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NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

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One day International Conference EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

th 27 October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on "Emerging Trends in Science and Technology (ETIST-2021)" is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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Soft semi weakly g^* -closed sets

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Abstract - In this paper, we introduce semi weakly g^* -closed sets in soft topological spaces and present its related properties.

Keywords Soft sets, soft topological spaces, soft semi weakly g^* -closed sets, soft semi weakly g^* -open sets. 2010 Subject classification: 54A05, 54A20, 54C05, 54C08

1 Introduction

Molodtsov [7] introduced the concept of soft set theory as a mathematical tool for deal with uncertainities like modeling the problems in science and engineering in 1999. Many researchers like Maji et al.[6] have further improved the theory of soft sets. In 2011, Cagman et al.[3] introduced soft topology on a soft set and defined soft topological space. Kannan [4] defined soft generalized closed and open sets in soft topological spaces which are defined over an initial universe with fixed set of parameters. since then this method of generalizing sets was adopted by many topologists. In this paper, we introduce semi weakly g^{*}-closed sets in soft topological spaces and study some of its properties.

2 Preliminaries

Definition 2.1. [7] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F: A \rightarrow P(U)$.

Definition 2.2. [6] Let U be an universal set and E be an universe set of parameters. Let (F, A) and (G, B) be soft sets over a common universe set U and $A, B \subseteq E$. Then (F, A) is a subset of (G, B) , denoted by $(F, A) \subseteq (G, B)$, if

(i) $A \subseteq B$,

(ii) for all $e \in E$, $F(e) \subset G(e)$.

 (F, A) equals (G, B) , denoted by $(F, A) = (G, B)$, if $(F, A) \subseteq (G, B)$ and $(G, B) \subseteq (F, A)$.

Definition 2.3. [6] Two soft set (F, A) and (G, B) over a common universe U are said to be equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A) .

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Definition 2.4. [6] A soft set (F, A) over U is said to be a null soft set, denoted by $\widetilde{\phi}$, if $\forall e \in A, F(e) = \phi$. **Definition 2.5.** [6] A soft set (F, A) over U is said to be a absolute soft set, denoted by \widetilde{U} , if $\forall e \in$ $A, F(e) = U.$

Definition 2.6. [6] Union of two soft sets of F , A) and (G, B) over the common universe U is the soft set (H, C) , where $C = A \cup B$, and $\forall e \in C$,

$$
H(e) = \begin{cases} F(e), & if \ e \in A - B \\ G(e), & if \ e \in B - A \\ F(e) \cup G(e), & if \ e \in A \cap B \end{cases}
$$

Definition 2.7. [6] Intersection of two soft sets F , A) and (G, B) over the common universe U is the soft set (H, C) , where $C = A \cap B$, and $\forall e \in C$, $H(e) = F(e)$ or $G(e)$. We write $(F, A) \cap (G, B) = (H, C)$.

Definition 2.8. [1] The complement of a soft set (F, A) , denoted by $(F, A)^c$, is defined by $(F, A)^c$ $(F^c, A), F^c: A \to P(X)$ is mapping given by $F^c(e) = X - F(e), \forall e \in A$ and F^c is called the soft complement function of F.

Definition 2.9. [5] Let X be a universe and E a set of attributes. Then the collection of all soft sets over X with attributes from E is called a soft class and is denoted as (X, E) .

Definition 2.10. [10] Let $\tilde{\tau}$ be the collection of soft sets over X. Then $\tilde{\tau}$ is said to be a soft topology on X if

- (i) $(\widetilde{\phi}, A), (\widetilde{X}, A) \in \widetilde{\tau}$, where $\widetilde{\phi}(\alpha) = \phi$ and $\widetilde{X}(\alpha) = X, \forall A$,
- (ii) the intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$,
- (iii) the union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.

The triple $(X, \tilde{\tau}, A)$ is called a soft topological space over X. The members of $\tilde{\tau}$ are said to be soft open sets in X.

Definition 2.11. [8] A soft set (E, A) over X is said to be soft element if there exists $\alpha \in A$ such that $E(\alpha)$ is a singleton, say $\{x\}$, and $E(\beta) = \phi$, $\forall \beta (\neq \alpha) \in A$ such a soft element is denoted by E^x_α .

Definition 2.12. [11] A soft set (G, A) in a soft topological space $(U, \tilde{\tau}, A)$ is called a soft neighborhood of the soft point $e_F \tilde{\in} U_A$ if there exists a soft open set (H, A) such that $e_F \tilde{\in} (H, A) \tilde{\subseteq} (G, A)$.

The neighborhood system of a soft point e_F , denoted by $N_\tau(e_F)$, is the family of all its neighborhoods.

