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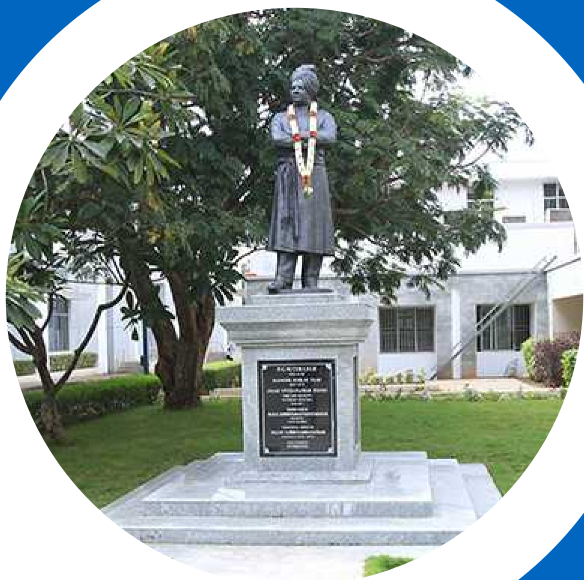

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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

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The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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STRONGER FORM OF SOFT CLOSED SETS

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Abstract - The aim of this paper is to introduce a stronger form of soft closed sets which is lie between soft regular closed sets and soft closed sets. And also we study its properties in soft topological spaces.

Keywords *Soft sets, soft topological spaces, soft δ-closed sets and soft δ-open sets.*
2010 Subject classification: *54A05, 54A10*

1 Introduction

Molodtsov [5] introduced the theory of soft sets, which can be seen as a new mathematical approach to vagueness. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. In recent years, the development in the fields of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parametrization expressed by a soft set.

In this paper, we introduce δ-closed sets and δ-open sets in soft topological spaces and study some of its properties.

2 Preliminaries

Definition 2.1. *[5] Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.*

Definition 2.2. *[4] Let U be an universal set and E be a set of parameters. Let (F, A) and (G, B) be soft sets over a common universe set U and $A, B \subseteq E$. Then (F, A) is a subset of (G, B), denoted by $(F, A) \subseteq (G, B)$, if*
(i) $A \subseteq B$,
(ii) for all $e \in E$, $F(e) \subseteq G(e)$.

(F, A) equals (G, B), denoted by $(F, A) = (G, B)$, if $(F, A) \tilde{\subseteq} (G, B)$ and $(G, B) \tilde{\subseteq} (F, A)$.

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Definition 2.3. *[3] Let X be a universe and E a set of attributes. Then the collection of all soft sets over X with attributes from E is called a soft class and is denoted as (X, E).*

Definition 2.4. *[4] A soft set (F, A) over U is said to be a null soft set, denoted by $\widetilde{\phi}$, if $\forall\ e \in A,\ F(e) = \phi$.*

Definition 2.5. *[4] A soft set (F, A) over U is said to be a absolute soft set, denoted by $\widetilde{U}$, if $\forall\ e \in A,\ F(e) = U$,*

Definition 2.6. *[4] Union of two soft sets of $F, A)$ and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$, and $\forall\ e \in C$,*

$$H(e) = \begin{cases} F(e), & if \quad e \in A - B \\ G(e), & if \quad e \in B - A \\ F(e) \cup G(e), & if \quad e \in A \cap B \end{cases}$$

Definition 2.7. *[4] Intersection of two soft sets (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A\ \cap\ B$, and $\forall\ e \in C$, $H(e) = F(e)$ or $G(e)$. We write $(F, A)\ \widetilde{\cap}\ (G, B) = (H, C)$.*

Definition 2.8. *[2] The complement of a soft set (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$, $F^c : A \to P(X)$ is mapping given by $F^c(e) = X - F(e), \forall\ e \in A$ and F^c is called the soft complement function of F.*

Definition 2.9. *[6] A soft set (E, A) over X is said to be soft element if there exists $\alpha \in A$ such that $E(\alpha)$ is a singleton, say $\{x\}$, and $E(\beta) = \phi,\ \forall\ \beta(\neq \alpha) \in A$ such a soft element is denoted by E^x_α.*

