\Delta_m^j\left(x_n + p_n x_{n-k} - q_n x_{n-l}\right) + f\left(x_{n-\tau}\right) = 0,\ \ n \in N(a),\ a \in N,\ k,\ l,\ m, \tau \in Z^+,$ j=1,2 and j$\in${3,5,7,9,…, r}, r is an odd positive integer.

Keywords: fixed point, nonoscillatory, difference equation, generalized difference.

1. INTRODUCTION & PRELIMINAIRES

1.1 Introduction

The basic theory of difference equations is based on the forward difference operator Δ defined by $\Delta^0 x_n = x_n$, $\Delta^j x_n = \sum_{s=0}^{j} (-1)^{j-s} \binom{j}{s} x_{n+s}$, $j \geq 1$, for $n = 0,1,2, \ldots$. Later the following definition was suggested for Δ_m by [1, 5, 3]:

$$\Delta_m x_n = x_{n+m} - x_n,\ n \in N,\ m \in R - \{0\}. \tag{1.1.1}$$

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Recently, equation (1.1.1) was reconsidered and its inverse was defined by Δ_m^{-1} [6]. By extending the sequences of complex numbers and m to be real, some new qualitative properties were studied for the solutions of difference equations involving Δ_m.

In the year 2011, N. Parhi[13] has studied the oscillatory and nonoscillatory solutions of the following second order difference equations, involving generalized difference operator,

$$\Delta_a(p(n-1)\Delta_a(y(n-1))+q(n)y(n)=0,\ n\geq 1 \tag{1.1.2}$$

and $$\Delta_a(p(n-1)\Delta_a(y(n-1))+q(n)y(n)=f(n),\ n\geq 1, \tag{1.1.3}$$

where Δ_a is defined by $\Delta_a y(n)=y(n+1)-ay(n),\ a\neq 0.$ Also he obtained necessary and sufficient conditions for the first equation (1.1.2).

In the year 2012, M. Maria Susai Manuel et al.[8] have studied the following second order generalized difference equation:

$$\Delta_l^2(u(k))+f(k,u(k))=0,\ k\in[a,\infty),\ a>0,\ l\in(0,\infty), \tag{1.1.4}$$

and also proved the condition for nonexistence of non-trivial $l_{2(l)}$ and $c_{0(l)}$ solutions of equation (1.1.4). Further they presented some formulae and examples to find the values of finite and infinite series in number theory as application of Δ_l. Further, in the year 2013, M. Maria Susai Manuel et al.[9] have studied the same generalized difference equation (1.1.4), and extended the applications of Δ_l in number theory.

Motivated by the papers [13], [8] and [9], in this article, we have studied the higher order nonlinear generalized difference equation with delay terms of the form:

$$\Delta_m^j(x_n+p_nx_{n-k}-q_nx_{n-l})+f(x_{n-\tau})=0, \tag{1.1.5}$$

where $n\in N(a),\ a\in N,\ \ k,l,m,\tau\in Z^+$, j=1,2 and $j\in\{3,5,7,9,\ldots,r\}$, r is an odd positive integer, Δ_m is the generalized forward difference operator defined by $\Delta_m^0 x_n=x_n$,

$$\Delta_m^j x_n=\sum_{s=0}^{j}(-1)^{j-s}\binom{j}{s}x_{n+ms},\ \ 1\leq j\leq n,\ 1\leq m\leq n\ ,\ j,m\in Z^+.$$

Further, it is assumed that

(C1) $p_n>0,\ q_n=0$,

(C2) $p_n=0,\ 0<q_n\leq 1$,

(C3) $p_n\geq 0,\ q_n>1$,

(C4) $p_n>0,\ 0<q_n\leq 1$,

(C5) $f:R-\{0\}\to R^+$ is continuously differentiable in its domain for $u\in R-\{0\}$.

Our objective here is to proceed further in this direction to obtain the asymptotic stability of fixed points of equation (1.1.5) which include and generalize some earlier results cited there in references. For applications of difference equations one can refer [1, 2, 3, 5, 7, 9, 10, 11, 12, 16, 17, 26].