Definition 2.13. A soft set (A, E) of a soft topological space $(X, \tilde{\tau}, E)$ is called

- (a) a soft generalized closed (briefly soft g-closed) [4] if $cl_s(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft open in $(X, \widetilde{\tau}, E)$,
- (b) a soft w-closed [9] if $cl_s(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft semi open in $(X, \widetilde{\tau}, E),$

Definition 2.14. [2] Let $(X, \tilde{\tau}, E)$ be a soft topological space and $(F, E), (G, E)$ be semi closed sets in X such that $(F, E) \cap (G, E) = \emptyset$. If there exist semi open soft sets (F_1, E) and (F_2, E) such that $(F, E) \subseteq (F_1, E), (G, E) \subseteq (F_2, E)$ and $(F_1, E) \cap (F_2, E) = \widetilde{\phi}$, then $(X, \widetilde{\tau}, E)$ is called a soft semi normal space.

Through out this paper we denote soft elements of (X, E) by X_e^x and soft elements of s soft set (A, E) by A_e^x .

3 Soft semi weakly g^* -closed sets

Definition 3.1. A soft set (A, E) in a soft topological space $(X, \tilde{\tau}, E)$ is called soft semi weakly g^{*}-closed
set (briefly soft sever^{*} closed set) if sel $(A, F) \tilde{\subset} (U, F)$ whenever $(A, F) \tilde{\subset} (U, F)$ and (U, F) is set (briefly soft swg^{*}-closed set) if $gcl_s(A, E) \subseteq (U, E)$ whenever $(A, E) \subseteq (U, E)$ and (U, E) is soft semiopen set in $(X, \widetilde{\tau}, E)$.

We denote the class of soft swg^{*}-closed sets in $(X, \tilde{\tau}, E)$ by $SSWG^*CS(X, \tilde{\tau}, E)$.

Example 3.2. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $F_1(e_1) =$ ${x_1}, F_1(e_2) = {\phi}, F_2(e_1) = {x_2}, F_2(e_2) = {\phi}$ and $F_3(e_1) = {X}, F_3(e_2) = {\phi}.$ Then a soft set (A, E) such that $A(e_1) = \{x_2\}, A(e_2) = \{x_1\}$ is a soft swg^{*}-closed set.

Proposition 3.3. 1. Every soft closed set is a soft swg^{*}-closed set.

- 2. Every soft w-closed set is a soft swg^{*}-closed set.
- 3. Every soft g-closed set is a soft swg^{*}-closed set.

Proof. 1. Let (A, E) be a soft closed set and (U, E) be a soft semi-open set such that $(A, E) \subseteq (U, E)$. Then we have $cl(A, E) = (A, E)$. Since every soft closed set is a soft q-closed set, (A, E) . Since every soft closed set is a soft q-closed set, $gcd_s(A, E) \subseteq cl_s(A, E) \subseteq (U, E)$. Hence (A, E) is a soft swg^{*}-closed set.

2. Let (A, E) be a soft w-closed set and (U, E) be a soft semi-open set such that $(A, E) \subseteq (U, E)$. Then we have $gcl_s(A, E) \subseteq cl_s(A, E) \subseteq (U, E)$, since every soft closed set is a soft g-closed set. Therefore (A, E) is a soft swg[∗] -closed set.

3. Let (A, E) be a soft q-closed set and (U, E) be a soft semi-open set such that $(A, E) \tilde{\subset} (U, E)$. Then by assumption $gcl_s(A, E) = (A, E)$ and $sogcl_s(A, E) \subseteq (U, E)$. Thus (A, E) is a soft swg^{*}-closed set.

Remark 3.4. Converse of the above proposition need not be true as seen from the following example.

Example 3.5. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $F_1(e_1) =$ ${\phi}, F_1(e_2) = {x_2}, F_2(e_1) = {x_1}, F_2(e_2) = {x_2}$ and $F_3(e_1) = {x_2}, F_3(e_2) = {X}.$ Then the soft set (A, E) such that $A(e_1) = \{x_1\}, A(e_2) = \{x_2\}$ is a soft swg^{*}-closed set but not soft closed, not soft ω -closed and not soft g-closed.

Proposition 3.6. If (A, E) and (B, E) are soft swg^{*}-closed sets then $(A, E) \cup (B, E)$ is soft swg^{*}-closed.

Proof. Presume that (A, E) $\widetilde{\cup}$ (B, E) $\widetilde{\subset}$ (U, E) and (U, E) is soft semi-open. Then (A, E) $\widetilde{\subset}$ (U, E) and $(B, E) \subseteq (U, E)$. Since (A, E) and (B, E) are soft swg^{*}-closed, $gcl_s(A, E) \subseteq (U, E)$ and $gcl_s(B, E) \subseteq (U, E)$ and hence $gcl_s((A, E) \ \widetilde{\cup} \ (B, E)) = gcl_s(A, E) \ \widetilde{\cup} \ gd_s(B, E) \subseteq (U, E)$. Thus $(A, E) \ \widetilde{\cup} \ (B, E)$ is swg^* -closed soft.

Remark 3.7. Intersection of two soft swg^{*}-closed sets is generally not soft swg^{*}-closed. In Example 3.5, soft sets (A, E) such that $A(e_1) = \{x_2\}, A(e_2) = \{X\}$ and (B, E) such that $B(e_1) = \{X\}, B(e_2) = \{x_2\}$ are soft swg[∗]-closed but their intersection $(C, E) = (A, E) \cap (B, E)$ such that $C(e_1) = \{x_2\}$, $C(e_2) = \{x_2\}$ is not a soft swg[∗] -closed set.

