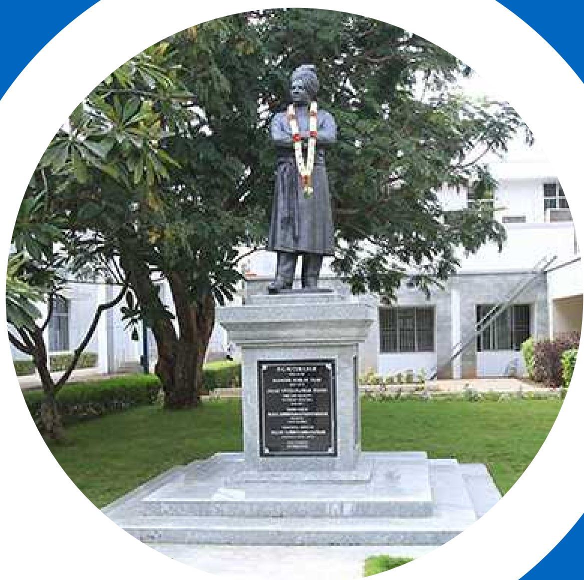
VOLUME XI
ISBN No.: 978-93-94004-00-9
Physical Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

Pollachi-642001



SUPPORTED BY

Riyasaa Labs

Sakthi as pure as mother's love

ABT Industries Ltd.,
Dairy Division

PROCEEDING

One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, Pollachi-642001.

Estd. 1957

Proceeding of the

One day International Conference on

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

ISBN No: 978-93-94004-00-9

978- 93- 94004- 00- 9

Nallamuthu Gounder Mahalingam College

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

www.ngmc.org

ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

International Conference on Emerging Trends in Science and Technology (ETIST 2021) Jointly Organized by Department of Biological Science, Physical Science and Computational Science Nallamuthu Gounder Mahalingam College, Affiliated to Bharathiar University, Tamilnadu, India.
Published by NGMC - November 2021

Generalized Regular Closed Sets in Soft Multi Topological Spaces

V. Inthumathi[1],**A. Gnanasoundari**[2] and **M. Maheswari**[3]

Abstract: This paper introduces regular open soft multisets and regular closed soft multisets. The notions of interior and closure are generalized using these sets. And also we introduce and study generalized regular closed sets and its properties in soft multi topological spaces.

Keywords : Soft Multiset, Soft Multi topology, Regular closed soft multiset, Generalized regular closed soft multiset.
2010 Subject classification: 54A05, 54B05, 54C05, 54D20

1 Introduction

In classical set theory, there is no repetition of the set members. However, in some cases, repetition of element of set may helphful, This set is called a multiset which is a collection of objects in which repetition of elements is significant. Multiset theory was introduced by Cerf et al. [2] in 1971.Then Yager [11] initiated further contributions to it. Girish and John [9] introduced the concept of mutiset topology. Molodtsov [10] introduced the concept of soft set theory as a Mathematical tool for dealing with uncertainities in 1999. In 2013, Babitha and John [1] was introduced the concept of soft multisets as a combination of soft sets and multisets. Moreover, in [3] [4] the soft multi topology and its basic properties was given. S. A. El-Sheikh et al.[6] introduced the notions of Some types of open soft multisets and some types of mappings in soft multi topological spaces. The same author [5] introduced the concept of generalized closed soft multisets in soft multi topological spaces. In this paper, we introduced the concept of regular closed soft mset and generalized regular closed soft mset in soft multi topological spaces and discuss some important properties in detail.

2 Preliminaries

Definition 2.1. *[8] An mset M drawn from the set X is represented by a function Count M or C_M defined as $C_M : X \to N$ where N represents the set of non negative integers. The word 'multiset' often shortened to 'mset'.*

[1]Associate Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Coimbatore, Tamilnadu, India.E.mail: inthumathi65@gmail.com
[2]Assistant Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Coimbatore, Tamilnadu, India.E.mail: gnana.ngm@gmail.com
[3]Assistant Professor, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Coimbatore, Tamilnadu, India.E.mail: mahimrk@gmail.com

Definition 2.2. *[8] A domain X, is defined as a set of elements from which msets are constructed. The mset space $[X]^m$ is the set of all msets whose elements are in X such that no element in the mset occurs more than m times.*
If $X = \{x_1, x_2, ...x_k\}$ then $[X]^m = \{\{m_1/x_1, m_2/x_2, ...m_k/x_k\}$ for $i = 1, 2, 3...k$; $m_i \in \{0, 1, 2, ...m\}\}$. Henceforth M stands for a mset drawn from the mset space $[X]^m$.