Definition 2.10. *[8] Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X and (A, E) be a soft set in $(X, \widetilde{\tau}, E)$ is called soft regular open set if $(A, E) = cl_s(int_s((A, E)))$.*

Definition 2.11. *[9] A soft topological space $(X, \widetilde{\tau}, E)$ is soft compact if each soft open cover of (X, E) has a finite subcover.*

Definition 2.12. *[7] Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X and (G, E) be a soft closed set in $(X, \widetilde{\tau}, E)$ and $X^x_e\ \widetilde{\in}\ (X, E)$ such that $X^x_e\ \widetilde{\notin}\ (G, E)$. If there exist soft open sets (F_1, E) and (F_2, E) such that $X^x_e\ \widetilde{\in}\ (F_1, E)$, $(G, E)\ \widetilde{\subseteq}\ (F_2, E)$ and $(F_1, E)\ \widetilde{\cap}\ (F_2, E) = \widetilde{\phi}$. Then $(X, \widetilde{\tau}, E)$ is called a soft regular space.*

Definition 2.13. *[7] Let $(X, \widetilde{\tau}, E)$ be a soft topological space over X, (F, E) and (G, E) be soft closed sets in $(X, \widetilde{\tau}, E)$ such that $(F, E)\ \widetilde{\cap}\ (G, E) = \widetilde{\phi}$. If there exist soft open sets (F_1, E) and (F_2, E) such that $(F, E)\ \widetilde{\subseteq}\ (F_1, E)$ $(G, E)\ \widetilde{\subseteq}\ (F_2, E)$ and $(F_1, E)\ \widetilde{\cap}\ (F_2, E) = \widetilde{\phi}$. Then $(X, \widetilde{\tau}, E)$ is called a soft normal space.*

Through out this paper we denote soft elements of (X, E) by X^x_e and soft elements of a soft set (A, E) by A^x_e.

3 Soft δ-closed sets and Soft δ-open sets

Definition 3.1. *A soft point $X_e^x \widetilde{\in} (X, E)$ is called soft δ-cluster point of a soft subset (A, E) in the soft topological space $(X, \widetilde{\tau}, E)$ if the soft interior of each soft closed neighborhood of X_e^x intersects (A, E) and is denoted by $A(e)$.*

$$i.e.,\ A(e) = \begin{cases} x \in X, & int_s(cl_s((U,E))) \widetilde{\cap} (A,E) \neq \widetilde{\phi}, \quad where (U,\ E)\ is\ the\ open \\ & neighborhood\ of\ X_e^x \widetilde{\in} (X,E) \quad for\ all \quad e \in E \\ \phi, & otherwise \end{cases}$$

Definition 3.2. *The collection of all soft δ-cluster points of (A, E) is called the soft δ-closure of (A, E) and is denoted by $\delta cl_s((A, E))$.*
Symbolically, $\delta cl_s((A, E)) = \{(e, A(e)) \widetilde{\in} (X, E)$ for all $e \in E\}$

Definition 3.3. *A soft subset (A, E) is called soft δ-closed if $\delta cl_s((A, E)) = (A, E)$.*
The complement of a soft δ-closed set of $(X, \widetilde{\tau}, E)$ is called soft δ-open.

Example 3.4.

1. *Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\ \widetilde{\phi}\ ,\ \widetilde{X}\ , (F_1, E), (F_2, E),$ $(F_3, E)\}$ where $(F_1, E) = \{(e_1, \{x_1\}), (e_2, \{\phi\})\}$, $(F_2, E) = \{(e_1, \{x_2\}), (e_2, \{\phi\})\}$, $(F_3, E) = \{(e_1, \{X\}), (e_2,$ Then soft δ-closed sets are $\widetilde{\phi}$, $\widetilde{X}$, $\{(e_1, \{x_1\}), (e_2, \{X\})\}$, $\{(e_1, \{x_2\}), (e_2, \{X\})\}$ and $\{(e_1, \{\phi\})(e_2, \{X\})\}$.*

2. *Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\ \widetilde{\phi}\ ,\ \widetilde{X}\ , (G_1, E), (G_2, E), (G_3, E)\}$ where $(G_1, E) = \{(e_1, \{\phi\}), (e_2, \{x_2\})$, $(G_2, E) = \{(e_1, \{x_1\}), (e_2, \{x_2\})$, $(G_3, E) = \{(e_1, \{x_2\}), (e_2, \{X\})$. Then soft δ-closed sets are $\widetilde{\phi}$ and $\widetilde{X}$.*

Proposition 3.5. *Every soft δ-closed set is soft closed*

Remark 3.6. *Converse of the above proposition need not be true. In Example 3.4(2), the soft set $\{(e_1, \{x_1\}), (e_2, \{\phi\})\}$ is soft closed but not a soft δ-closed set.*

Proposition 3.7.