Proposition 3.8. In a soft topological space $(X, \tilde{\tau}, E)$, if $SSO(X, \tilde{\tau}, E) = {\tilde{\phi}, \tilde{X}}$, then every soft subset of (X, E) is a soft swg^{*}-closed set.

Proof. Let $SSO(X, \tilde{\tau}, E) = {\tilde{\phi}, \tilde{X}}$ and (A, E) be any soft subset of (X, E) . Presume that $(A, E) = \tilde{\phi}$, then (A, E) is a soft swg^{*}-closed set. Presume that $(A, E) \neq \emptyset$, then X is the only soft semi-open set containing (A, E) and so $gcl_s(A, E) \subseteq X$. Hence (A, E) is soft swg^{*}-closed.

Remark 3.9. Converse of the above proposition need not be true. In Example 3.2, every soft subset of (X, E) is soft swg^{*}-closed set but $SSO(X, \tau, E) \neq {\tilde{\phi}, \tilde{X}}.$

Proposition 3.10. Every soft subset of (X, E) is soft swq^* -closed if $SSO(X, \widetilde{\tau}, E) \subseteqq {\{F, E\}} \subseteqq (X, E) : (F, E)^c \in SGO(X, \widetilde{\tau}, E)$, where $SGO(X, \widetilde{\tau}, E)$ is the set of all soft a green sets soft g-open sets.

Proof. Assume that $SSOX_E \subseteq \{(F, E) \subseteq (X, E) : (F, E)^c \in SGO(X, \tilde{\tau}, E)\}$. Let (A, E) be any soft
subset of (X, τ, E) such that $(A, E) \subseteq (U, E)$ where (U, E) is a soft some pop set. Then (U, E) is soft subset of (X, τ, E) such that $(A, E) \subseteq (U, E)$ where (U, E) is a soft semi-open set. Then (U, E) is soft g-closed and $gcl_s(A, E) \subseteq (U, E)$. Hence (A, E) is a swg^{*}-closed set.

Proposition 3.11. If (A, E) is a soft swg^{*}-closed subset of $(X, \tilde{\tau}, E)$ such that $(A, E) \tilde{\subset} (B, E) \tilde{\subset}$ cal (A, E) then (B, E) is a soft such closed set in $(X, \tilde{\tau}, E)$ $(A, E) \subseteq (B, E) \subseteq \text{gcl}_s(A, E)$, then (B, E) is a soft swg^{*}-closed set in $(X, \widetilde{\tau}, E)$.

Proof. Let $(B, E) \subseteq (U, E)$ and (U, E) be soft semi-open. Then $(A, E) \subseteq (U, E)$. Since (A, E) is soft swg^* -closed, $gcl_s(A, E) \subseteq (U, E)$. Now, $gcl_s(B, E) \subseteq gd_s(gcl_s(A, E)) = gcl_s(A, E) \subseteq (U, E)$. Therefore (B, E) is soft swg^{*}-closed in $(X, \tilde{\tau}, E)$.

Remark 3.12. Converse of the above proposition need not be true as seen from the following example.

Example 3.13. Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{\phi}, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where $F_1(e_1) = \{x_1\}, F_1(e_2) = \{x_2\}, F_2(e_1) = \{x_1\}, F_2(e_2) = \{x_1, x_2\}, F_3(e_1) = \{x_1, x_2\}, F_3(e_2) = \{x_2\}, F_4(e_2) = \{x_3\}$ $F_4(e_1) = \{x_1, x_2\}, F_4(e_2) = \{x_1, x_2\}.$ Let (S, E) such that $S(e_1) = \{\phi\}, S(e_2) = \{x_3\}$ and (T, E) such that $T(e_1) = \{x_1\}$, $T(e_2) = \{x_1, x_3\}$ be two soft swg^{*}-closed sets. Then $(S, A) \subseteq (T, E) \nsubseteq gcl_s(S, E)$. Thus any soft swg^{*}-closed set need not be lie between soft swg^{*}-closed set and its soft g-closure.

Proposition 3.14. If a soft subset (A, E) of (X, E) is soft swg^* -closed, then $\gcd_s(A, E) - (A, E)$ contains no nonempty soft semi-closed set.

Proof. Assume that (A, E) is a soft swg^{*}-closed set. Let (U, E) be a soft semi-closed subset contained in $gcl_s(A, E) - (A, E)$. Then $(A, E) \subseteq (U, E)^c$. Since (A, E) is soft swg^{*}-closed, we have $gcl_s(A, E) \subseteq (U, E)^c$ and so $(U, E) \subseteq (gcl_s(F, E))^c$. Hence $(U, E) \subseteq gcl_s(A, E) \cap (gcl_s(A, E))^c = \phi$. Therefore $gcl_s(A, E) - (A, E)$ contains no nonempty soft semi-closed set.

Corollary 3.15. If a soft subset (A, E) of (X, E) is soft swg^* -closed, then $qcl_s(A, E) - (A, E)$ contains no nonempty soft closed set.