Definition 2.3. *[10] Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$*

Definition 2.4. *[4] Let U be an universal multiset, E be a set of parameters and $A \subseteq E$. Then, an ordered pair (F, A) is called a soft multiset where F is a mapping given by $F : A \to P^*(U); P^*(U)$ is the power set of a mset U. For all $e \in A$, F(e) multiset represents by count function $C_{F(e)} : U^* \to N$ where N represents the set of non-negative integers and U^* represents the support set of U.*

Let $U = \{2/x, 3/y, 2/z\}$ be a multiset. Then, the support set of U is $U^* = \{x, y, z\}$.

Definition 2.5. *[4] For two soft multisets (F, A) and (G, B) over U, we say that (F, A) is a sub soft multiset of (G, B) if:*
1. $A \subseteq B$.
2.$C_{F(e)}(x) \leq C_{G(e)}(x), \forall\ x \in U^*, \forall e \in A \cap B$.
We write$(F, A)\widetilde{\subseteq}(G, B)$.

Definition 2.6. *[4] The union of two soft multisets (F, A) and (G, B) over U is the soft multiset (H, C), where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}$, $\forall\ e \in A \cup B, \forall x \in U^*$. We write $(F, A)\widetilde{\cup}(G, B)$.*

Definition 2.7. *[4] The intersection of two soft multisets (F, A) and (G, B) over U is the soft multiset (H, C), where $C = A \cap B$and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}$, $\forall\ e \in A \cap B, \forall x \in U^*$. We write$(F, A)\widetilde{\cap}(G, B)$.*

Definition 2.8. *[4] A soft multiset (F, A) over U is said to be a null soft multiset denoted by $\widetilde{\phi}$ if for all $e \in A, F(e) = \phi$.*

Definition 2.9. *[4] A soft multiset (F, A) over U is said to be an absolute soft multiset denoted by $\widetilde{U}$ if for all $e \in A$, $F(e) = U$.*

Definition 2.10. *[4] The complement of a soft multiset (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to P^*(U)$ is mapping given by $F^c(e) = U \backslash F(e)$ for all $e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x), \forall x \in U^*$*

Definition 2.11. *[4] Let X be an universal multiset and E be a set of parameters. Then, the collection of all soft multisets over X with parameters from E is called a soft multi class and is denoted as $SMS(X)_E$.*

Definition 2.12. *[4] Let $\tau \subseteq SMS(X)_E$, then τ is said to be a soft multi topology on X if the following conditions hold:*
1. $\widetilde{\phi}$, $\widetilde{X}$ belong to τ .

2. The union of any number of soft multisets in τ belongs to τ .
3. The intersection of any two soft multisets in τ belongs to τ .
τ is called a soft multi topology over X and the triple (X, τ, E) is called a soft multi topological space over X. Also, the members of τ are said to be open soft multisets in X.
A soft multiset (F, E) in $SMS(X)_E$is said to be a closed soft multiset in X, if its complement $(F, E)^c$ belongs to τ.

Definition 2.13. *[4] Let X be universal mset, E be the set of parameters. Then:*
1. $\tau = \{\widetilde{\phi}, \widetilde{X}\}$ is called the indiscrete soft multi topology on X and (X, τ, E) is said to be an indiscrete soft multi space over X.
2. Let τ be the collection of all soft multisets over X. Then, τ is called the discrete soft multi topology on X and (X, τ, E) is said to be a discrete soft multi space over X.

Definition 2.14. *[4] Let (X, τ, E) be a soft multi topological space over X and (F, E) be a soft multiset over X. Then, the soft multi closure of (F, E), denoted by cl(F, E) [or $\overline{(F, E)}$] is the intersection of all closed soft multiset containing (F, E).*

Definition 2.15. *[4] Let (X, τ, E) be a soft multi topological space over X and (F, E) be a soft multiset over X. Then, the soft multi interior of (F, E), denoted by int(F, E) [or $(F, E)^o$] is the union of all open soft multiset contained in (F, E).*