1. *Every soft regular open set is soft δ-open.*

2. *Every soft δ-open set is the union of a collection of soft regular open sets.*

Proof.1. Let (A, E) be soft regular open. Then for each $A_e^x \widetilde{\in} (A, E)$, $(A, E) \widetilde{\cap} (A, E)^c = \widetilde{\phi}$ and so $A_e^x \widetilde{\notin} \delta cl_s((A, E)^c)$. That is $A_e^x \widetilde{\notin} (A, E)^c$ implies $A_e^x \widetilde{\notin} \delta cl_s((A, E)^c)$. This implies that $\delta cl_s((A, E)^c) \widetilde{\subseteq} (A, E)^c$. But $(A, E)^c \widetilde{\subseteq} \delta cl_s((A, E)^c)$ always. Thus, we have $(A, E)^c = \delta cl_s((A, E)^c)$. Hence (A, E) is soft δ-open. 2. Let (A, E) be a soft δ-open set. Then $(A, E)^c = \delta cl_s((A, E)^c)$. For each $A_e^x \widetilde{\in} (A, E)$, $A_e^x \widetilde{\notin} \delta cl_s((A, E)^c)$ and there exists a soft open neighborhood (N, E) of A_e^x such that $int_s(cl_s((N, E))) \widetilde{\cap} (A, E)^c = \widetilde{\phi}$. Then $A_e^x \widetilde{\in} (N, E) \widetilde{\subseteq} int_s(cl_s((N, E))) \widetilde{\subseteq} (A, E)$ and hence $(A, E) = \widetilde{\cup} \{int_s(cl_s((N, E))) : A_e^x \widetilde{\in} (A, E)\}$. Since $int_s(cl_s((N, E)))$ is soft regular open for each $A_e^x \widetilde{\in} (A, E)$, hence (A, E) is the union of a collection of soft regular open sets.

Remark 3.8. *Converse of the Proposition 3.7 (1) need not be true. In Example* 3.4(1), *the soft set $\{(e_1, \{X\}), (e_2, \{\phi\})\}$ is soft δ-open but not a soft regular open set.*

Proposition 3.9. *For any subsets (A, E) and (B, E) of (X, E), the following properties hold:*

1. $(A,E) \widetilde{\subseteq} \delta cl_s((A,E))$.
2. If $(A,E) \widetilde{\subseteq} (B,E)$ then $\delta cl_s((A,E)) \widetilde{\subseteq} \delta cl_s((B,E))$.

Proof. 1. We have $\widetilde{\phi} \neq (A,E) \widetilde{\cap} (N,E) \widetilde{\subseteq} (A,E) \widetilde{\cap} int_s(cl_s((N,E)))$, for any $A_e^x \widetilde{\in} (A,E)$ and any soft open neighborhood (N,E) of A_e^x and so $A_e^x \widetilde{\in} \delta cl_s((A,E))$. Thus $(A,E) \widetilde{\subseteq} \delta cl_s((A,E))$.

2. Assume that $(A,E) \widetilde{\subseteq} (B,E)$. Presume that $X_e^x \widetilde{\notin} \delta cl_s((B,E))$. Then there exists a soft open neighborhood (N,E) of X_e^x such that $int_s(cl_s((N,E))) \widetilde{\cap} (B,E) = \widetilde{\phi}$ and so $int_s(cl_s((N,E))) \widetilde{\cap} (A,E) = \widetilde{\phi}$. Thus $X_e^x \widetilde{\notin} \delta cl_s((A,E))$. Hence $\delta cl_s((A,E)) \widetilde{\subseteq} \delta cl_s((B,E))$.