Proof. Let (A, E) be soft swg^{*}-closed in (X, τ, E) and (F, E) be a soft closed subset of $gcl_s(A, E)$ – (A, E) . Then $(A, E) \subseteq (F, E)^c$. Since every soft open set is soft semi-open and (A, E) is soft swg^{*}-closed, $gcd_s(A, E) \subseteq (F, E)^c$. Consequently $(F, E) \subseteq (gcd_s(F, E))^c$. Thus $(F, E) \subseteq gcd_s(A, E) \cap (gcd_s(A, E))^c$. Hence $qcl_s(A, E) - (A, E)$ contains no nonempty soft closed set.

Corollary 3.16. If a soft swg^{*}-closed set (A, E) is soft semi-closed then $\mathrm{scl}_s(A, E) - (A, E)$ is soft semiclosed.

Proof. Since (A, E) is soft semi-closed, then $scl_s(A, E) - (A, E) = \phi$ and so $scl_s(A, E) - (A, E)$ is a soft semi-closed set.

Proposition 3.17. If (A, E) is a soft swg^{*}-closed set then for each $X_e^x \in \mathcal{G}cl_s(A, E)$, $\operatorname{scl}_s(X_e^x) \widetilde{\cap} (A, E) \neq \phi.$

Proof. Assume that $X_{\epsilon}^{x} \in gel_{s}(A, E)$ and $sel_{s}(X_{\epsilon}^{x}) \cap (A, E) = \phi$. Then $(A, E) \subseteq (sel_{s}(X_{\epsilon}^{x}))^{c}$ and $(sel_{s}(X_{\epsilon}^{x}))^{c}$ is set come Since (A, E) is set comet shoot and $(A, E) \subseteq (sel_{s}(X_{\epsilon}^{x}))^{c}$ and $(scl_s(X_e^x))^c$ is soft semi-open. Since (A, E) is soft swg^{*}-closed, $gcl_s(A, E) \subseteq (scl_s(X_e^x))^c$, which is a contradiction.

Proposition 3.18. If $\operatorname{scl}_s(X_e^x)$ $\widetilde{\cap}$ $(A, E) \neq \phi$, for each $X_e^x \widetilde{\in}$ $\operatorname{gcd}_s(A, E)$, then $qcl_s(A, E) - (A, E)$ contains no nonempty soft semi-closed sets.

Proof. Let $(F, E) \subseteq \text{gcl}_s(A, E) - (A, E)$, where (F, E) be soft semi-closed. If there is a soft point $X_{e}^{x} \widetilde{\in} (F, E),$ then $X_{e}^{x} \widetilde{\in} gcl_{s}(A, E)$ and so by assumption, $\phi \neq scl_{s}(X_{e}^{x}) \widetilde{\cap} (A, E)$
 $\widetilde{\in} (F, E) \widetilde{\in} (A, E) \widetilde{\in} (A, E) \widetilde{\in} (A, E)$ $\widetilde{\subseteq}$ (F, E) $\widetilde{\cap}$ (A, E) \subseteq $(gcl_s(A, E) - (A, E))$ $\widetilde{\cap}$ $(A, E) = \widetilde{\phi}$, a contradiction. Therefore $(F, E) = \widetilde{\phi}$.

Definition 3.19. A soft subset (A, E) is said to be soft nowhere dense if $int_s(cl_s(A, E)) = \widetilde{\phi}$.

Remark 3.20. A soft singleton set of (X, E) is denoted by $\{x\}$ for all $e \in E$ and $x \in X$.

Lemma 3.21. Let X_e^x be a soft point of (X, E) . Then $\{x\}_e$ is soft nowhere dense or soft pre-open.

Proof. Presume that $\{x\}_e$ is not soft nowhere dense. Then $int_s(cl_s(\{x\}_e) \neq \phi$, and so $X_e^x \in int_s(cl_s(\{x\}_e))$. Thus ${x\}_e \subseteq int_s(cl_s({x\}_e)$. Hence ${x\}_e$ is soft pre-open.

Remark 3.22. We may consider the decomposition of a soft topological space $(X, \tilde{\tau}, E)$, namely (X, E) = $(X_1, E) \cup (X_2, E)$, where $(X_1, E) = \{X_e^x \in (X, E) : \{x\}_e$ is nowhere dense $\}$ and $(X_2, E) = \{X_e^x \in (X, E) : \{x\}_e \in (E, E) \}$ ${x}_e$ is pre open}

Definition 3.23. The intersection of all soft semi-open subsets of (X, τ, E) containing (A, E) is said to be the soft semi-kernal of (A, E) and is denoted by ssker (A, E) . *i.e.*, $ssker(A, E) = \bigcap \{(F, E) : (A, E) \subseteq (F, E) \text{ where } (F, E) \in SSO(X, E)\}.$

Theorem 3.24. A soft subset (A, E) of (X, E) is soft swg^{*}-closed if and only if $\gcd_s(A, E) \subseteq \operatorname{ssker}(A, E).$

Proof. Assume that (A, E) is soft swg^{*}-closed. Let $X_e^x \in \text{gcd}_s(A, E)$. Presume that $X_e^x \notin \text{ssker}(A, E)$, then X_e^x does not belong to soft semi-open sets which contains (A, E) . By assumption $gcl_s(A, E) \subseteq (F, E)$ where (F, E) is soft semi-open set which contains (A, E) and so $X_e^x \notin \mathcal{G}cl_s(A, E)$, which is a contradiction.

