VOLUME XI
ISBN No.: 978-93-94004-00-9
Physical Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,
Pollachi-642001

SUPPORTED BY

as pure as mother's love
ABT Industries Ltd.,
Dairy Division

PROCEEDING

One day International Conference
EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)
27th October 2021
Jointly Organized by
Department of Biological Science, Physical Science and Computational Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, Pollachi-642001.

Estd. 1957

Proceeding of the

One day International Conference on

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

ISBN No: 978-93-94004-00-9

978- 93- 94004- 00- 9

Nallamuthu Gounder Mahalingam College

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

www.ngmc.org

ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

International Conference on Emerging Trends in Science and Technology (ETIST 2021) Jointly Organized by Department of Biological Science, Physical Science and Computational Science Nallamuthu Gounder Mahalingam College, Affiliated to Bharathiar University, Tamilnadu, India. Published by NGMC - November 2021

Generalized Semi Closed Soft Multisets

V. Inthumathi[1],**A. Gnanasoundari**[2] and **M. Maheswari**[3]

Abstract - In this paper, we introduce the concepts of generalized semi closed sets in soft multi topological spaces and some of its properties are studied.

Keywords *Soft multiset, Soft multi topology, Generalized semi closed soft multiset.*
2010 Subject classification: *03E70, 54A05, 54B05*

1 Introduction

Multiset theory was introduced by Cerf et al. [3] in 1971. Then Yager [15] and Blizard [2] initiated further contributions to it. Continuing this study, multiset relation and multiset function was introduced by Girish and John [8][9]. In addition, these authors [10] [11] using multiset relations gave multiset topology. In 1999, Molodtsov [13] introduced the concept of soft set theory as a mathematical tool for dealing with uncertainties. In 2013, Babitha and John [1] was introduced the concept of soft multisets as a combination of soft sets and multisets. Moreover, in [4] [5] the soft multi topology and its basic properties was given. S. A. El-Sheikh et al.[7] introduced the notions of Some types of open soft multisets and some types of mappings in soft multi topological spaces. The same author [6] introduced the concept of generalized closed soft multisets in soft multi topological spaces. In this paper we introduce the concept of generalized semi closed soft multisets in soft multi topological spaces and discuss some important properties in detail.

2 Preliminaries

Definition 2.1. *[12] An mset M drawn from the set X is represented by a function Count M or C_M defined as $C_M : X \to N$ where N represents the set of non negative integers. The word 'multiset' often shortened to 'mset'.*

Definition 2.2. *[12] A domain X, is defined as a set of elements from which msets are constructed. The mset space $[X]^m$ is the set of all msets whose elements are in X such that no element in the mset occurs more than m times. If $X = \{x_1, x_2, ...x_k\}$ then $[X]^m = \{\{m_1/x_1, m_2/x_2, ...m_k/x_k\}$ $for\ i = 1, 2, 3...k; m_i \in \{0, 1, 2, ...m\}\}$. Henceforth M stands for a mset drawn from the mset space $[X]^m$.*

Definition 2.3. *[13] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

[1]Associate Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Coimbatore, Tamilnadu, India.E.mail: inthumathi65@gmail.com
[2]Assistant Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Coimbatore, Tamilnadu, India.E.mail: gnana.ngm@gmail.com
[3]Assistant Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Coimbatore, Tamilnadu, India.E.mail: mahimrk@gmail.com

Definition 2.4. *[5] Let U be an universal mset, E be a set of parameters and $A \subseteq E$. Then, an ordered pair (F, A) is called a soft mset where F is a mapping given by $F : A \to P^*(U); P^*(U)$ is the power set of a mset U.*

For all $e \in A, F(e)$ mset represents by count function $C_{F(e)} : U^* \to N$where N represents the set of non-negative integers and U^* represents the support set of U.

Let $U = \{2/x, 3/y, 2/z\}$ be a mset. Then, the support set of U is $U^* = \{x, y, z\}$.

Definition 2.5. *[5] For two soft msets (F, A) and (G, B) over U, we say that (F, A) is a sub soft mset of (G, B) if:*
1. $A \subseteq B$,
2.$C_{F(e)}(x) \leq C_{G(e)}(x),\ \forall\ x \in U^*, \forall e\ \in\ A \cap B$.

We write$(F, A)\widetilde{\subseteq}(G, B)$.

Definition 2.6. *[5] Two soft msets (F, A) and (G, B) over U are said to be equal if (F, A) is a sub soft mset of (G, B) and (G, B) is a sub soft mset of (F, A).*

Definition 2.7. *[5] The union of two soft msets (F, A) and (G, B) over U is the soft mset (H, C), where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}$, $\forall\ e \in A \cup B, \forall x \in U^*$. We write $(F, A)\widetilde{\cup}(G, B)$.*

Definition 2.8. *[5] The intersection of two soft msets (F, A) and (G, B) over U is the soft mset (H, C), where $C = A \cap B$and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}$, $\forall\ e \in A \cap B, \forall x \in U^*$. We write$(F, A)\widetilde{\cap}(G, B)$.*

Definition 2.9. *[5] A soft mset (F, A) over U is said to be a null soft mset denoted by $\widetilde{\phi}$ if for all $e \in A, F(e) = \phi$.*

Definition 2.10. *[5] A soft mset (F, A) over U is said to be an absolute soft mset denoted by $\widetilde{U}$ if for all $e \in A,\ F(e) = U$.*

Definition 2.11. *[5] The difference (H, E) between two soft msets (F, E) and (G, E) over U, denoted by $(F, E)\backslash(G, E)$, is defined as $H(e) = F(e)\backslash G(e)$ for all $e \in E$where $C_{H(e)}(x) = max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}$, $\forall x \in U^*$.*