1.2 Preliminaries

Difference equations usually describe the evolution of some certain phenomena over time and are also important in describing dynamics for fundamentally discrete system. The population dynamics have discrete generation; the size of the *(n+1)^th^* generation *x(n+1)* is a function of the n^{th} generation *x(n)*[5]. This can be expressed as difference equation of the form

$$x(n+1) = g(x(n)) \tag{1.2.1}$$

The concept of difference equations with many examples in applications such as asymptotic behavior of solutions of difference equations were studied extensively by S. N. Elaydi[3]. Further, by a solution of difference equation (1.2.1), we mean a real sequence $\{x_n\}$, $n = 0,1,2,\ldots$, which satisfies the difference equation (1.2.1), for all $n \geq n_0$, $n_0 \geq 0$. A point x^* is said to be a fixed point of the difference equation (1.2.1) if $g(x^*) = x^*$. Let x be a point in the domain of g. If there exists a positive integer μ and a fixed point x^* of (1.2.1) such that $g^{\mu}(x) = x^*$, $g^{\mu-1}(x) \neq x^*$, then x is an eventually fixed point of equation (1.2.1). The fixed point x^* of equation (1.2.1) is stable if given $\varepsilon > 0$ there exists $\delta > 0$ such that $\left|x_0 - x^*\right| < \delta$ implies $\left|g^m(x_0) - x^*\right| < \varepsilon$, for all $m > 0$. If x^* is not stable, then it is called unstable. The fixed point x^* of equation (1.2.1) is asymptotically stable if it is stable and there exists $\eta > 0$ such that $\left|x_0 - x^*\right| < \eta$ implies $\lim_{n\to\infty} x_n = x^*$. If $\eta = \infty$, then the fixed point x^* is said to be globally asymptotically stable.

A fixed point is also referred to as a fixed solution or critical point or equilibrium point or stationary point or rest point or singular point or limit point)[1]. If x^* is a fixed point of equation (1.2.1) [or equation (1.1.5)], then obviously $\{x_n\} = \{x^*\}$ is a solution of equation (1.2.1) [or equation (1.1.5)]. Equation (1.1.5) is also referred as nonautonomous or time-variant whereas equation (1.2.1) is called autonomous or time-invariant [3]. Several authors have been studying time-variant systems in the area of dynamical systems, but in this paper we have studied the equation (1.1.5) in the discrete manner.

Throughout this paper we use the notations $N = \{0,1,2,\ldots\}$, $R^+ =$ set of all positive real numbers. $Z^+ =$ set of all positive integers, $R =$ set of all real numbers, $Z =$ set of all integers, $N(a) = \{a, a+1, a+2, \ldots\}$, where $a \in N$. .

2. MAIN RESULTS

In this section, we give some new criteria for asymptotic stability of fixed points (or fixed solution) of equation (1.1.5). The following definitions and theorems are main tools in this section. In this direction, we introduce the following definitions for asymptotic stability of fixed points (or fixed solution) of equation (1.1.5).

Definition 2.1 The generalized difference operator Δ_m^j is defined as $\Delta_m^0 x_n = x_n$; $\Delta_m^j x_n = \sum_{s=0}^{j} (-1)^{j-s} \binom{j}{s} x_{n+m}$, $j \ge 1$; for all $m \in Z^+$.

Definition 2.2 The generalized difference operator of j[th] kind is defined as $\Delta_m^j = \Delta_m \left(\Delta_m^{j-1}\right)$.

Definition 2.3 Suppose that sequence $\{x_n\}$ be a sequence as defined in Definition 2.1, then we define the following:

$$\sum_{s=n_0}^{n} \Delta_m x_s = \sum_{s=n_0}^{m} \left(x_{n+s} - x_s\right),\ \sum_{s=n_0}^{n} \Delta_m^2 x_s = \sum_{s=n_0}^{m} x_s - \sum_{s=n_0}^{m} \left(x_{m+s} + x_{n+s}\right) + \sum_{s=m+1}^{2m} x_{n+s}$$, for all $n \in N(n_0)$,

$$\sum_{s=n_0}^{n} \Delta_m^j x_s = (-1)^{j+1} \sum_{s=n_0}^{m} \left(x_{n+s} - x_s\right) + (-1)^{j+1} (j-1) \left(\sum_{s=n_0}^{m} x_{m+s} - \sum_{s=m+1}^{jm} x_{n+s} \right)$$, for all $n \in N(n_0)$,