Conversely, assume that $gcl_s(A, E) \subseteq ssker(A, E)$. Let (F, E) be any soft semi-open set containing (A, E) . Then $gcl_s(A, E) \subseteq \text{ssker}(A, E) \subseteq (F, E)$, by definition. Therefore (A, E) is soft swg^{*}closed.

Proposition 3.25. If (A, E) is soft swg^{*}-closed then $(X_1, E) \cap \text{gcl}_s(A, E) \subseteq (A, E)$.

Proof. Assume that (A, E) is soft swg^{*}-closed. Let $X_e^x \in (X_1, E) \cap \text{gcd}_s(A, E)$.

Then $int_s (cl_s({x\})_e) = \phi$. Therefore ${x\}_e$ is soft semi-closed. Presume that if $X_e^x \notin (A, E)$ and if $(U, E) = X - \{x\}_e$, then (U, E) is a soft semi-open set containing (A, E) and so $gcl_s(A, E) \subseteq U, E$, which is a contradiction.

Proposition 3.26. If a soft compact topological space $(X, \tilde{\tau}, E)$ is a soft $T_{1/2}$ -space and suppose that (A, E) is a soft swg^{*}-closed subset of (X, E) , then (A, E) is soft compact.

Proof. Presume that $C = \bigcup_i (G_i, E)$ be a soft open cover of (A, E) . Then $\bigcup_i (G_i, E)$ is soft semi open, since arbitrary union of soft semi open set is soft semi open. By hypothesis, we have $\mathfrak{gcl}_s(A, E) \subseteq \bigcup_{A \in \mathbb{R}} (G_i, E)$. Also $gcl_s(A, E) = cl_s(A, E)$ is soft compact in $(X, \tilde{\tau}, E)$. Therefore $(A, E) \subseteq \text{gcl}_s(A, E) = cl_s(A, E) \subseteq$ $\bigcup_{i=1,2,\dots,n}(G_i,A)$, where $G_1, G_2, \dots, G_n \in \mathbb{C}$. Hence (A, E) is soft compact.

Proposition 3.27. If $(X, \tilde{\tau}, E)$ is soft semi-normal and soft $T_{1/2}$ -space and $(F, E) \tilde{\cap} (A, E) = \tilde{\phi}$, where (F, E) is soft semi-closed and (A, E) is soft swg^{*}-closed then there exist disjoint soft semi-open sets (U_1, E) and (U_2, E) such that $(A, E) \subseteq (U_1, E)$ and $(F, E) \subseteq (U_2, E)$.

Proof. Since (F, E) is soft semi-closed and (F, E) $\widetilde{\cap}$ $(A, E) = \phi$. We have $(A, E) \subseteq (F^c, E)$ and so $gcl_s(A, E) \subseteq (F^c, E)$. Thus $gcl_s(A, E) \cap (F, E) = \phi$. Since $gcl_s(A, E) = cl_s(A, E)$, $cl_s(A, E)$ and (F, E) are soft semi-closed. Then there exist soft semi open sets (U_1, E) and (U_2, E) such that $gcl_s(A, E) \subseteq (U_1, E)$ and $(F, E) \subseteq (U_2, E)$. Therefore $(A, E) \subseteq (U_1, E)$ and $(F, E) \subseteq (U_2, E)$.

Corollary 3.28. If $(A, E) = (F, E) - (N, E)$, where (F, E) is soft closed and (N, E) contains no non empty soft semi-closed set then (A, E) is soft swg^{*}-closed.

Proof. Presume that $(A, E) = (F, E) - (N, E)$. Let $(A, E) \subseteq (U, E)$ such that (U, E) is soft semi-open. Then $(F, E) \tilde{\cap} (U, E)^c$ is soft semi-closed and it is a subset of (N, E) . But by hypothesis $(F, E) \tilde{\cap} (U, E)^c =$
 $\tilde{\cap}$ F . $\tilde{\cap}$ F $\widetilde{\phi}$. Thus $(F, E) \subset (U, E)$ and so $qcl_s(A, E) \subset \overline{qcl_s(F, E)} \subset cl_s(F, E) = (F, E) \subset \overline{C}$ (U, E) . Therefore (A, E) is soft swg[∗] -closed.

4 Soft semi weakly g^* -open sets

Definition 4.1. A sub soft set (A, E) in (X, E) is called soft semi weakly g^{*}-open (briefly soft swg^{*}-open) if (A^c, E) is soft swg^{*}-closed.

We denote the class of soft swg^{*}-open sets in $(X, \tilde{\tau}, E)$ by $SSWG^*OS(X, \tilde{\tau}, E)$.