Definition 2.12. *[5] The complement of a soft mset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to P^*(U)$ is mapping given by $F^c(e) = U\backslash F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x), \forall x \in U^*$.*

Definition 2.13. *[5] Let X be an universal mset and E be a set of parameters. Then, the collection of all soft msets over X with parameters from E is called a soft multi class and is denoted as $SMS(X)_E$.*

Definition 2.14. *[5] Let $\tau \subseteq SMS(X)_E$, then τ is said to be a soft multi topology on X if the following conditions hold:*
1. $\widetilde{\phi}$, $\widetilde{X}$ belong to τ,
2. The union of any number of soft msets in τ belongs to τ,
3. The intersection of any two soft msets in τ belongs to τ.
τ is called a soft multi topology over X and the triple (X, τ, E) is called a soft multi topological space over X. Also, the members of τ are said to be open soft msets in X. Furthermore, $OSM(X)_E$ is the set of all open sub soft msets of $\widetilde{X}$. A soft mset (F, E) in $SMS(X)_E$is said to be a closed soft mset in X, if its complement $(F, E)^c$ belongs to τ.

Definition 2.15. *[5] Let X be universal multiset, E be the set of parameters. Then:*
1. $\tau = \{\widetilde{\phi}, \widetilde{X}\}$ is called the indiscrete soft multi topology on X and (X, τ, E) is said to be an indiscrete soft multi space over X.
2. Let τ be the collection of all soft multisets over X. Then, τ is called the discrete soft multi topology on X and (X, τ, E) is said to be a discrete soft multi space over X.

Definition 2.16. *[4] Let (X, τ, E) be a soft multi topological space over X and (F, E) be a soft multiset over X. Then, the soft multi closure of (F, E), denoted by $cl(F, E)$ [or $\overline{(F, E)}$] is the intersection of all closed soft multiset containing (F, E).*

Definition 2.17. *[4] Let (X, τ, E) be a soft multi topological space over X and (F, E) be a soft multiset over X. Then, the soft multi interior of (F, E), denoted by $int(F, E)$ [or $(F, E)^o$] is the union of all open soft multiset contained in (F, E).*

Definition 2.18. *[4] Let (X, τ, E) be a soft multi topological space over X and (G, E) be a soft multiset over X and $x \in X$. Then, (G, E) is said to be a soft multi neighbourhood of x if there exists a soft multi open set (F, E) such that $x \in (F, E) \subseteq (G, E)$. The set of all soft multi neighbourhood of α, denoted by $\widetilde{N}(\alpha)$, is called the family of soft multi neighbourhoods of α,*
i.e. $\widetilde{N}(\alpha) = \{(G, E) : (G, E) \in T, \alpha \in (G, E)\}$.

Definition 2.19. *[7] Let (X, τ, E) be a soft multi topological space. A mapping $\gamma : SMS(X)_E \to SMS(X)_E$ is said to be an operation on $OSM(X)_E$, if $N_E \widetilde{\subseteq} \gamma(N_E)$ for all $N_E \in OSM(X)_E$. The family of all $\gamma-$open soft multisets is denoted by $OSM(\gamma) = \{N_E : N_E \widetilde{\subseteq} \gamma(N_E), N_E \in SMS(X)_E\}$. Also, the complement of $\gamma-$open soft multiset is called a $\gamma-$closed soft multiset and the set of all $\gamma-$closed soft multisets denoted by $CSM(\gamma)$.*

Definition 2.20. *[7] Let (X, τ, E) be a soft multi topological space. Different cases of $\gamma-$operations on $SMS(X)_E$ are as follows:*
(i) If $\gamma = int(cl)$, then γ is called a pre-open soft multi operator. The family of all pre-open soft multisets is denoted by $POSM(X)_E$ and the family of all pre-closed soft multisets is denoted by $PCSM(X)_E$.
(ii) If $\gamma = int(cl(int))$, then γ is called an $\alpha-$open soft multi operator. The family of all $\alpha-$open soft multisets is denoted by $\alpha OSM(X)_E$ and the family of all $\alpha-$closed soft multisets is denoted by $\alpha CSM(X)_E$.
(iii) If $\gamma = cl(int)$, then γ is called a semi open soft multi operator. The family of all semi open soft multisets is denoted by $SOSM(X)_E$ and the family of all semi closed soft multisets is denoted by $SCSM(X)_E$.
(iv) If $\gamma = cl(int(cl))$, then γ is called a $\beta-$open soft multi operator. The family of all $\beta-$open soft multisets is denoted by $\beta OSM(X)_E$ and the family of all $\beta-$closed soft multisets is denoted by $\beta CSM(X)_E$.