Proposition 3.10. *For any soft subset (A,E) of (X,E),*

1. *$\delta cl_s((A,E))$ is the smallest soft δ-closed set containing (A,E).*
2. *(A,E) is soft δ-closed if and only if $\delta cl_s((A,E)) = (A,E)$.*

Proof.

1. Let $\{(F_i,E) : i \in \Lambda\}$ be the collection of all soft δ-closed sets containing (A,E). Then $\delta cl_s((A,E)) = \widetilde{\bigcap}\{(F$ $i \in \Lambda\}$ and so $\delta cl_s((A,E))$ is soft δ-closed, since arbitrary intersection of all soft δ-closed sets is soft δ-closed. Since $(A,E) \widetilde{\subseteq} (F_i,E)$ for each i, we have $(A,E) \widetilde{\subseteq} \widetilde{\bigcap}\{(F_i,E), i \in \Lambda\} = \delta cl_s((A,E))$. Moreover $\delta cl_s((A,E))$ is the intersection of all soft δ-closed set containing (A,E), $\delta cl_s((A,E))$ is contained in every soft δ-closed set containing (A,E). Hence $\delta cl_s((A,E))$ is the smallest soft δ-closed set containing (A,E).
2. Assume that (A,E) is soft δ-closed. Then the smallest soft δ-closed set containing (A,E) is (A,E) itself and so $\delta cl_s((A,E)) = (A,E)$. Conversely, assume that $\delta cl_s((A,E)) = (A,E)$ then $\delta cl_s((A,E))$ is soft δ-closed and so (A,E) is soft δ-closed.

Theorem 3.11. *For each subset (S,E), the soft δ-closure of (S,E) is the intersection of all soft δ-closed sets containing (S,E).*

Proof. Let $X_e^x \widetilde{\in} \delta cl_s((S,E))$. For any soft open neighborhood (N,E) of X_e^x and any soft δ-closed set (A,E) containing (S,E), we have $\widetilde{\phi} \neq int_s(cl_s((N,E))) \widetilde{\cap} (S,E) \widetilde{\subseteq} int_s(cl_s((N,E))) \widetilde{\cap} (A,E)$ and hence $X_e^x \widetilde{\in} \delta cl_s((A,E)) = (A,E)$. Thus, we have $X_e^x \widetilde{\in} \widetilde{\cap}\{(A,E) : (S,E) \widetilde{\subseteq} (A,E)$ and (A,E) is soft δ-closed$\}$.

Conversely, presume that $x \widetilde{\notin} \delta cl_s((S,E))$. Then there exists a soft open neighborhood (N,E) of X_e^x such that $int_s(cl_s((N,E))) \widetilde{\cap} (S,E) = \widetilde{\phi}$. Since every soft regular open set is soft δ-open, $(int_s(cl_s((N,E))))^c$ is a soft δ-closed set which contains (S,E) and does not contain X_e^x. Thus $X_e^x \widetilde{\notin} \widetilde{\cap}\{(A,E) : (S,E) \widetilde{\subseteq} (A,E)$ and (A,E) is soft δ-closed$\}$. Hence the proof.

Proposition 3.12. *Arbitrary intersection of soft δ-closed sets is a soft δ-closed set.*

Proof. Let (A_i,E) be a soft δ-closed sets in $(X,\widetilde{\tau},E)$ for each $i \in \Lambda$. Then $\delta cl_s[\bigcap_{i\in\Lambda}(A_i,E)] \widetilde{\subseteq} \delta cl_s(A_i,E) = (A_i,E)$ and so $\delta cl_s[\bigcap_{i\in\Lambda}(A_i,E)] \widetilde{\subseteq} \bigcap_{i\in}(A_i,E)$. This implies $\bigcap_{i\in}(A_i,E)$ is soft δ-closed.

Proposition 3.13. *For a soft subset (A,E), $\delta cl_s((A,E))$ is soft δ-closed.*

Proof. Proof is obvious from Theroem 3.11 and Proposition 3.12.

Theorem 3.14. *Let $(X, \widetilde{\tau}, E)$ be a soft topological space and let $\widetilde{\tau}_\delta$=$\{(A, E) \widetilde{\subseteq} (X, E) : (A, E)$ is a soft δ-open set of $(X, \widetilde{\tau}, E)\}$. Then $\widetilde{\tau}_\delta$ is a soft topology.*

Proof.