Definition 2.16. *[7] Let f_E be a soft mset over X_E. f_E is called a soft multi point over X, if there exists $e \in E$ and $n/x \in X, 1 \leq n \leq m$ such that*

$$f(\epsilon) = \begin{cases} \{n/x\} & if\ \epsilon = e, 1 \leq n \leq m \\ \phi & if\ \epsilon \in E - \{e\} \end{cases}$$

We denote f_E by $[(n/x)_e]_E$. In this case, x is called support point of $[(n/x)_e]_E$, $\{x\}$ is called support set of $[(n/x)_e]_E$ and e is called the expressive parameter of $[(n/x)_e]_E$. The family of all soft multi points over X is denoted by $P(X, E)$ or P.
i.e. $P(X, E) = \{[(n/x_i)_{e_j}]_E : x_i \in X, e_j \in E, 1 \leq n \leq m\}$.

Definition 2.17. *[6] Let (X, τ, E) be a soft multi topological space. A mapping $\gamma : SMS(X)_E \rightarrow SMS(X)_E$ is said to be an operation on $OSM(X)_E$, $if N_E \widetilde{\subseteq} \gamma(N_E)$ for all $N_E \in OSM(X)_E$. The family of all $\gamma-$open soft multisets is denoted by $OSM(\gamma) = \{N_E : N_E \widetilde{\subseteq} \gamma(N_E), N_E \in SMS(X)_E\}$.Also, the complement of $\gamma-$open soft multiset is called a γ-closed soft multiset and the set of all γ-closed soft multisets denoted by CSM(γ).*

Definition 2.18. *[6] Let (X, τ, E) be a soft multi topological space. Different cases ofγ-operations on $SMS(X)_E$ are as follows:*
(i) If γ = int(cl), then γ is called a pre-open soft multi operator. The family of all pre-open soft multisets is denoted by $POSM(X)_E$ and the family of all pre-closed soft multisets is denoted by $PCSM(X)_E$.
(ii) If γ = int(cl(int)), then γ is called an α-open soft multi operator. The family of all α-open soft multisets is denoted by $\alpha OSM(X)_E$ and the family of all α-closed soft multisets is denoted by $\alpha CSM(X)_E$.
(iii) If γ = cl(int), then γ is called a semi open soft multi operator. The family of all semi open soft multisets is denoted by $SOSM(X)_E$ and the family of all semi closed soft multisets is denoted by $SCSM(X)_E$.

(iv) If $\gamma = cl(int(cl))$, then γ is called a β-open soft multi operator. The family of all β-open soft multisets is denoted by $\beta OSM(X)_E$ and the family of all β-closed soft multisets is denoted by $\beta CSM(X)_E$.

Definition 2.19. *[5] A soft multiset (A, E) in a soft multiset topological space (X, τ, E) is said to be a generalized closed (for short, g-closed) soft multiset if $C_{cl(A)(e)}(x) \leq C_{B(e)}(x) whenever C_{A(e)}(x) \leq C_{B(e)}(x)$ for all $x \in X^*, e \in E$ and (B, E) is open soft multiset in X.*

3 Regular Closed Soft Multisets

Definition 3.1. *A soft mset S_E in a soft multiset topological space (X, τ, E) is said to be a regular closed soft mset if $C_{cl(int(S))(e)}(x) = C_{S(e)}(x) \ \ \forall x \in X^*, \ e \in E$. The set of all regular closed soft mset is denoted by $RCSMS(X)_E$. The complement of regular closed soft multiset is regular open soft multiset. The set of all regular open soft mset is denoted by $ROSMS(X)_E$.*

Example 3.2. *Let $X = \{1/a, 2/b, 1/c\}$ be a mset, $E = \{e_1, e_2\}$ be a set of parameters and $\tau = \{\widetilde{\phi}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ be a soft multi topological space on (X, τ, E), where $F_1(e_1) = \{1/a, 1/c\}, F_1(e_2) = \{1/a, 1/c\}$, $F_2(e_1) = \{1/a\}, F_2(e_2) = \{1/c\}$, $F_3(e_1) = \{2/b\}, F_3(e_2) = \{2/b\}$, $F_4(e_1) = \{1/a, 2/b\}, F_4(e_2) = \{2/b, 1/c\}$. Let S_E be a sub soft mset of X_E such that $S(e_1) = \{1/a, 1/c\}, S(e_2) = \{1/a, 1/c\}$. Then S_E is a regular closed soft mset in (X, τ, E).*

Proposition 3.3. *Every regular closed soft mset is a closed(resp. α- closed, semi closed, pre closed, β-closed) soft mset.*

Proof: 1. Let S_E be a regular closed soft mset. Then we have $C_{cl(int(S))(e)}(x) = C_{S(e)}(x)$, $C_{cl(int(S))(e)}(x) = C_{cl(S)(e)}(x)$ implies that $C_{S(e)}(x) = C_{cl(S)(e)}(x)$. Hence S_E is a closed soft mset.
By (1) and since every closed soft mset is α- closed(resp. semi closed, pre closed and β-closed) soft mset, we will get other proofs.