Definition 2.21. *[6] A soft multiset (A, E) in a soft multi topological space (X, τ, E) is said to be a generalized closed (for short, g−closed) soft multiset if $C_{cl(A)(e)}(x) \leq C_{B(e)}(x)$ whenever $C_{A(e)}(x) \leq C_{B(e)}(x)$ for all $x \in X^*$, $e \in E$ and (B, E) is open soft multiset in X.*

Definition 2.22. *[14] A sub soft mset S_E of (X, τ, E) is called clopen soft mset if it is both open soft mset and closed soft mset.*

Definition 2.23. *[14] Let (X, τ, E) be soft multi topological space. A soft multiset (F, A) is called compact if each soft multi open cover of (F, A) has a finite subcover. Also soft multi topological space (X, τ, E) is called compact if each soft multi open cover of $\widetilde{X}$ has a finite subcover.*

3 Generalized semi closed soft msets

Definition 3.1. *A soft mset S_E in a soft multi topological space (X, τ, E) is said to be a generalized semi closed soft multiset(briefly gscsmset)if $C_{scl(S)(e)}(x) \leq C_{U(e)}(x)$ whenever $C_{S(e)}(x) \leq C_{U(e)}(x)$ for all $x \in X^*$, $e \in E$ and $U_E \in OSM(X)_E$. The set of all gscs mset is denoted by $gscs(X)_E$.*

Example 3.2. *Let (X, τ, E) be a soft multi topological space with mset $X = \{1/x_1, 2/x_2, 1/x_3\}$, parameter set $E = \{e_1, e_2\}$ and topology $\tau = \{\widetilde{\phi}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ where $F_1(e_1) = \{1/x_1, 1/x_3\}, F_1(e_2) = \{1/x_1, 1/x_3\}, F_2(e_1) = \{1/x_1\}, F_2(e_2) = \{1/x_3\}, F_3(e_1) = \{2/x_2\}, F_3(e_2) = \{2/x_2\}, F_4(e_1) = \{1/x_1, 2/x_2\}, F_4(e_2) = \{2/x_2, 1/x_3\}$. Then a soft mset S_E such that $S(e_1) = \{1/x_1, 1/x_3\}$, $S(e_2) = \{1/x_1, 1/x_3\}$ is a gscsmset in (X, τ, E).*

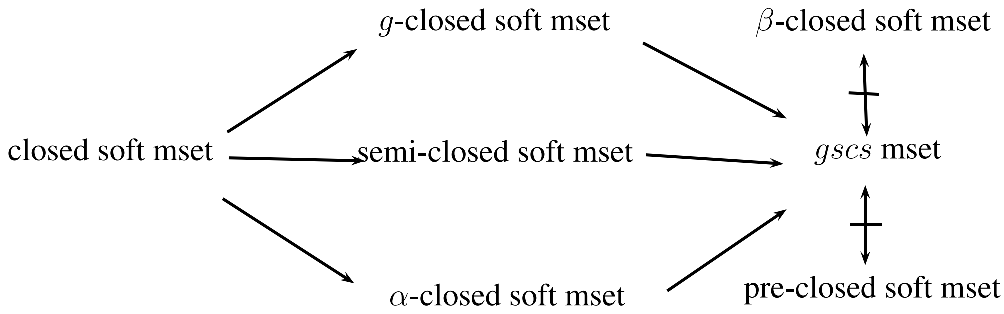
Proposition 3.3. *Every closed(resp. α-closed, semi-closed and g-closed) soft mset is a gscs mset.*

Proof. 1. Let S_E be a closed soft mset and $U_E \in OSM(X)_E$ such that $C_{S(e)}(x) \leq C_{U(e)}(x)$. Then $C_{scl(S)(e)}(x) \leq C_{cl(S)(e)}(x) = C_{S(e)}(x) \leq C_{U(e)}(x)$, by hypothesis and every closed soft mset is a semi-closed soft mset. Hence S_E is a *gscs* mset.
2. Let S_E be an α-closed soft mset and $U_E \in OSM(X)_E$ such that $C_{S(e)}(x) \leq C_{U(e)}(x)$. Since every α-closed soft mset is a semi-closed soft mset, we have $C_{scl(S)(e)}(x) \leq C_{\alpha cl(S)(e)}(x) = C_{S(e)}(x) \leq C_{U(e)}(x)$. Hence S_E is a *gscs* mset.
3. Let S_E be a semi-closed soft mset and $U_E \in OSM(X)_E$ such that $C_{S(e)}(x) \leq C_{U(e)}(x)$. Then we have $C_{scl(S)(e)}(x) = C_{S(e)}(x) \leq C_{U(e)}(x)$ because S_E is a semi-closed soft mset. Hence S_E is a *gscs* mset.
4. Let S_E be a g-closed soft mset and $U_E \in OSM(X)_E$ such that $C_{S(e)}(x) \leq C_{U(e)}(x)$. By assumption and every closed soft mset is a semi-closed soft mset, we have $C_{scl(S)(e)}(x) \leq C_{cl(S)(e)}(x) \leq C_{U(e)}(x)$. This implies S_E is a *gscs* mset.

Remark 3.4. *Converse of the above proposition need not be true. 1. In Example 3.2, the soft mset S_E with $S(e_1) = \{1/x_3\}$, $S(e_2) = \{1/x_3\}$ is a gscs mset but not a closed(resp. α-closed, semi-closed) soft mset in (X, τ, E).*
2. Let (X, τ, E) be a soft multi topological space with mset $X = \{2/x_1, 2/x_2, 1/x_3\}$, parameter set $E = \{e_1, e_2\}$ and topology $\tau = \{\widetilde{\phi}, \widetilde{X}, (F_1, E), (F_2, E)\}$ where $F_1(e_1) = \{2/x_1\}$, $F_1(e_2) = \{2/x_1\}$, $F_2(e_1) = \{2/x_1, 2/x_2\}$, $F_2(e_2) = \{2/x_1, 2/x_2\}$. Then a soft mset S_E such that $S(e_1) = \{2/x_2\}$, $S(e_2) = \{2/x_2\}$ is a gscs mset but not a g-closed soft mset.