1. Clearly $\widetilde{\phi}$, $\widetilde{X} \widetilde{\in} \widetilde{\tau}_\delta$.

2. Let $(A_i, E) \widetilde{\in} \widetilde{\tau}_\delta$ for each $i \in \Lambda$. Then $(A_i, E)^c$ is soft δ-closed for each $i \in \Lambda$. Then $\widetilde{\bigcap}_{i\in\Lambda}(A_i, E)^c$ is soft δ-closed by Proposition 3.12. Thus $\widetilde{\bigcup}_{i\in\Lambda}(A_i, E)$ is soft δ-open.

3. Let $(A, E),\ (B, E) \widetilde{\in} \widetilde{\tau}_\delta$. By Proposition 3.7(2), $(A, E) = \widetilde{\bigcup}_{i\in\Lambda_1}(A_i, E)$ and $(B, E) = \widetilde{\bigcup}_{j\in\Lambda_2}(B_j, E)$, where A_i and B_j are soft regular open sets for each $i \in \Lambda_1$ and $j \in \Lambda_2$. Thus $(A, E) \widetilde{\cap} (B, E) = \widetilde{\bigcup} \{(A_i, E) \widetilde{\cap}$ $\Lambda_1,\ j \in \Lambda_2\}$. Since $(A_i, E) \widetilde{\cap} (B_j, E)$ is soft regular open, $(A, E) \widetilde{\cap} (B, E)$ is a soft δ-open set.
   Hence$\widetilde{\tau}_\delta$ is a soft topology.

Corollary 3.15. $\widetilde{\tau}_\delta \widetilde{\subseteq} \widetilde{\tau}$.

Proof. It is immediate from Proposition 3.5.

Proposition 3.16. *For a soft subsets (A, E) and (B, E), we have the following properties.*

1. $\delta cl_s(\widetilde{\phi}) = \widetilde{\phi}$ *and* $\delta cl_s(\widetilde{X}) = \widetilde{X}$.

2. $\delta cl_s(\delta cl_s((A, E))) = \delta cl_s((A, E))$.

3. $\delta cl_s((A, E) \widetilde{\cap} (B, E)) \widetilde{\subseteq} \delta cl_s((A, E)) \widetilde{\cap} \delta cl_s((B, E))$.

4. $\delta cl_s((A, E) \widetilde{\cup} (B, E)) = \delta cl_s((A, E)) \widetilde{\cup} \delta cl_s((B, E))$.

Proof. 1. Since $\widetilde{\phi}$ and $\widetilde{X}$ are soft δ-closed sets, we have $\delta cl_s(\widetilde{\phi}) = \widetilde{\phi}$ and $\delta cl_s(\widetilde{X}) = \widetilde{X}$.
2. Let $(A, E) \widetilde{\subseteq} (B, E)$ and (B, E) be soft δ-closed. Then $\delta cl_s((A, E) \widetilde{\subseteq} (B, E))$ and so $\delta cl_s(\delta cl_s((A, E))) \widetilde{\subseteq} (B, E)$
Therefore $\delta cl_s(\delta cl_s((A, E))) \widetilde{\subseteq} \bigcap\{(B, E) : (A, E) \widetilde{\subseteq} (B, E)$ where (B, E) is soft δ-closed$\} = \delta cl_s((A, E))$. Then we have $\delta cl_s((A, E)) \widetilde{\subseteq} \delta cl_s(\delta cl_s((A, E)))$. Thus $\delta cl_s(\delta cl_s((A, E))) = \delta cl_s((A, E))$.
3. Since $(A, E) \widetilde{\cap} (B, E) \widetilde{\subseteq} (A, E)$ and $(A, E) \widetilde{\cap} (B, E) \widetilde{\subseteq} (B, E)$, we have $\delta cl_s((A, E) \widetilde{\cap} (B, E)) \widetilde{\subseteq} \delta cl_s((A, E))$ and $\delta cl_s((A, E) \widetilde{\cap} (B, E)) \widetilde{\subseteq} \delta cl_s((B, E))$. This implies $\delta cl_s((A, E) \widetilde{\cap} (B, E)) \widetilde{\subseteq} \delta cl_s((A, E)) \widetilde{\cap} \delta cl_s((B, E))$.
4. Since $(A, E) \widetilde{\subseteq} (A, E) \widetilde{\cup} (B, E)$ and $(B, E) \widetilde{\subseteq} (A, E) \widetilde{\cup} (B, E)$, we have $\delta cl_s((A, E)) \widetilde{\subseteq} \delta cl_s((A, E) \widetilde{\cup} (B, E))$ and $\delta cl_s((B, E)) \widetilde{\subseteq} \delta cl_s((A, E) \widetilde{\cup} (B, E))$ and so $\delta cl_s((A, E)) \widetilde{\cup} \delta cl_s((B, E)) \widetilde{\subseteq} \delta cl_s((A, E) \widetilde{\cup} (B, E))$. We know that $(A, E) \widetilde{\subseteq} \delta cl_s((A, E))$ and $(B, E) \widetilde{\subseteq} \delta cl_s((B, E))$. This implies $(A, E) \widetilde{\cup} (B, E) \widetilde{\subseteq} \delta cl_s((A, E)) \widetilde{\cup} \delta cl_s((B$ and $\delta cl_s((A, E) \widetilde{\cup} (B, E)) \widetilde{\subseteq} \delta cl_s((A, E)) \widetilde{\cup} \delta cl_s((B, E))$. Hence $\delta cl_s((A, E) \widetilde{\cup} (B, E)) = \delta cl_s((A, E)) \widetilde{\cup} \delta cl_s((B, E$