Remark 3.4. *Converse of the above proposition need not be true as seen from the following example.*

Example 3.5. *In Example 3.2, let S_E be a sub soft mset of X_E such that $S(e_1) = \{2/b, 1/c\}, S(e_2) = \{1/a, 2/b\}$. Then S_E is closed, semi closed, pre closed, α closed and βclosed soft mset of (X, τ, E) but not an regular closed soft mset.*

Definition 3.6. *Let (X, τ, E) be a soft multi topological space and S_E be a sub soft mset of X_E. Then*
(i) The regular closure of S_E is denoted by $rcl(S_E)$ and it is defined as
$rcl(S_E) = \widetilde{\cap}\{G_E : S_E \widetilde{\subseteq} G_E, G_E \in RCSMS(X)_E\}$.
(ii)The regular interior of S_E is denoted by $rint(S_E)$ and it is defined as
$rint(S_E) = \widetilde{\cup}\{G_E : G_E \widetilde{\subseteq} S_E, G_E \in ROSMS(X)_E\}$

Theorem 3.7. *If S_E is sub soft mset of X_E then $C_{cl(S)(e)}(x) \leq C_{rcl(S)(e)}(x)$.*

Proof: Obvious from Theorem 3.3.

Theorem 3.8. *If S_E is regular closed soft mset then $C_{S(e)}(x) = C_{rcl(S)(e)}(x)$.*

Theorem 3.9. *Let (X, τ, E) be a soft multi topological space. Then, the following hold*
(i) $C_{rcl(\widetilde{X})(e)}(x) = C_{\widetilde{X}(e)}(x)$ *and* $C_{rcl(\widetilde{\phi})(e)}(x) = C_{\widetilde{\phi}(e)}(x)$.
(ii) $C_{S(e)}(x) \leq C_{rcl(S)(e)}(x)$.
(iii) If $C_{S(e)}(x) \leq C_{T(e)}(x)$ *then* $C_{rcl(S)(e)}(x) \leq C_{rcl(T)(e)}(x)$.
(iv)$C_{rcl(S\widetilde{\cap}T)(e)}(x) \leq C_{(rcl(S)\widetilde{\cap}rcl(T))(e)}(x)$.
(v)$C_{rcl(S\widetilde{\cup}T)(e)}(x) = C_{(rcl(S)\widetilde{\cup}rcl(T))(e)}(x)$.
(vi)$C_{rcl(rcl(S))(e)}(x) = C_{rcl(S)(e)}(x)$.