Remark 3.5. *The following example shows that gscs msets are independent of pre-closed soft msets and β-closed soft msets.*

Example 3.6. *Let (X, τ, E) be a soft multi topological space with mset $X = \{2/x_1, 2/x_2, 2/x_3\}$, parameter set $E = \{e_1, e_2\}$ and topology $\tau = \{\widetilde{\phi}, \widetilde{X}, (F_1, E)\}$ where $F_1(e_1) = \{2/x_1\}$, $F_1(e_2) = \{2/x_1\}$. Let S_E be a sub soft mset of X_E such that $S(e_1) = \{2/x_1, 2/x_2\}$, $S(e_2) = \{2/x_1, 2/x_2\}$. Then S_E is a gscs mset but not pre-closed and β-closed soft mset in (X, τ, E). Let T_E be a sub soft mset of X_E such that $T(e_1) = \{1/x_1\}$, $T(e_2) = \{1/x_1\}$. Then T_E is pre-closed and β-closed soft mset but not a gscs mset in X_E.*



Remark 3.7. *1. The union of gscs msets need not be a gscs mset.*
2. The intersection of gscs msets need not be a gscs mset.

Example 3.8. *Let (X, τ, E) be a soft multi topological space with mset $X = \{2/x_1, 2/x_2, 2/x_3\}$, parameter set $E = \{e_1, e_2\}$ and topology $\tau = \{\widetilde{\phi}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ where $F_1(e_1) = \{2/x_2\}, F_1(e_2) = \{2/x_2\}$, $F_2(e_1) = \{2/x_3\}$, $F_2(e_2) = \{2/x_3\}$, $F_3(e_1) = \{2/x_2, 2/x_3\}$, $F_3(e_2) = \{2/x_2, 2/x_3\}$. Let S_E and T_E be two sub soft msets of X_E such that $S(e_1) = \{2/x_2\}$, $S(e_2) = \{2/x_2\}$ and $T(e_1) = \{2/x_3\}$, $T(e_2) = \{2/x_3\}$. Then S_E and T_E are gscs msets but $S_E \,\widetilde{\cup}\, T_E$is not a gscs mset in (X, τ, E).*

Example 3.9. *In Example 3.2, let S_E and T_E be two sub soft msets of X_E such that $S(e_1) = \{1/x_1, 2/x_2\}$, $S(e_2) = \{1/x_1\}$ and $T(e_1) = \{1/x_1, 1/x_2, 1/x_3\}, T(e_2) = \{1/x_2\}$. Then S_E and T_E are gscs msets but $S_E \,\widetilde{\cap}\, T_E$ is not a gscs mset in (X, τ, E).*

Proposition 3.10. *Every sub soft mset of X_E is gscs-mset iff $OSM(X)_E \subseteq \{F_E \subseteq X_E : F_E^c \in SOSM(X)_E\}$.*

Proof. Suppose that every sub soft mset of X_E is a *gscs*-mset. Let $U_E \in OSM(X)_E$. Since $C_{U(e)}(x) \leq C_{U(e)}(x)$ and U_E is a *gscs*-mset, $C_{scl(U)(e)}(x) \leq C_{U(e)}(x)$. Hence $U_E \in \{F_E \subseteq X_E : F_E^c \in SOSM(X)_E\}$. Conversely, assume that $OSM(X)_E \subseteq \{F_E \subseteq X_E : F_E^c \in SOSM(X)_E\}$. Let S_E be any sub soft mset of X_E such that $C_{S(e)}(x) \leq C_{U(e)}(x)$, where U_E is open soft mset. Thus U_E is semi-closed and so $C_{scl(S)(e)}(x) \leq C_{U(e)}(x)$. Hence S_E is a *gscs*-mset.

Remark 3.11. *Difference of two gscs-msets is not generally gscs-mset. In Example 3.2, let S_E and T_E be two sub soft msets of X_E such that $S(e_1) = \{1/x_1, 1/x_2, 1/x_3\}$, $S(e_2) = \{1/x_1\}$ and $T(e_1) = \{1/x_3\}, T(e_2) = \{1/x_3\}$. Then S_E and T_E are gscs msets but $S_E \setminus T_E$ is not a gscs mset in (X, τ, E).*

Definition 3.12. *Let (X, τ, E) be a soft multi topological space and S_E be a sub soft mset of X_E. Then S_E is a nowhere dense soft mset if and only if $C_{int(cl(S))(e)}(x) = C_{\phi(e)}(x)$ for all $x \in X^*,\ e \in E$.*

Definition 3.13. *Let f_E be a soft mset over X_E. f_E is called a soft multi point over X, if there exists $e \in E$ and $n/x \in X, 1 \leq n \leq m$ such that*

$$f(\epsilon) = \begin{cases} \{n/x\} & \text{if } \epsilon = e, 1 \leq n \leq m \\ \phi & \text{if } \epsilon \in E - \{e\} \end{cases}$$

We denote f_E by $[(n/x)_e]_E$. In this case, x is called support point of $[(n/x)_e]_E$, $\{x\}$ is called support set of $[(n/x)_e]_E$ and e is called the expressive parameter of $[(n/x)_e]_E$. The family of all soft multi points over X is denoted by $P(X, E)$ or P.

i.e. $P(X, E) = \{[(n/x_i)_{e_j}]_E : x_i \in X, e_j \in E, 1 \leq n \leq m\}.$

Definition 3.14. *A sub soft mset (F, E) which is soft multi point is said to be singleton soft mset and is denoted by $(F, E)_e^{n/x}$, for all $e \in E,\ x \in X^*$ and $1 \leq n \leq m$.*

Lemma 3.15. *Let $[(n/x)_e]_E \in P(X, E)$. Then $(F, E)_e^{n/x}$ is either nowhere dense soft mset or pre-open soft mset.*

Proof. Suppose that $(F, E)_e^{n/x}$ is not a nowhere dense soft mset. Then $C_{int(cl(\{n/x\}))(e)}(x) \neq C_{\phi(e)}(x)$ and so $[(n/x)_e]_E \in C_{int(cl(\{n/x\}))(e)}(x)$. This implies that $C_{\{n/x\}(e)}(x) \leq C_{int(cl(\{n/x\}))(e)}(x)$. Hence $(F, E)_e^{n/x}$ is a pre-open soft mset.