$j=1,\ 2,\ 1 \le m \le n$.

$$\sum_{s=n_0}^{n} \Delta_m^j x_s = (-1)^{j+1} \sum_{s=n_0}^{m} \left(x_{n+s} - x_s\right) - x_j + (-1)^{j+1} (j-1) \left(\sum_{s=n_0}^{j-1} (-1)^{s+1} x_{m+s} - \sum_{s=m+1}^{(j-1)m} (-1)^{s+1} x_{n+s} \right) + x_{j+n}$$, for all $n \in N(n_0)$, $3 \le j \le n$, $1 \le m \le n$.

Theorem 2.1 If one of the conditions (C1), (C2), (C4) is satisfied along with the condition (C5), then the fixed solution $\{x^*\}$ of equation (1.1.5) is asymptotically stable.

Proof: Without loss of generality we can assume that $\{x_n\}$ be an eventually positive and nondecreasing solution of (1.1.5), then there exists an integer $a_1 \in N(a)$ such that $x_n > 0$, for $n \in N(a_1)$. It follows that both x_{n-k}, $x_{n-l} > 0$, for $n \in N(a_1)$, $a_1 \in N(a)$.

Let $z_n = x_n + p_n x_{n-k} - q_n x_{n-l}$, for $n \in N(a_1)$, $a_1 \in N(a)$. (2.1)

First, we consider the condition (C1). Then from equation (1.1.5) and from condition (C5) we have

$$\Delta_m^j z_n = -f\left(x_{n-\tau}\right) < 0 \ \ and \ \ z_n > 0, \text{ for } n \in N(a_1),\ a_1 \in N(a). \quad (2.2)$$

Next we shall show that

(i) $\{x_n\} = \{x^*\}$ is a fixed solution of equation (1.1.5),

(ii) $\lim_{n \to \infty} x_n = x^*$.

Case (i) To show that $\{x_n\} = \{x^*\}$ is a fixed solution of equation (1.1.5).

Since $\{x_n\}$ is eventually fixed solution of equation (1.1.5), we have there exists a positive integer $a_2 \in N(a_1)$ such that $f^{a_2}(x_{n-\tau}) = x^*$, $f^{a_2-1}(x_{n-\tau}) \neq x^*$, for $n \in N(a_2)$. This implies that $\{x_n\}$ is a periodic solution of equation (1.1.5). It remains to show that $f^{n_2-1}(x_{n-\tau}) = x^*$, for $n \in N(a_2)$, $a_2 \in N(a_1)$.

Suppose that $f^{a_2}(x_{n-\tau}) = x^*$, $f^{a_2-1}(x_{n-\tau}) \neq x^*$, for $n \in N(a_2)$, $a_2 \in N(a_1)$. Therefore, either $f^{a_2-1}(x_{n-\tau}) < x^*$ or $f^{a_2-1}(x_{n-\tau}) > x^*$, for all $n \in (a_2, \infty)$, $a_2 \in N(a_1)$. This implies that $\left| f^{a_2-1}(x_{n-\tau}) - x^* \right| < 0$, for $n \in N(a_2)$, $a_2 \in N(a_1)$, which is a contradiction since distance between two points is not negative. Now Consider $f^{a_2-1}(x_{n-\tau}) > x^*$, for $n \in N(a_2)$, $a_2 \in N(a_1)$ $\Rightarrow f^{a_2-1}(x_{n-\tau}) > f^{a_2}(x_{n-\tau})$, for $n \in N(a_2)$, $a_2 \in N(a_1)$. This shows that there exists a sufficiently small real number $\alpha > 0$ such that $x_{n-\alpha} > x_n$, for $n \in N(a_2)$, $a_2 \in N(a_1)$, which is a contradiction to our assumption. Thus from both the cases we conclude that $f^{a_2-1}(x_{n-\tau}) = x^*$, for all $x_{n-\alpha} > x_n$. Hence we proved that $\{x_n\} = \{x^*\}$ is a fixed solution of equation (1.1.5).