Proposition 4.2. Every soft open (resp. soft g-open and soft ω -open) set is soft swg^{*}-open.

Proof. Follows from Proposition 3.3.

Theorem 4.3. A soft set (A, E) is soft swg^{*}-open if and only if $(F, E) \subseteq$ gint_s (A, E) whenever (F, E) is soft semi-closed and $(F, E) \subseteq (A, E)$.

Proof. Presume that (A, E) is soft swg^{*}-open such that $(F, E) \subseteq (A, E)$ and (F, E) is soft semiclosed. Then (F^c, E) is soft semi-open and $(A^c, E) \subseteq (F^c, E)$. Therefore $gcl_s(A^c, E) \subseteq (F^c, E)$ and so $(F, E) \subseteq qint_s(A, E).$

Presume that $(F, E) \subseteq \text{gint}_s(A, E)$, where (F, E) is soft semi-closed and $(F, E) \subseteq (A, E)$. Let $(A^c, E) \subseteq (U, E)$ and (U, E) be soft semi-open. Then $(U^c, E) \subseteq (A, E)$ and (U^c, E) is soft semi-closed. Therefore $(U^c, E) \subseteq gint_s (A, E)$ Then we have $gcl_s(A^c, E) \subseteq (U, E)$ and so (A^c, E) is soft swg^{*}-closed. Hence (A, E) is soft swg^{*}-open.

5 Soft swg^* -operators

Definition 5.1. A soft set (N, E) in a soft topological space $(X, \tilde{\tau}, E)$ is said to be soft swg^{*}-neighbourhood (briefly soft swg^{*} neighbourhood) of $X^x \tilde{\epsilon}$ (X, E) if there exists a soft swg^{*} apen set (A, E) (briefly soft swg^{*}-nbd.) of $X_c^x \in (X, E)$ if there exists a soft swg^{*}-open set (A, E) such that $X_c^x \in (A, E) \subseteq (N, E)$.
The set of all soft swe^{*} wickles when set of a soft with $X_c^x \in (X, E)$ is salled the soft wea^{*} The set of all soft swg[∗]-neighbourhoods of a soft point $X_e^x \n\t\widetilde{\in} (X, E)$ is called the soft swg^{*}-neighbourhood
term of X_e^x and is denoted by swe $\widetilde{\in} (X_x^x)$ system of X_e^x , and is denoted by swg^{*}- $N(X_e^x)$.

Example 5.2. Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{\phi}, \tilde{X}, (F, E)\}$ where $F(e_1) = \{x_1\}$, $F(e_2) = \{x_2\}$. The soft swg^{*}-neighbourhood system of $X_{e_1}^{x_1}$ is $swg^* \text{-} N(X_{e_1}^{x_1}) = \{ \{ (e_1, \{x_1\}), (e_2, \{x_2\}) \}, \{ (e_1, \{x_1\}), (e_2, \{X\}) \}, \{ (e_1, \{X\}), (e_2, \{x_2\}) \},$ $\{(e_1, \{X\}), (e_2, \{X\})\}.$ The soft swg^{*}-neighbourhood system of $X^{x_2}_{e_1}$ is $swg^* \text{-} N(X_{e_1}^{x_2}) = \{ \{ (e_1, \{x_2\}), (e_2, \{X\}) \} \{ (e_1, \{X\}), (e_2, \{X\}) \} \}.$ The soft swg^{*}-neighbourhood system of $X^{x_1}_{e_2}$ is $swg^* \text{-} N(X_{e_2}^{x_1}) = \{ \{ (e_1, \{x_2\}), (e_2, \{X\}) \} \{ (e_1, \{X\}), (e_2, \{X\}) \} \}.$ The soft swg^{*}-neighbourhood system of $X_{e_2}^{x_2}$ is $swg^* \text{-} N(X_{e_2}^{x_2}) = \{ \{ (e_1, \{\phi\}), (e_2, \{x_2\}) \}, \{ (e_1, \{\phi\}), (e_2, \{X\}) \}, \{ (e_1, \{x_1\}), (e_2, \{x_2\}) \},$ $\{(e_1,\{x_1\}), (e_2,\{X\})\}, \{(e_1,\{x_2\}), (e_2,\{x_2\})\}, \{(e_1,\{x_2\}), (e_2,\{X\})\}, \{(e_1,\{X\}), (e_2,\{x_2\})\},$ $\{(e_1, \{X\}), (e_2, \{X\})\}.$

Proposition 5.3. Let $(X, \tilde{\tau}, E)$ be a soft topological space and let swg^{*}- $N(X_e^x)$ be a swg^{*}-neighbourhood system of X^x . Then system of X_e^x . Then

- 1. For all $X_e^x \in (X, E)$, swg^* - $N(X_e^x) \neq \phi$.
- 2. For every $(A, E) \in swg^*N(X_e^x), X_e^x \in (A, E)$.
- 3. $(A, E) \in swg^* \cdot N(X_e^x)$ and $(A, E) \subseteq (B, E)$ implies $(B, E) \in swg^* \cdot N(X_e^x)$.
- 4. The intersection of two soft swg^{*}-nbds. of $X_e^x \tilde{\in} (X, E)$ is also a soft swg^{*}-nbd. of X_e^x .
- 5. If (N, E) is a soft swg^{*}-nbd. of X_e^x , then there exists a soft swg^{*}-nbd. (M, E) of X_e^x such that $(M, E) \subseteq (N, E)$ and (M, E) is a soft swg^{*}-nbd. of each of its points.