Remark 3.16. *In the notion of Lemma 3.15, we may consider the following decomposition of a given soft multi topological space (X, τ, E), namely, $P = P_1 \tilde{\cup} P_2$ where $P_1 = \{[(n/x)_e]_E \in P(X, E) : (F, E)_e^{n/x}$ is nowhere dense soft mset$\}$ and $P_2 = \{[(n/x)_e]_E \in P(X, E) : (F, E)_e^{n/x}$ is pre-open soft mset$\}$.*

Definition 3.17. *The intersection of all open soft msets of (X, τ, E) containing S_E is called soft multi kernel of S_E and is denoted by $smker(S)_E$.*

Theorem 3.18. *A soft mset S_E of X_E is a gscs mset if and only if $C_{scl(S)(e)}(x) \leq C_{smker(S)(e)}(x)$.*

Proof. Assume that S_E is a *gscs* mset. Let $[(n/x)_e]_E \in C_{scl(S)(e)}(x)$. If $[(n/x)_e]_E \notin C_{smker(S)(e)}(x)$, then there is an open soft mset U_E containing S_E such that $[(n/x)_e]_E \notin C_{U(e)}(x)$. Then we have $C_{scl(S)(e)}(x) \leq C_{U(e)}(x)$, by hypothesis. Since U_E is an open soft mset containing S_E, we have $[(n/x)_e]_E \notin C_{scl(S)(e)}(x)$, which is a contradiction. Therefore $C_{scl(S)(e)}(x) \leq C_{smker(S)(e)}(x)$.

Conversely, let $C_{scl(S)(e)}(x) \leq C_{smker(S)(e)}(x)$ and U_E be an open soft mset containing S_E. Then $C_{scl(S)(e)}(x) \leq C_{smker(S)(e)}(x) \leq C_{U(e)}(x)$. Hence S_E is a *gscs* mset.

Proposition 3.19. *For any soft mset S_E of X_E, $C_{(P_2 \tilde{\cap} scl(S))(e)}(x) \leq C_{smker(S)(e)}(x)$.*

Proof. Let $[(n/x)_e]_E \in C_{(P_2 \tilde{\cap} scl(S))(e)}(x)$ and suppose that $[(n/x)_e]_E \notin C_{smker(S)(e)}(x)$. Then there is an open soft mset U_E containing S_E such that $[(n/x)_e]_E \notin C_{U(e)}(x)$. If $C_{A(e)}(x) = C_{(X-U)(e)}(x)$, then A_E is a closed soft mset and so semi-closed soft mset. That is $C_{scl(\{n/x\})(e)}(x) = C_{(\{n/x\} \tilde{\cup} int(cl(\{n/x\})))(e)}(x) \leq C_{A(e)}(x)$. Since $C_{cl(\{n/x\})(e)}(x) \leq C_{scl(S)(e)}(x) \leq C_{cl(S)(e)}(x)$, we have $C_{int(cl(\{n/x\}))(e)}(x) \leq C_{S \tilde{\cup} int(cl(S))(e)}(x)$. Also $[(n/x)_e]_E \in C_{P_2(e)}(x)$, we have $[(n/x)_e]_E \notin C_{P_1(e)}(x)$ and so $C_{int(cl(\{n/x\}))(e)}(x) \neq C_{\phi(e)}(x)$. Therefore there must be some point $[(n/y)_e]_E \in C_{(S \tilde{\cap} int(cl(\{n/x\})))(e)}(x)$ and hence $[(n/y)_e]_E \in C_{(A \tilde{\cap} S)(e)}(x)$, which is a contradiction. That is $C_{(P_2 \tilde{\cap} scl(S))(e)}(x) \leq C_{smker(S)(e)}(x)$.

Proposition 3.20. *If $C_{(P_1 \tilde{\cap} scl(S))(e)}(x) \leq C_{S(e)}(x)$ then S_E is a gscs mset.*

Proof. Given that $C_{(P_1 \tilde{\cap} scl(S))(e)}(x) \leq C_{S(e)}(x)$. Then $C_{(P_1 \tilde{\cap} scl(S))(e)}(x) \leq C_{smker(S)(e)}(x)$, since $C_{S(e)}(x) \leq C_{smker(S)(e)}(x)$. Now, $C_{scl(S)(e)}(x) = C_{(P \tilde{\cap} scl(S))(e)}(x) = C_{((P_1 \tilde{\cup} P_2) \tilde{\cap} scl(S))(e)}(x) = C_{((P_1 \tilde{\cap} scl(S)) \tilde{\cup} (P_2 \tilde{\cap} scl(S)))(e)}(x) \leq C_{smker(S)(e)}(x)$, since $C_{(P_1 \tilde{\cap} scl(S))(e)}(x) \leq C_{smker(S)(e)}(x)$ and by Proposition 3.19. Thus S_E is a *gscs* mset.