Case (ii) To show that $\lim\limits_{n\to\infty} x_n = x^*$.

Suppose that $\lim\limits_{n\to\infty} x_n \neq x^*$. Then there exists an infinite subsequence $\{n^{(i)}\} \subseteq \{n\}$ such that $\left| x_{n^{(i)}} - x^* \right| \geq \varepsilon$. Therefore we can take a sequence of subsets $N(i) \subseteq N$ such that $n^{(i)} \in (i, \infty)$, $i \in N(i)$, for $i \in N$. So there exists $i_1 \in N(i)$ such that $n \in N(i)$, $i \in N$ and $x_n > \dfrac{\varepsilon}{2}$. Thus

$$f(x_{n-\tau}) > f\left(\frac{\varepsilon}{2}\right), \text{ for } n \in N(i),\ i \in N(i_1). \tag{2.3}$$

Therefore from inequalities (2.2) and (2.3), we have

$$\Delta_m^j z_n < -f\left(\frac{\varepsilon}{2}\right), \text{ for } n \in N(i),\ i \in N(i_1). \tag{2.4}$$

Summing the inequality (2.4) from a_2 to n, we obtain the following

$$(-1)^{j+1}\sum_{s=a_2}^{m}(z_{n+s} - z_s) + (-1)^{j+1}(j-1)\left(\sum_{s=a_2}^{m} z_{m+s} - \sum_{s=m+1}^{jm} z_{n+s}\right) < -(n-a_2)f\left(\frac{\varepsilon}{2}\right), \text{ for}$$

$n \in N(i)$, $i \in N(a_2)$, j=1,2, $1 \leq m \leq n$, and

$$(-1)^{j+1}\sum_{s=a_2}^{m}(z_{n+s}-z_s)-z_j+(-1)^{j+1}(j-1)\left(\sum_{s=a_2}^{j-1}(-1)^{s+1}z_{m+s}-\sum_{s=m+1}^{(j-1)m}(-1)^{s+1}z_{n+s}\right)+z_{j+n}<-(n-a_2)f\left(\frac{\varepsilon}{2}\right)$$

, for $n\in N(i),\ i\in N(a_2)$, $3\le j\le n,\ 1\le m\le n$.

Case (a) when j=1, we have

$$\sum_{s=a_2}^{m}(z_{n+s}-z_s)<-(n-a_2)f\left(\frac{\varepsilon}{2}\right),\text{ for } n\in N(i),\ i\in N(a_2),\ 1\le m\le n.$$

$$\Rightarrow \sum_{s=a_2}^{m}z_{n+s}<-(n-a_2)f\left(\frac{\varepsilon}{2}\right),\text{ for } n\in(i,\ \infty),\ i\in N(a_2),\ 1\le m\le n.$$

Case (b) when j=2, we have

$$\sum_{s=a_2}^{m}(z_s-z_{n+s})+\left(\sum_{s=m+1}^{2m}z_{n+s}-\sum_{s=a_2}^{m}z_{m+s}\right)<-(n-a_2)f\left(\frac{\varepsilon}{2}\right),\text{ for } n\in N(i),\ i\in N(a_2),\ 1\le m\le n.$$

$$\Rightarrow \sum_{s=a_2}^{m}z_s+\sum_{s=m+1}^{2m}z_{n+s}<-(n-a_2)f\left(\frac{\varepsilon}{2}\right),\text{ for } n\in N(i),\ i\in N(a_2),\ 1\le m\le n.$$

Case (c) when j is an odd positive integer, we have

$$\sum_{s=a_2}^{m}(z_{n+s}-z_s)-z_j+(j-1)\left(\sum_{s=a_2}^{j-1}(-1)^{s+1}z_{m+s}-\sum_{s=m+1}^{(j-1)m}(-1)^{s+1}z_{n+s}\right)+z_{j+n}<-(n-a_2)f\left(\frac{\varepsilon}{2}\right),\text{ for}$$