Proof: (i) We know that, if S_E is a regular closed soft mset, then the smallest regular closed soft mset containing S_E is itself. Therefore $C_{S(e)}(x) = C_{rcl(S)(e)}(x)$. By reason of $\widetilde{X}$ *and* $\widetilde{\phi}$ are regular closed then $C_{rcl(\widetilde{X})(e)}(x) = C_{\widetilde{X}(e)}(x)$ and $C_{rcl(\widetilde{\phi})(e)}(x) = C_{\widetilde{\phi}(e)}(x)$.
(ii) From Definition 3.6, $C_{S(e)}(x) \leq C_{rcl(S)(e)}(x)$.
(iii) Let $C_{S(e)}(x) \leq C_{T(e)}(x)$. Then $C_{S(e)}(x) \leq C_{T(e)}(x) \leq C_{rcl(T)(e)}(x)$. But $rcl(S_E)$ is the smallest regular closure of S_E. Therefore $C_{rcl(S)(e)}(x) \leq C_{rcl(T)(e)}(x)$.
(iv) Since $C_{(S\widetilde{\cap}T)(e)}(x) \leq C_{S(e)}(x)$ and $C_{(S\widetilde{\cap}T)(e)}(x) \leq C_{T(e)}(x)$, by (iii) $C_{rcl(S\widetilde{\cap}T)(e)}(x) \leq C_{rcl(S)(e)}(x)$ and $C_{rcl(S\widetilde{\cap}T)(e)}(x) \leq C_{rcl(T)(e)}(x)$. Therefore $C_{rcl(S\widetilde{\cap}T)(e)}(x) \leq C_{(rcl(S)\widetilde{\cap}rcl(T))(e)}(x)$.
(v) Since $C_{S(e)}(x) \leq C_{(S\widetilde{\cup}T)(e)}(x)$ and $C_{T(e)}(x) \leq C_{(S\widetilde{\cup}T)(e)}(x)$. Then $C_{rcl(S)(e)}(x) \leq C_{rcl(S\widetilde{\cup}T)(e)}(x)$ and $C_{rcl(T)(e)}(x) \leq C_{rcl(S\widetilde{\cup}T)(e)}(x)$. Therefore $C_{(rcl(S)\widetilde{\cup}rcl(T))(e)}(x) \leq C_{rcl(S\widetilde{\cup}T)(e)}(x)$. Let $[(n/x)_e]_E \in C_{rcl(S\widetilde{\cup}T)(e)}(x)$ and suppose that $[(n/x)_e]_E \notin C_{(rcl(S)\widetilde{\cup}rcl(T))(e)}(x)$. Then there exists an regular closed soft msets A_E and B_E with $C_{S(e)}(x) \leq C_{A(e)}(x)$, $C_{T(e)}(x) \leq C_{B(e)}(x)$ and $[(n/x)_e]_E \notin C_{(A\cup B)(e)}(x)$. Then $C_{(S\widetilde{\cup}T)(e)}(x) \leq C_{(A\widetilde{\cup}B)(e)}(x)$, $C_{(A\widetilde{\cup}B)(e)}(x)$ is a regular closed soft mset such that $[(n/x)_e]_E \notin C_{(A\widetilde{\cup}B)(e)}(x)$. Thus $[(n/x)_e]_E \notin C_{rcl(S\widetilde{\cup}T)(e)}(x)$ which is a contradiction. Therefore $[(n/x)_e]_E \in C_{rcl(S\widetilde{\cup}T)(e)}(x)$. Hence $C_{rcl(S\widetilde{\cup}T)(e)}(x) = C_{(rcl(S)\widetilde{\cup}rcl(T))(e)}(x)$.

(vi) Let $C_{S(e)}(x) \leq C_{F(e)}(x)$, F_E is a regular closed soft mset. Then $C_{rcl(S)(e)}(x) \leq C_{F(e)}(x)$,and $C_{rcl(rcl(S))(e)}(x) \leq C_{F(e)}(x)$. Since $C_{rcl(rcl(S))(e)}(x) \leq C_{F(e)}(x)$, $rcl(rcl(S_E)) \subseteq \widetilde{\cap}\{T_E : C_{S(e)}(x) \leq C_{T(e)}(x), T_E$ *is regular closed*$\} = rcl(S_E)$ and so $C_{rcl(rcl(S))(e)}(x) \leq C_{rcl(S)(e)}(x)$. But $C_{rcl(S)(e)}(x) \leq C_{rcl(rcl(S))(e)}(x)$. Therefore $C_{rcl(rcl(S))(e)}(x) = C_{rcl(S)(e)}(x)$.

Theorem 3.10. *Let (X, τ, E) be a soft multi topological space. Then, the following hold*
(i) $C_{rint(\widetilde{X})(e)}(x) = C_{\widetilde{X}(e)}(x)$ *and* $C_{rint(\widetilde{\phi})(e)}(x) = C_{\widetilde{\phi}(e)}(x)$.
(ii) $C_{S(e)}(x) \leq C_{rint(S)(e)}(x)$.
(iii) If $C_{S(e)}(x) \leq C_{T(e)}(x)$ *then* $C_{rint(S)(e)}(x) \leq C_{rint(T)(e)}(x)$.
(iv)$C_{rint(S\widetilde{\cap}T)(e)}(x) = C_{(rint(S)\widetilde{\cap}rint(T))(e)}(x)$.
(v)$C_{rint(S\widetilde{\cup}T)(e)}(x) \geq C_{(rint(S)\widetilde{\cup}rint(T))(e)}(x)$.
(vi)$C_{rint(rint(S))(e)}(x) = C_{rint(S)(e)}(x)$.