Proposition 3.21. *Let S_E be a gscs mset. Then $C_{(scl(S)-S)(e)}(x)$ does not contain any nonempty closed soft mset.*

Proof. Suppose that F_E is a closed soft mset contained in $C_{(scl(S)-S)(e)}(x)$. Then $C_{S(e)}(x) \leq C_{F^c(e)}(x)$. Since F_E^c is an open soft mset and S_E is a *gscs* mset $C_{scl(S)(e)}(x) \leq C_{F^c(e)}(x)$. Consequently, $C_{F(e)}(x) \leq C_{(scl(S))^c(e)}(x)$, we have $C_{F(e)}(x) \leq C_{scl(S)(e)}(x)$. Thus $C_{F(e)}(x) \leq C_{(scl(S) \cap (scl(S))^c)(e)}(x) = C_{\phi(e)}(x)$ and hence F_E is empty.

Remark 3.22. *The converse of the above proposition need not be true. In Example 3.2, let S_E be a sub soft mset of X_E such that $S(e_1) = \{1/x_1\}$, $S(e_2) = \{1/x_2\}$. Then $C_{(scl(S)-S)(e)}(x)$ does not contain any nonempty closed soft mset but S_E is not a gscs mset.*

Corollary 3.23. *Let S_E be a gscs mset in X_E. Then S_E is a semi closed soft mset if and only if $C_{(scl(S)-S)(e)}(x)$ is a closed soft mset.*

Proof. Let S_E be a semi-closed soft mset. Then $C_{scl(S)}(x) = C_{S(e)}(x)$ and so $C_{(scl(S)-S)(e)}(x) = C_{\phi(e)}(x)$ which is a closed soft mset by Proposition 3.21. Conversely, suppose that $C_{(scl(S)-S)(e)}(x)$ is a closed soft mset. Then $C_{(scl(S)-S)(e)}(x) = C_{\phi(e)}(x)$, since S_E is a *gscs* mset. That is $C_{scl(S)(e)}(x) = C_{S(e)}(x)$ and so S_E is a semi-closed soft mset.

Proposition 3.24. *If S_E is a gscs mset then for each $[(n/x)_e]_E \in C_{scl(S)}(x)$, $C_{(cl(\{m/x\})\tilde{\cap}S)(e)}(x) \neq C_{\phi(e)}(x)$.*

Proof. Suppose $[(n/x)_e]_E \in C_{scl(S)(e)}(x)$ and $C_{(cl(\{n/x\})\tilde{\cap}S)(e)}(x) = C_{\phi(e)}(x)$. Then $C_{S(e)}(x) \leq C_{(cl(\{n/x\}))^c(e)}(x)$ and $(cl((F,E)_e^{n/x}))^c$ is an open soft mset. By assumption $C_{scl(S)(e)}(x) \leq C_{(cl(\{n/x\}))^c(e)}(x)$, which is a contradiction to $[(n/x)_e]_E \in C_{scl(S)(e)}(x)$. Therefore $C_{(cl(\{n/x\})\tilde{\cap}S)(e)}(x) \neq C_{\phi(e)}(x)$.

Proposition 3.25. *If $C_{(cl(\{n/x\})\tilde{\cap}S)(e)}(x) \neq C_{\phi(e)}(x)$ for each $[(n/x)_e]_E \in C_{scl(S)(e)}(x)$, then $C_{(scl(S)-S)(e)}(x)$ does not contain any nonempty closed soft mset.*

Proof. Let $C_{A(e)}(x) \leq C_{(scl(S)-S)(e)}(x)$ such that A_E be a closed soft mset. If there is an element $[(n/x)_e]_E \in C_{A(e)}(x)$, then $[(n/x)_e]_E \in C_{scl(S)}(x)$ and so $C_{(cl(\{n/x\})\tilde{\cap}S)(e)}(x) \leq C_{(A\tilde{\cap}S)(e)}(x) \leq C_{((scl(S)-S)\tilde{\cap}S)(e)}(x) = C_{\phi(e)}(x)$, which is a contradiction. Therefore $C_{A(e)}(x) = C_{\phi(e)}(x)$.

Proposition 3.26. *If S_E is a gscs mset of X_E such that $C_{S(e)}(x) \leq C_{T(e)}(x) \leq C_{scl(S)(e)}(x)$, then T_E is also a gscs mset of X_E.*

Proof. Let $U_E \in OSM(X)_E$ such that $C_{T(e)}(x) \leq C_{U(e)}(x)$. Then $C_{S(e)}(x) \leq C_{U(e)}(x)$. Since S_E is a *gscs* mset, we have $C_{scl(S)(e)}(x) \leq C_{U(e)}(x)$. Now $C_{scl(T)(e)}(x) \leq C_{scl(scl(S))(e)}(x) = C_{scl(S)(e)}(x) \leq C_{U(e)}(x)$. Therefore T_E is also a *gscs* mset of X_E.

Example 3.27. *Converse of the above proposition need not be true. In Example 3.2, let S_E and T_E be a gscs mset of X_E such that $S(e_1) = \{1/x_3\}$, $S(e_2) = \{1/x_1\}$ and $T(e_1) = \{1/x_1, 2/x_2\}, T(e_2) = \{1/x_1\}$. Then $C_{T(e)}(x) \nleq C_{scl(S)(e)}(x)$. Thus any gscs mset of X_E need not be lie between gscs mset and its semi-closure.*

Proposition 3.28. *For each $[(n/x)_e]_E \in X_E$, either $(F,E)_e^{n/x}$ is a closed soft mset or $((F,E)_e^{n/x})^c$ is a gscs mset.*

Proof. Suppose that $(F,E)_e^{n/x}$ is not a closed soft mset. Then $((F,E)_e^{n/x})^c$ is not an open soft mset and the only open soft mset containing $((F,E)_e^{n/x})^c$ is $\widetilde{X}$ itself. Therefore $C_{scl(\{n/x\}^c)(e)}(x) \leq C_{X(e)}(x)$ and so $((F,E)_e^{n/x})^c$ is a *gscs* mset.