Proposition 3.17. *For a soft subsets (A, E) and (B, E), we have the following properties.*

1. $\delta int_s((A, E)) \widetilde{\subseteq} (A, E)$.

2. $\delta int_s(\widetilde{\phi}) = \widetilde{\phi}$ *and* $\delta int_s(\widetilde{X}) = \widetilde{X}$.

3. *If* $(A, E) \widetilde{\subseteq} (B, E)$ *then* $\delta int_s((A, E)) \widetilde{\subseteq} \delta int_s((B, E))$.

4. $\delta int_s(\delta int_s((A, E))) = \delta int_s((A, E))$.

5. *$\delta int_s((A,E)\ \widetilde{\cap}\ (B,E)) = \delta int_s((A,E))\ \widetilde{\cap}\ \delta int_s((B,E))$.*

6. *$\delta int_s((A,E))\ \widetilde{\cup}\ \delta int_s((B,E))\ \widetilde{\subseteq}\ \delta int_s((A,E)\ \widetilde{\cup}\ (B,E))$.*

7. *$\delta int_s((A,E))$ is the largest soft δ-open set contained in (A,E).*

8. *(A,E) is soft δ-open if and only if $\delta int_s((A,E)) = (A,E)$.*

Theorem 3.18. *For a soft topological space $(X,\widetilde{\tau},E)$, the following are equivalent.*

1. *$(X,\widetilde{\tau},E)$ is soft normal.*

2. *For any soft δ-closed set (A,E) and a soft open set (B,E) containing (A,E), there exists a soft open set (V,E) such that $(A,E)\ \widetilde{\subseteq}\ (V,E)$ and $cl_s((V,E))\ \widetilde{\subseteq}\ (B,E)$.*

Proof. 1⇒2. Assume that $(X,\widetilde{\tau},E)$ is soft normal and (B,E) is a soft open set containing the soft δ-closed set (A,E). Then $(B,E)^c\ \widetilde{\cap}\ (A,E) = \widetilde{\phi}$ and so $(B,E)^c$ is a soft closed set disjoint from the soft closed set (A,E), since every soft δ-closed set is soft closed. By hypothesis, there exist disjoint soft open sets (U,E) and (V,E) such that $(B,E)^c\ \widetilde{\subseteq}\ (U,E)$ and $(A,E)\ \widetilde{\subseteq}\ (V,E)$. This implies $cl_s((V,E))\ \widetilde{\subseteq}\ (U,E)^c\ \widetilde{\subseteq}\ (B,E)$. Thus (V,E) is a soft open set such that $(A,E)\ \widetilde{\subseteq}\ (V,E)$ and $cl_s((V,E))\ \widetilde{\subseteq}\ (B,E)$.