Proposition 5.4. If a soft subset of a soft topological space is soft swg^{*}-open then it is a soft swg^{*}-nbd. of each of ite points.

Proof. Let (A, E) be a soft swg^{*}-open set in a soft topological space $(X, \tilde{\tau}, E)$. Then for each $A_e^x \tilde{\in} (A, E)$
there exists a set (A, E) such that $A^x \tilde{\in} (A, E) \tilde{\in} (A, E)$. Thus $(A, E) \tilde{\in}$ such $N(A^x)$ there exists a swg^{*}-open set (A, E) such that $A_e^x \tilde{\in} (A, E) \subseteq (A, E)$. Thus $(A, E) \tilde{\in} swg^*$ - $N(A_e^x)$.

Proposition 5.5. Let (S, E) be a soft swg^{*}-closed subset of $(X, \tilde{\tau}, E)$. Then there exists a soft swg^{*}-nbd.
(N, E), of X^x , such that $(X, E) \underset{\tilde{\tau}}{\sim} (S, E)$, $\widetilde{\tau}$ (N, E) of X_e^x such that $(N, E) \cap (S, E) = \phi$.

Proof. Since $(S, E)^c$ is a soft swg^{*}-open set containing X_e^x , we have $(S, E)^c$ is a soft swg^{*}- nbd. of X_e^x . Let $(S, E)^c = (N, E)$. Then (N, E) is a soft swg^{*}- nbd. of X_e^x such that $(N, E) \cap (S, E) = \phi$.

Proposition 5.6. For the soft swg^{*}-neighbourhood system swg^{*}-N(X_e^x), the following hold.

- 1. For all $X_e^x \in (X, E)$, swg^* - $N(X_e^x) \neq \phi$.
- 2. If $(A, E) \in swg^*$ - $N(X_e^x)$ then $X_e^x \in (A, E)$.
- 3. If $(A, E) \in swg^* \cdot N(X_e^x)$ and $(A, E) \subseteq (B, E)$ then $(B, E) \in swg^* \cdot N(X_e^x)$.
- 4. If $(A, E), (B, E) \in swg^*N(X_e^x)$, then $(A, E) \cap (B, E) \in swg^*N(X_e^x)$.
- 5. If $(A, E) \in swg^* N(X_e^x)$ then there exists $(B, E) \in swg^* N(X_e^x)$ such that $(B, E) \subseteq (A, E)$ and $(B, E) \subseteq swg^* N(X_e^x)$ for some $X_v^y \in (B, E)$ $(B, E) \in swg^*N(X_e^x)$ for every $X_e^y \in (B, E)$.

Definition 5.7. Let $(X, \tilde{\tau}, E)$ be a soft topological space and $(A, E) \tilde{\in} (X, E)$. Then the intersection of all soft swg^{*}-closed sets containing (A, E) is said to be soft swg^{*}-closure of (A, E) (briefly swg^{*}-cl_s (A, E)). i.e. swg^* - $cl_s((A, E)) = \bigcap \{ (F, E) : (A, E) \subseteq (F, E), (F, E) \in SSWG^*CS(X, \tilde{\tau}, E) \}.$

Proposition 5.8. For any $(A, E) \subseteq (X, E)$, $(A, E) \subseteq swg^*$ - $cl_s((A, E)) \subseteq cl_s((A, E))$.

Proposition 5.9. For any $(A, E) \nsubseteq (X, E)$, 1. swg^* - $cl_s((A, E))$ is the smallest soft swg^{*}-closure of (A, E) . 2. If (A, E) is soft swg^{*}-closed then swg^{*}-cl_s $((A, E)) = (A, E)$

Remark 5.10. Converse of the above proposition need not be true. In Example 5.2 swg^{*}-cl_s({ e_1 , { ϕ },(e_2 , { x_2 })}) ${e_1, \{\phi\}, (e_2, \{x_2\})\}$ but ${e_1, \{\phi\}, (e_2, \{x_2\})\}$ is not a soft swg^{*}-closed set.