$n\in N(i),\ i\in N(a_2)$, $3\le j\le n,\ 1\le m\le n$.

$$\Rightarrow \sum_{s=a_2}^{m}(z_{n+s}-z_s)-z_j+(j-1)\left(\sum_{s=a_2}^{j-1}(-1)^{s+1}z_{m+s}-\sum_{s=m+1}^{(j-1)m}(-1)^{s+1}z_{n+s}\right)+z_{j+n}<-(n-a_2)f\left(\frac{\varepsilon}{2}\right)$$

From both the cases (a) and (b), we see that $z_n<0$, for $n\in N(i),\ i\in N(i_1),\ n\ge a_2$, where $a_2\in N(i_1)$, which is a contradiction to the fact that $z_n>0$, for $n\in N(a_2),\ a_2\in N(a_1)$. Hence we proved that $\lim_{n\to\infty}x_n=x^*$.

Second, we consider the condition (C2). Then from equation (2.1) and from condition (C2), we have

$$z_n=x_n-q_nx_{n-l},\text{ for } n\in N(a_1),\ a_1\in N(a). \tag{2.6}$$

It follows that there is an integer $a_2\in N(a_1)$ such that

$$\Delta_m^j z_n=-f(x_{n-\tau})<0,\text{ for } n\in N(a_2),\ a_2\in N(a_1),\ j\ge 1. \tag{2.7}$$

First we have to prove that $z_n>0$, for $n\in N(a_2),\ a_2\in N(a_1)$. Suppose that $z_n\le 0$, for $n\in N(a_2),\ a_2\in N(a_1)$. Then there exists a positive integer $a_3\in N(a_2)$ and a real number $K>0$ such that $z_n\le -K$, for $n\in N(a_3),\ a_3\in N(a_2)$. Therefore form equation (2.6), we obtain

$x_n \le -K + q_n x_{n-l}$, for $n \in N(a_3),\ a_3 \in N(a_2)$. (2.8)

Case (I) Suppose $\{x_n\}$ is unbounded. *i.e.,* $\limsup\limits_{n\to\infty} x_n = \infty$. Then there exists a subsequence $\{n_i\}_{i=1}^{\infty} \subseteq N$ such that $n_i \ge a_3 + l$ and $n_i \to \infty$ as $i \to \infty$, $x_{n_i} = \max\limits_{l}(x_{n_i - l})$. In view of the inequality (2.8), we have $x_{n_i} \le -K + q_n x_{n_i - l} \le -K + q_n x_{n_i}$, which is a contradiction.

Case (II) Suppose $\{x_n\}$ is bounded. *i.e.,* $\limsup\limits_{n\to\infty} x_n = \varepsilon,\ 0 < \varepsilon < \infty$. Then there exists a subsequence $\{n_i^{*}\}_{i=1}^{\infty} \subseteq N$ such that $x_{n_i^*} \to \infty$ and $n_i^{*} \to \infty$ as $i \to \infty$, $x_{n_i^*} = \max\limits_{l}(x_{n_i^* - l})$. It follows that $\limsup\limits_{i\to\infty} x_{n_i^*} \le \varepsilon$. In view of the inequality (2.8), we have $x_{n_i^*} \le -K + q_n x_{n_i^* - l} \le -K + q_n x_{n_i^*}$, which implies $\limsup\limits_{i\to\infty} x_{n_i^*} \le -K + q_n \limsup\limits_{i\to\infty} x_{n_i^* - l} \le -K + q_n \limsup\limits_{i\to\infty} x_{n_i^*}$. *i.e.,* $\varepsilon \le -K + q_n \varepsilon$, which is a contradiction.

Thus in both cases we obtained the contradiction to the inequality (2.8). Therefore our assumption, namely, $z_n \le 0$, for $n \in N(a_2),\ a_2 \in N(a_1)$ is wrong. Hence we proved that inequalities remains hold for this condition also. Proof of the remaining part is same as that of condition (C1), and hence we omitted.