Definition 3.29. *A soft multi topological space (X,τ,E) is said to be soft multi partition space if every sub soft mset of (X,τ,E) is a clopen soft mset.*

Proposition 3.30. *If (X,τ,E) is a soft multi partition space then every sub soft mset is a gscs mset.*

Proof. Let $U_E \in OSM(X)_E$ and S_E be any sub soft mset of (X, τ, E) such that $C_{S(e)}(x) \leq C_{U(e)}(x)$. Since U_E is a closed soft mset in a soft multi partition space (X, τ, E) and so $C_{scl(S)(e)}(x) \leq C_{scl(U)(e)}(x) \leq C_{cl(U)(e)}(x) = C_{U(e)}(x)$. Therefore S_E is a *gscs* mset.

Definition 3.31. *A soft multi topological space (X, τ, E) is said to be soft multi regular space iff for every closed sub soft mset F_E of X_E and every soft multi point $[(n/x)_e]_E \notin F_E$, there exists open soft msets G_E and H_E such that $C_{F(e)}(x) \leq C_{G(e)}(x)$, $[(n/x)_e]_E \in H_E$, and $C_{(G\tilde{\cap}H)(e)}(x) = C_{\phi(e)}(x)$.*

Lemma 3.32. *A soft multi topological space (X, τ, E) is soft multi regular iff for each $[(n/x)_e]_E \in X_E$ and each soft multi neighbourhood $\widetilde{N}_E^{n/x}$ of $[(n/x)_e]_E$ there exists $\widetilde{M}_E^{n/x}$ of $[(n/x)_e]_E$ such that $C_{cl(\widetilde{M})(e)}(x) \leq C_{(\widetilde{N})(e)}(x)$.*

Proof. Let (X, τ, E) be a soft multi regular space. Let $[(n/x)_e]_E \in X_E$ and let $\widetilde{N}_E^{n/x}$ be a soft multi neighbourhood of $[(n/x)_e]_E$. Then there exists an open soft mset G_E such that $[(n/x)_e]_E \in C_{G(e)}(x) \leq C_{\widetilde{N}(e)}(x)$. Consequently, G_E^c is a closed soft mset and $[(n/x)_e]_E \notin G_E^c$. So, by regularity of (X, τ, E), there exists open soft msets S_E and T_E such that $C_{G^c(e)}(x) \leq C_{S(e)}(x)$, $[(n/x)_e]_E \in T_E$ and $C_{(S \cap T)(e)}(x) = C_{\phi(e)}(x)$. Now, $C_{G^c(e)}(x) \leq C_{S(e)}(x)$ implies $C_{T(e)}(x) \leq C_{S^c(e)}(x)$ and so $C_{cl(T)(e)}(x) \leq C_{cl(S^c)(e)}(x) = C_{S^c(e)}(x)$. Thus $\widetilde{M}_E^{n/x} = T_E$ is a soft multi neighbourhood of $[(n/x)_e]_E$ such that $C_{cl(M)(e)}(x) \leq C_{cl(T)(e)}(x) \leq C_{S^c(e)}(x) \leq C_{G(e)}(x) \leq C_{N(e)}(x)$. Conversely, let every soft multi neighbourhood of any soft multi point of X_E contain the closure of another soft multi neighbourhood of that soft multi point. Let F_E be any closed sub soft mset of X_E and let $[(n/x)_e]_E \notin F_E$. Then F_E^c is open soft multi neighbourhood of $[(n/x)_e]_E$. so, by given hypothesis there exists an open soft multi neighbourhood G_E of $[(n/x)_e]_E$ such that $[(n/x)_e]_E \in C_{cl(G)(e)}(x) \leq C_{F^c(e)}(x)$. But $C_{cl(G)(e)}(x) \leq C_{F^c(e)}(x)$ implies $C_{F(e)}(x) \leq C_{(cl(G))^c(e)}(x)$. Thus G_E and $(cl(G_E))^c$ are open soft msets such that $C_{F(e)}(x) \leq C_{(cl(G))^c(e)}(x)$, $[(n/x)_e]_E \in G_E$ and $C_{((cl(G))^c \cap G)(e)}(x) = C_{\phi(e)}(x)$. Hence (X, τ, E) is a soft multi regular space.

Lemma 3.33. *Let (X, τ, E) be a soft multi regular space and let A_E be a compact sub soft mset of X_E. If G_E is an open sub soft mset of X_E containing A_E, then there exists a closed soft mset F_E such that $C_{A(e)}(x) \leq C_{F(e)}(x) \leq C_{G(e)}(x)$.*