Proof: Proof is obvious.

Theorem 3.11. *Finite union of regular closed soft msets is a regular closed soft mset.*

Proof: Let S_E and T_E be any two regular closed soft msets. Then we have $C_{cl(int(S))(e)}(x) = C_{S(e)}(x)$ and $C_{cl(int(T))(e)}(x) = C_{T(e)}(x)$. Now, $C_{(S\widetilde{\cup}T)(e)}(x) = C_{cl(int(S))\widetilde{\cup}cl(int(T))(e)}(x) = C_{cl(int(S)\widetilde{\cup}int(T))(e)}(x) \leq C_{cl(int(S\widetilde{\cup}T))(e)}(x)$. Since S_E and T_E are regular closed soft msets and so they are closed soft msets, We have $C_{(S\widetilde{\cup}T)(e)}(x) = C_{cl(int((S\widetilde{\cup}T)))(e)}(x)$.

Remark 3.12. *Finite intersection of regular closed soft msets is not a regular closed soft mset.*

Example 3.13. *In Example 3.2,let S_E and T_E be two sub soft mset of X_E such that $S(e_1) = \{2/b, 1/c\}, S(e_2) = \{2/b, 1/c\}$, $T(e_1) = \{1/a, 1/c\}, T(e_2) = \{1/a, 1/c\}$. Then $S_E \cap T_E$ is not a regular closed soft mset in (X, τ, E).*

4 Generalized regular closed soft msets

Definition 4.1. *A soft mset S_E in a soft multi topological space (X, τ, E) is said to be a generalized regular closed (briefly gr-closed) soft mset if $C_{rcl(S)(e)}(x) \leq C_{U(e)}(x)$ whenever $C_{S(e)}(x) \leq C_{U(e)}(x)$ for all $x \in X^*$, $e \in E$ and U_E is a open soft mset in X_E. The complement of gr-closed soft mset is gr-open soft mset.*

Example 4.2. *In Example 3.2, the soft mset S_E is a gr-closed soft mset in (X, τ, E).*

Proposition 4.3. *Every regular closed soft mset is a gr-closed soft mset.*

Proof. Let S_E be a regular closed soft mset and $C_{S(e)}(x) \leq C_{U(e)}(x)$ where U_E is open soft mset. Then $C_{rcl(S)(e)}(x) = C_{S(e)}(x) \leq C_{U(e)}(x)$, by hypothesis. Thus S_E is a *gr*-closed soft mset.

Remark 4.4. *Converse of the above proposition need not be true. In Example 3.2, the soft mset S_E with $S(e_1) = \{2/b, 1/c\}$, $S(e_2) = \{1/a, 2/b\}$ is a gr-closed soft mset but not a regular closed soft mset in (X, τ, E).*

Proposition 4.5. *Every gr-closed soft mset is a g-closed soft mset.*

Proof. Let S_E be a *gr*-closed soft mset and let $C_{S(e)}(x) \leq C_{U(e)}(x)$ where U_E is open soft mset in (X, τ, E). Then $C_{cl(S)(e)}(x) \leq C_{rcl(S)(e)}(x) \leq C_{U(e)}(x)$. Hence S_E is a *g*-closed soft mset.

Remark 4.6. *Converse of the above proposition need not be true. In Example 3.2, let S_E be a sub soft mset of X_E such that $S(e_1) = \{1/a\}, S(e_2) = \{\phi\}$. Then S_E is a g-closed soft mset but not a gr-closed soft mset in (X, τ, E).*

Remark 4.7. *The following example shows that gr-closed soft mset is independent of closed soft mset.*

Example 4.8. *In Example 3.2, the soft mset S_E with $S(e_1) = \{2/b, 1/c\}$, $S(e_2) = \{1/a, 2/b\}$ is a gr-closed soft mset but not a closed soft mset in (X, τ, E). The soft mset S_E with $S(e_1) = \{1/c\}$, $S(e_2) = \{1/a\}$ is a closed soft mset but not a gr-closed soft mset in (X, τ, E).*

Proposition 4.9. *If S_E is an open soft mset and gr-closed soft mset of (X, τ, E), then it is a closed soft mset.*

Proof: Let S_E be a *gr*-closed soft mset and an open soft mset of(X, τ, E). Therefore $C_{S(e)}(x) = C_{int(S)(e)}(x)$. consequently from definition $C_{cl(S)(e)}(x) \leq C_{rcl(S)(e)}(x) \leq C_{S(e)}(x)$. But we know $C_{S(e)}(x) \leq C_{cl(S)(e)}(x)$, so $C_{S(e)}(x) = C_{cl(S)(e)}(x)$. Hence S_E is a closed soft mset.