Proposition 5.11. Let $(X, \tilde{\tau}, E)$ be a soft topological space and $(A, E), (B, E)$ be subsets of (X, E) . 1. $swg^*cl_s(\phi) = (\phi)$ and $swg^*cl_s(\tilde{X}) = (\tilde{X})$

- 2. swg^* - $cl_s((A, E)) = swg^*$ - $cl_s((A, E))$.
- 3. If $(A, E) \subseteq (B, E)$, then $swg^* \text{-} cl_s((A, E)) \subseteq swg^* \text{-} cl_s((B, E)),$
- \mathcal{L} . swg^{*}-cl_s((A, E)) $\widetilde{\Box}$ (B, E)) $\widetilde{\Box}$ swg^{*}-cl_s((A, E)) $\widetilde{\Box}$ swg^{*}-cl_s((B, E)).
- 5. $swg^* cl_s((A, E) \ \tilde{\cup} \ (B, E)) = swg^* cl_s((A, E)) \ \tilde{\cup} \ swg^* cl_s((B, E)).$

Proposition 5.12. For a soft element $X_e^x \in (X, E)$, the following are equivalent 1. $X_e^x \tilde{\in} \, swg^* \text{-} cl_s((A, E)).$

2. For every soft swg^{*}-open set (U, E) containing X_e^x , $(U, E) \cap (A, E) \neq \emptyset$.

Proof. $1 \Rightarrow 2$. Let $X^x \in swg^* - cl_s((A, E))$ for any $X^x \in (X, E)$. Suppose that there exists a soft swg^{*}open set (U, E) containing X_e^x such that $(U, E) \cap (A, E) = \phi$. Then $(A, E) \subseteq (U, E)^c$. Since $(U, E)^c$ is a soft swg^{*}-closed set containing (A, E) , swg^{*}-cl_s((A, E)) $\subseteq (U, E)^c$ which implies $X_e^x \notin swg^*$ -cl_s((A, E)), a contradiction. Thus $(U, E) \tilde{\cap} (A, E) \neq \phi$.

2 ⇒ 1. Presume that $X_e^x \notin swg^*$ - $cl_s((A, E))$. Then there exists a soft swg^{*}-closed set (B, E) containing (A, E) such that $X_e^x \notin (B, E)$. Thus $X_e^x \widetilde{\in} (B, E)^c$ and $(B, E)^c$ is soft swg^{*}-open. Also $(B, E)^c \widetilde{\cap} (A, E) = \widetilde{\phi}$, which is a contradiction. Therefore $X_e^x \tildot{=} swg^* \tcdot cl_s((A, E)).$

Definition 5.13. Let $(X, \tilde{\tau}, E)$ be a soft topological space and $(A, E) \tilde{\epsilon}(X, E)$. Then the union of all soft swg^* -open sets contained in (A, E) is said to be soft swg^{*}-interior of (A, E) (briefly swg^{*}-int_s((A, E)) i.e. $swg^*-int_s(A, E) = \bigcup \{(S, E) : (S, E) \subseteq (A, E), (S, E) \in SSWG^*OS(X, \tilde{\tau}, E)\}.$

Proposition 5.14. For any soft subset $(A, E) \subseteq (X, E)$, $int_s((A, E)) \subseteq swg^*$ - $cl_s((A, E))$.

Proposition 5.15. Let (A, E) and (B, E) be subsets of (X, E) . Then we have the following. 1. $\text{swg}^*\text{-}int_s(\phi) = (\phi)$ and $\text{swg}^*\text{-}int_s(\tilde{X}) = (\tilde{X})$ 2. If (A, E) is soft swg^{*}-open then swg^{*}-int_s $((A, E)) = (A, E)$

- 3. $swg^* int_s(swg^* int_s((A, E))) = swg^* int_s((A, E)).$
- 4. If $(A, E) \subseteq (B, E)$, then $swg^* \text{-}int_s((A, E)) \subseteq swg^* \text{-}int_s((B, E))$,
- 5. $swg^*-int_s((A, E) \widetilde{\cap} (B, E)) = swg^*-int_s((A, E) \widetilde{\cap} swg^*-int_s((B, E)).$
- 6. swg^{*}-int_s((A, E)) $\widetilde{\cup}$ swg^{*}-int_s((B, E)) $\widetilde{\subseteq}$ swg^{*}-int_s((A, E) $\widetilde{\cup}$ (B, E)).

Proposition 5.16. A soft set (A, E) is soft swg^{*}-open in X then $(U, E) = X$ whenever (U, E) is soft semi-open and gint_s (A, E) $\widetilde{\cup}$ $(A, E)^c$ $\widetilde{\subseteq}$ (U, E) .

Proof. Assume that (A, E) is soft swg^{*}-open and $gint_s(A, E) \widetilde{\cup} (A, E)^c \widetilde{\subseteq} (U, E)$, where (U, E) be a soft supplying $(U, E)^c \widetilde{\subseteq}$ [sint $(A, E)^c \widetilde{\subseteq} (A, E)^c$] $(A, E)^c$ (A, E)^c (A, E)^c (A, E)^c (A, E)^c) semi-open set. This implies $(U, E)^c \subseteq [gint_s(A, E)]^c \cap [(A, E)^c]^c = gcl_s(A, E)^c - (A, E)^c$. Since $(A, E)^c$ is soft swg^{*}-closed and (U, E^c) is soft semi-closed, then by Proposition 3.14, $(U, E)^c = \phi$. Thus $\tilde{X} = (U, E)$.

Corollary 5.17. If $(U, E) = \tilde{X}$ whenever (U, E) is soft semi-open and $int_s(A, E) \tilde{\cup} (A, E)^c \tilde{\subseteq} (U, E)$ then (A, E) is soft swg^{*}-open.

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