Finally, we consider the condition (C4). Then from equation (2.1) and from condition (C5), we have

$z_n = x_n + p_n x_{n-k} - q_n x_{n-l}$, for $n \in N(a_1),\ a_1 \in N(a)$. (2.9)

Then from equation (1.3), condition (C5), and proof of the theorem for condition (C2) we have

$\Delta_m^j z_n = -f(x_{n-\tau}) < 0$ *and* $z_n > 0$, for $n \in N(a_2),\ a_2 \in N(a_1)$, $j \ge 1$.

The remaining part of the proof is the same as that of condition (C1), and hence we omitted.

Hence the theorem is completely proved.

Remark 2.1 [5] Intuitively, a fixed solution $\{x^*\}$ is stable if solutions close to $\{x^*\}$ do not wander far from $\{x^*\}$ under all iterations f in equation (1.2.1). Asymptotically stability of $\{x^*\}$ requires the additional condition that all solutions of equation (1.2.1) that start near $\{x^*\}$ converge to $\{x^*\}$.

Corollary 2.1 If one of the conditions (C1), (C2), (C4) is satisfied along with the condition (C5), then the fixed solution $\{x^*\}$ of equation (1.1) is globally asymptotically stable.

Proof It is easily seen from the proof of the theorem 2.1 along with the remark 2.1.

Theorem 2.2 If the condition (C3) is satisfied along with the condition (C5), then the solution $\{x^*\}$ of equation (1.1.5) is not a fixed solution and hence it is not asymptotically stable.

Remark 2.2 The proof of the theorem 2.2 is similar to that of theorem 2.1, and hence omitted.

Corollary 2.2 If the condition (C3) is satisfied along with the condition (C5), then the solution $\{x^*\}$ of equation (1.3) is not a fixed solution and hence it is not globally asymptotically stable.

Proof It is easily seen from the proof of the theorem 2.2 along with the remark 2.1.

3. CONCLUSION

In this manuscript, we obtained new criteria for asymptotic behavior of fixed solutions of nonlinear delay difference equation (1.1,5).

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REFERENCES

[1] R. P. Agarwal, 'Difference Equations and Inequalities – Theory, Methods and Applications', 2nd edition, 2000.

[2] R. P. Agarwal, Martin Bohner, Said R. Grace, Donal O'Regan, Discrete Oscillation Theory, CMIA Book Series, Volume 1, ISBN: 977–5945–19–4, 2005.

[3] Saber N. Elaydi, An Introduction to Difference Equations, Springer-Verlag, Newyork, 3rd edition, 2005.

[4] R. A. C. Ferreia, D. F. M. Torres, *Fractional h-difference Equations Arising from the Calculus of Variations*, Appl. Annal. Discrete Math., 5(1), 110-121, 2011. doi:10.2298/AADM110131002F

[5] Walter G. Kelley, Allan C. Peterson, Difference Equations, An Introduction with Applications, Harcourt: Academic Press, San Diego, 2nd edition, 2001.

[6] M. M. S. Manuel, G. B. A. Xavier, G*eneralized difference calculus of sequences of real and complex numbers*, Int. J. Comput. Number. Anal. Appl., 6 (4), 401-415, 2004.

[7] M. M. S. Manuel, G. B. A. Xavier, E. Thandapani, T*heory of generalized difference operator and its applications*, Far East J. Math. Sci., 20 (2), 163–171, 2006.

[8] M. M. S. Manuel, A. Kilicman, G. B. A. Xavier, R. Pugalarasu, D. S. Dilip, *On the solutions of second order generalized difference equations*, Adv. Differ. Equ. 2012, 105, 2012. doi:10.1186/1687-1847-2012-105.

[9] M. M. S. Manuel, A. Kilicman, G. B. A. Xavier, R. Pugalarasu, D. S. Dilip, *An applications on the second-order generalized difference equations*, Adv. Differ. Equ. 2013, 35, 2013. doi:10.1186/1687-1847-2013-35

[10] S. Kaleeswari, B. Selvaraj, M. Thiyagarajan, *A new creation of mask from difference operator to image analysis*, Journal of Theoretical & Applied Information Technology, 69 (1), 2014.