2⇒1. Assume that for each soft δ-closed set (A,E) and a soft open set (B,E) containing (A,E) there exists a soft open set (V,E) such that $(A,E)\ \widetilde{\subseteq}\ (V,E)$ and $cl_s((V,E))\ \widetilde{\subseteq}\ (B,E)$. Let (C,E) and (A,E) be any two disjoint soft closed subsets of $(X,\widetilde{\tau},E)$. Then $(A,E)\ \widetilde{\subseteq}\ (C,E)^c$ and $(C,E)^c$ is a soft open set containing a soft δ-closed set (A,E). By assumption. there exists a soft open set (V,E) such that $(A,E)\ \widetilde{\subseteq}\ (V,E)$ and $cl_s((V,E))\ \widetilde{\subseteq}\ (C,E)^c$. Thus (V,E) and $[cl_s((V,E))]^c$ are disjoint soft open sets such that $(C,E)\ \widetilde{\subseteq}\ [cl_s((V,E))]^c$ and $(A,E)\ \widetilde{\subseteq}\ (V,E)$.

Proposition 3.19. *Every soft δ-closed subset of a soft compact topological space is soft compact.*

Proof. Let (A,E) be a soft δ-closed subset of a soft compact topological space $(X,\widetilde{\tau},E)$ and let $\{(U_i,E)\}$ be a soft open covering of (A,E). Then $(A,E)\ \widetilde{\subseteq}\ \bigcup_i(U_i,E)$. Consequently, $\widetilde{X} = (A,E)\ \widetilde{\cup}\ (A,E)^c\ \widetilde{\subseteq}\ \bigcup_i(U_i,E)\ \widetilde{\cup}\ ($ where $(A,E)^c$ is soft δ-open. Thus $\{\{(U_i,E)\},(A,E)^c\}$ is a soft open covering of $\widetilde{X}$. Since $\widetilde{X}$ is soft compact, $\widetilde{X}\ \widetilde{\subseteq}\ \bigcup_{i=1,2,\ldots,n}(U_i,E)$ and so $(A,E)\ \widetilde{\subseteq}\ \bigcup_{i=1,2,\ldots,n}(U_i,E)$. Hence (A,E) is soft compact.

Proposition 3.20. *Let (A,E) be a soft compact subset of a soft regular topological space $(X,\widetilde{\tau},E)$. If (U,E) is a soft δ-open subset containing (A,E), then there exists a soft closed set (F,E) such that $(A,E)\ \widetilde{\subseteq}\ (F,E)\ \widetilde{\subseteq}\ (U,E)$.*

Proof. Since (U,E) is a soft δ-open set containing a soft compact subset (A,E), we have (U,E) is a soft open nbd. of each point of (A,E). Then for each $A_e^x\ \widetilde{\in}\ (A,E)$, there exists a soft open nbd. $(V_{A_e^x},E)$ of A_e^x such that $cl_s((V_{A_e^x},E))\ \widetilde{\subseteq}\ (U,E)$, since $\widetilde{X}$ is soft regular. Therefore $\{(V_{A_e^x},E) : A_e^x\ \widetilde{\in}\ (A,E)\}$ is a soft open covering of (A,E) and so $(A,E)\ \widetilde{\in}\ \bigcup_{i=1,2,\ldots,n}(V_{A_e^{x_i}},E)$ and $cl_s((V_{A_e^{x_i}},E))\ \widetilde{\subseteq}\ (U,E)$ for each i. Set $(F,E) = \bigcup_{i=1,2,\ldots,n}[cl_s((V_{A_e^{x_i}},E))]$. Then (F,E) is soft closed and $(F,E)\ \widetilde{\subseteq}\ (U,E)$ and $(A,E)\ \widetilde{\subseteq}\ (F,E)$. Hence $(A,E)\ \widetilde{\subseteq}\ (F,E)\ \widetilde{\subseteq}\ (U,E)$.

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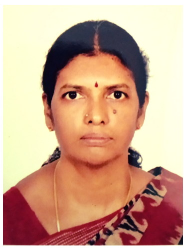
BIOGRAPHY



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