Proposition 4.10. *Finite union of gr-closed soft msets is a gr-closed soft mset.*

Proof: Assume that S_E and T_E are two gr-closed soft msets in (X, τ, E). Let $C_{(S\tilde{\cup}T)(e)}(x) \leq C_{U(e)}(x)$ and U_E is open soft mset. But S_E and $T_E$are gr-closed soft mset. Therefore $C_{rcl(S)(e)}(x) \leq C_{U(e)}(x)$ and $C_{rcl(T)(e)}(x) \leq C_{U(e)}(x)$. Moreover $C_{rcl(S\tilde{\cup}T)(e)}(x) = C_{(rcl(S)\tilde{\cup}rcl(T))(e)}(x) \leq C_{U(e)}(x)$. Hence $S_E\tilde{\cup}T_E$ is a gr-closed soft mset.

Remark 4.11. *Finite intersection of gr-closed soft msets is not gr-closed soft mset.*

Example 4.12. *In Example 3.2, let S_E and T_E be two sub soft mset of X_E such that $S(e_1) = \{2/b, 1/c\}, S(e_2) = \{2/b, 1/c\}$, $T(e_1) = \{1/a, 1/c\}, T(e_2) = \{1/a, 1/c\}$. Then $S_E \cap T_E$ is not a gr-closed soft mset in (X, τ, E).*

Proposition 4.13. *Let S_E and T_E be two regular closed soft mset of (X, τ, E), then $S_E\tilde{\cup}T_E$ is a gr-closed soft mset.*

Proof: Let $C_{(S\tilde{\cup}T)(e)}(x) \leq C_{U(e)}(x)$, U_E is open soft mset. Then finite union of regular closed soft mset is regular closed soft mset. This implies that $C_{rcl(S\tilde{\cup}T)(e)}(x) \leq C_{U(e)}(x)$. Hence $S_E\tilde{\cup}T_E$ is a gr-closed soft mset.

Proposition 4.14. *The intersection of a gr-closed soft mset and a closed soft mset is a gr-closed soft mset.*

Proof: Let S_E be a gr-closed soft mset of X_E. Let F_E be a closed soft mset, $C_{S(e)}(x) \leq C_{U(e)}(x)$ and U_E be open soft mset of X_E with $C_{(S\tilde{\cap}F)(e)}(x) \leq C_{U(e)}(x)$, then $C_{S(e)}(x) \leq C_{(U\tilde{\cup}F^c)(e)}(x)$, So $C_{rcl(S)(e)}(x) \leq C_{(U\tilde{\cup}F^c)(e))}(x)$. Since $C_{rcl(S)\tilde{\cap}F(e)}(x) \leq C_{U(e)}(x)$. Now $C_{cl(S\tilde{\cap}F)(e)}(x) \leq C_{cl(S)\tilde{\cap}cl(F)(e)}(x)$ and so $C_{cl(S\tilde{\cap}F)(e)}(x) \leq C_{rcl(S)\tilde{\cap}F(e)}(x) \leq C_{U(e)}(x)$ implies that $C_{cl(S\tilde{\cap}F)(e)}(x) \leq C_{U(e)}(x)$. Hence $S_E\tilde{\cap}T_E$ is a gr-closed soft mset.

Remark 4.15. *The intersection of a gr-closed soft mset and a regular closed soft mset is gr-closed soft mset. i.e. The intersection of two regular closed soft mset is a gr-closed soft mset.*

Proposition 4.16. *Let $C_{S(e)}(x) \leq C_{T(e)}(x) \leq C_{rcl(S)(e)}(x)$ and S_E is a gr-closed soft mset of X_E, then T_E is a gr-closed soft mset of X_E.*

Proof: Let $C_{T(e)}(x) \leq C_{U(e)}(x)$ and U_E is open. Since S_E is a gr-closed soft mset of X_E. So $C_{rcl(S)(e)}(x) \leq C_{U(e)}(x)$. Let $C_{S(e)}(x) \leq C_{T(e)}(x) \leq C_{rcl(S)(e)}(x)$. Now $C_{rcl(T)(e)}(x) \leq C_{rcl(rcl((S))(e)}(x) \leq C_{rcl(S)(e)}(x) \leq C_{U(e)}(x)$. Hence T_E is a gr-closed soft mset of (X, τ, E).