[11] S. Kaleeswari, B. Selvaraj, M. Thiyagarajan, *Removing noise through a nonlinear difference operator*, International Journal of Applied Engineering Research 9 (21), 5100-5106, 2014.

[12] S. Kaleeswari, B. Selvaraj, M. Thiyagarajan, *An application of certain third order difference equation in image enhancement*, Asian Journal of Information Technology 15 (23), 4945-4954, 2016.

[13] N. Parhi, *Oscillation and non-oscillation solutions of second order difference equations involving generalized difference*, Applied Mathematics and Computation, 218, 458-468, 2011.

[14] M. Raju, C. Selvamani, *Fixed solutions of second order nonlinear difference equations*, Journal of Physics: Conference Series, Special Issue of ICACSE 2020, 2021. doi:10.1088/1742-6596/1964/2/022020

[15] M. Raju, B. Selvaraj, *A note on nonoscillatory properties of certain third order nonlinear difference equations*, American International Journal of Research in Science, Technology, Engineering and Mathematics, Special Issue of 2nd ICCSPAM, 402-405, 2019.

[16] M. Raju, B. Selvaraj, M. Thiyagarajan, *Time complexity of algorithms and its difference equation representation*, Journal of Theoretical and Applied Information Technology, 69(1), 101-105, 2014.

[17] M. Raju, B. Selvaraj, M. Thiyagarajan, *A novel application of difference equation for fern leaf developer as an inverse problem*, International Journal of Advanced Engineering Technology, VII(II), 127-133, 2016.

[18] B. Selvaraj, G. Gomathi Jawahar, *Asymptotic behavior of fourth order non–linear delay difference equations,* Far East Journal of Mathematical Science(FJMS), 1 (7), 877–886, 2010.

[19] B. Selvaraj, S. Kaleeswari , *Oscillation of solutions of certain nonlinear difference equations*, Progress in Nonlinear Dynamics and Chaos, 1, 34-38, 2013.

[20] B. Selvaraj, S. Kaleeswari, *Oscillation of solutions of second order nonlinear difference equations*, Bulletin of Pure & Applied Sciences-Mathematics and Statistics 32 (1), 83-92, 2013.

[21] B. Selvaraj, M. Raju, *Oscillatory behavior of solutions of certain fourth order nonlinear neutral delay difference equations*, Progress in Nonlinear Dynamics and Chaos, 1, 27-33, 2013.

[22] B. Selvaraj, M. Raju, *Oscillatory properties of third-order quasilinear difference equations*, Malaya Journal of Matematik, 4(1), 142-147, 2013.

[23] B. Selvaraj, M. Raju, *Asymptotic and oscillatory behavior of certain second order neutral delay difference equations*, International Journal of Mathematical Analysis, 9(20), 965-977, 2015.

[24] B. Selvaraj, M. Raju, *Oscillatory solutions to certain second order neutral difference equations with quasi difference*, Far East Journal of Mathematical Sciences, 99(8), 1271-1281, 2016.

[25] Bothiah Selvaraj, Murugasamy Raju, *Oscillatory solutions to a certain third order difference equation*, Applied Sciences, 18, 108-112, 2016.

[26] B. Selvaraj, M. Raju, M. Thiyagarajan, *A novel difference equation representation for auto regressive time series*, Journal of Theoretical and Applied Information Technology, 67(1), 78-83, 2014.

[27] E. Thandapani, I. M. Arockiasamy – '*Oscillatory and asymptotic behavior of fourth order nonlinear neutral delay difference equations*, Indian J. pure appl. Math., 32(1), 109-123, 2001.

[28] E. Thandapani, B. Selvaraj, *Existence and asymptotic behavior of nonoscillatory solutions of certain nonlinear difference equations*, Far East Journal of Mathematical Sciences (FJMS), 14(1), 9-25, 2004.

[29] Yu-Ping Zhao, Xi-Lan Liu, *Asymptotic behavior for nonoscillatory solutions of nonlinear delay difference equations*, International Journal of Difference Equations, 5(2), 266-271, 2010.

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