Proof. Since G_E is an open soft mset containing A_E, it follows that G_E is an open nbd. of each soft multi point of A_E by [14]. Also by regularity of X_E and above lemma, for each $[(n/x)_e]_E \in A_E$, there is an open neighbourhood $\widetilde{N}_E^{n/x}$ of $[(n/x)_e]_E$ such that $C_{cl(\widetilde{N})(e)}(x) \leq C_{G(e)}(x)$. It is clear that the collection $\{\widetilde{N}_E^{n/x} : [(n/x)_e]_E \in A_E\}$ is an open covering of A_E. Since A_E is compact soft mset, there exists finitely many points $[(n_1/x_1)_e]_E, [(n_2/x_2)_e]_E, ...[(n_n/x_n)_e]_E$ of A_E such that $C_{A(e)}(x) \leq C_{\cup_{i=1}^{n}\widetilde{N}(e)}(x)$ and $C_{cl(\widetilde{N})(e)}(x) \leq C_{G(e)}(x)$ for each i. Let $C_{F(e)}(x) = C_{cl(\widetilde{N})(e)}(x)$. Then F_E is closed, being a finite union of closed sets. Also, $C_{cl(\widetilde{N})(e)}(x) \leq C_{G(e)}(x)$ for each i implies that $C_{\cup_{i=1}^{n}\widetilde{N}(e)}(x) \leq C_{G(e)}(x)$. Hence $C_{F(e)}(x) \leq C_{G(e)}(x)$. Moreover, $C_{(\widetilde{N})(e)}(x) \leq C_{cl(\widetilde{N})(e)}(x)$ implies that $C_{\cup_{i=1}^{n}\widetilde{N}(e)}(x) \leq C_{cl(\widetilde{N})(e)}(x)$. Hence $C_{A(e)}(x) \leq C_{F(e)}(x)$.

Proposition 3.34. *If S_E is compact sub soft mset of a soft multi regular space (X, τ, E), then S_E is a gscs mset.*

Proof. Let $C_{S(e)}(x) \leq C_{U(e)}(x)$, and $U_E \in OSM(X)_E$. Since S_E is compact sub soft mset of a soft multi regular space (X, τ, E),by above lemma, there exists a closed soft mset V_E such that $C_{A(e)}(x) \leq C_{V(e)}(x) = C_{cl(V)(e)}(x) \leq C_{U(e)}(x)$ and so $C_{scl(S)(e)}(x) \leq C_{U(e)}(x)$ for all $x \in X^*$, $e \in E$. Therefore S_E is a *gscs* mset.

References

[1] Babitha K. V. and Sunil Jocob John, On soft multisets, Ann. fuzzy Math. Inform., 2013, 5(1), 35-44.

[2] Blizard D., Multiset theory, Notre Dame Journal of logic,1989, 30, 36-65.

[3] Cerf V., Fernandez E., Gostelow K. and Volausky S., Formal control and low properties of a model of computation, Report ENG 7178, computer Science Department, University of California, LOs Angeles, CA, 1971, p.81.

[4] Deniz Tokat and Ismail Osmanoglu, On the soft multi topology and it's some properties, Nevsehir Universitesi Fen Bilimleri Enstitusu Dergisi Cilt, 2013, 2, 109-118.

[5] Deniz Tokat and Ismail Osmanoglu, Connectedness on soft multi topological spaces, Journal of New Results in Science, 2013, 2, 8-18.

[6] El-Sheikh S. A., Omar R. A-K. and Raafat M., Generalized closed soft multiset in soft multi topological spaces, Asian Journal of Mathematics and computer research, 2015, 9, 302-311.

[7] El-Sheikh S. A., Omar R. A-K. and Raafat M., Some types of open soft multisets and some types of mappings in soft multi topolgical spaces, Annals of fuzzy Mathematics and Informatics, 2016, 12(1), 15-30.

[8] Girish K. P. and Sunil Jacob John, Relations and functions in multiset context, Information Sciences, 2009, 179, 758-768.

[9] Girish K. P. and Sunil Jacob John, General relation between partially ordered multisets and their chains and antichains, Mathematical Communication, 2009, 14(2), 193-205.

[10] Girish K. P. and Sunil Jacob John, Multiset topologies induced by multiset relations, Information Sciences, 2012, 188, 298-313.

[11] Girish K. P. and Sunil Jacob John, On Multiset Topologies, Theory and Applications of Mathematics and Computer Science, 2012, 2(1), 37-52.

[12] Jena S. P., Ghosh S. K. and Tripathy B. K., On theory of bags and lists, Information Sciences, 2001, 132, 241-254.

[13] Molodtsov D., Soft set theory - first results, Comput. Math. Appl., 1999, 37(4-5) 19-31 .

[14] Muhammad Riaz, Naim Cagman, Nabeela Wali and Amna Mushtaq, Certain properties of soft multiset topology with applications in multi-criteria decision making, Decision Making: Applications in Management and Engineering, 3(2), 70-96, 2020.

[15] Yager R. R., On the theory of bags, International Journal General System, 1986, 13, 23-37.

BIOGRAPHY

V. Inthumathi received Doctoral degree in the field of Ideal topological spaces from Bharathiar University in 2012. She is working as an Associate Professor in Department of Mathematics in Nallamuthu Gounder Mahalingam college, Pollachi, India. She has 24 years of teaching experience. she has published about 45 research articles in reputed journals. She guided 23 M.Phil scholars and guiding 4 Ph.D. scholars. Currently she is doing research in the area of soft topological spaces and Nano ideal topological spaces.

A. Gnanasoundari doing Ph.D. degree in the field of multiset topological spaces under the guidance of Dr. V. Inthumathi, Associate professor and Head, PG and Research Department of Mathematics, Nallamuthu Gounder Mahalingam college, Pollachi, India. She has presented 4 articles in international conference and published one research article in reputed journal.

Dr. M. Maheswari is working as an Assistant Professor in the Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi. She has got her Doctoral degree in the field of Ideal bitopological spaces from Bharathiar University, Coimbatore. She has 6 years experience in teaching. She published 8 research papers in reputed journals. Her areas of interest are Algebra, Complex Analysis and Statistics.