Remark 4.17. *Let$C_{S(e)}(x) \leq C_{T(e)}(x) \leq C_{cl(S)(e)}(x)$ and S_E is a gr-closed soft mset of X_E, then T_E is a g-closed soft mset of (X, τ, E).*

Remark 4.18. *If S_E is a gr-closed soft mset of X_E, and since $C_{S(e)}(x) \leq C_{cl(S)(e)}(x) \leq C_{rcl(S)(e)}(x)$. So from above proposition $cl(S_E)$ is a gr-closed soft mset of (X, τ, E).*

Converse is need not be true as seen from the following example.

Example 4.19. *In Example 4.2, let S_E be a sub soft mset of X_E such that $S(e_1) = \{2/b\}, S(e_2) = \{2/b, 1/c\}$. Then $cl(S_E) =\widetilde{X}$, Obviously a gr-closed soft mset but S_E is not a gr-closed soft mset.*

References

[1] *K. V. Babitha and S. J. John, On soft multisets, Ann. fuzzy Math. Inform., 5(1), 35-44, 2013.*

[2] *V. Cerf, E. Fernandez, K. Gostelow, S. Volausky, Formal control and low properties of a model of computation, Report ENG 7178, computer Science Department, University of California, LOs Angeles, CA, December, p.81, 1971.*

[3] *Deniz Tokat and Ismail Osmanoglu, On the soft multi topology and it?s some properties, Nevsehir Universitesi Fen Bilimleri Enstitusu Dergisi Cilt 2, 109-118, 2013.*

[4] *Deniz Tokat and Ismail Osmanoglu, Connectedness on soft multi topological spaces, Journal of New Results in Science 2, 8-18, 2013.*

[5] *S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Generalized closed soft multiset in soft multi topological spaces, Asian Journal of Mathematics and computer research, 9, 302-311, 2015.*

[6] *S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Some types of open soft multisets and some types of mappings in soft multi topolgical spaces, Annals of fuzzy Mathematics and Informatics 12(1), 15-30, 2016.*

[7] *V. Inthumathi, A. Gnanasoundari and M. Maheswari, Generalized semi closed soft multisets(Communicated).*

[8] *S. P. Jena, S. K. Ghosh, B. K. Tripathy, On theory of bags and lists, Information Sciences, 132, 241-254, 2001.*

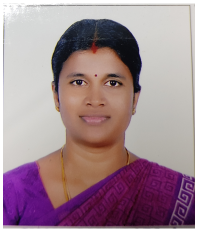
[9] *K. P. Girish, S. J. John, Multiset topologies induced by multiset relations, Information Sciences, 188, 298-313, 2012.*

[10] *D. Molodtsov, Soft set theory - first results Comput. Math. Appl. 37 (4-5) 19-31, 1999.*

[11] *R. R. Yager, On the theory of bags, International Journal General System, 13, 23-37, 1986.*

BIOGRAPHY

V. Inthumathi received Doctoral degree in the field of Ideal topological spaces from Bharathiar University in 2012. She is working as an Associate Professor in Department of Mathematics in Nallamuthu Gounder Mahalingam college, Pollachi, India. She has 24 years of teaching experience. she has published about 45 research articles in reputed journals. She guided 23 M.Phil scholars and guiding 4 Ph.D. scholars. Currently she is doing research in the area of soft topological spaces and Nano ideal topological spaces.



A. Gnanasoundari doing Ph.D. degree in the field of multiset topological spaces under the guidance of Dr. V. Inthumathi, Associate professor and Head, PG and Research Department of Mathematics, Nallamuthu Gounder Mahalingam college, Pollachi, India. She has presented 4 articles in international conference and published one research article in reputed journal.

Dr. M. Maheswari is working as an Assistant Professor in the Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi. She has got her Doctoral degree in the field of Ideal bitopological spaces from Bharathiar University, Coimbatore. She has 6 years experience in teaching. She published 8 research papers in reputed journals. Her areas of interest are Algebra, Complex Analysis and Statistics.