VOLUME XI
ISBN No.: 978-93-94004-00-9
Physical Science

Estd. 1957

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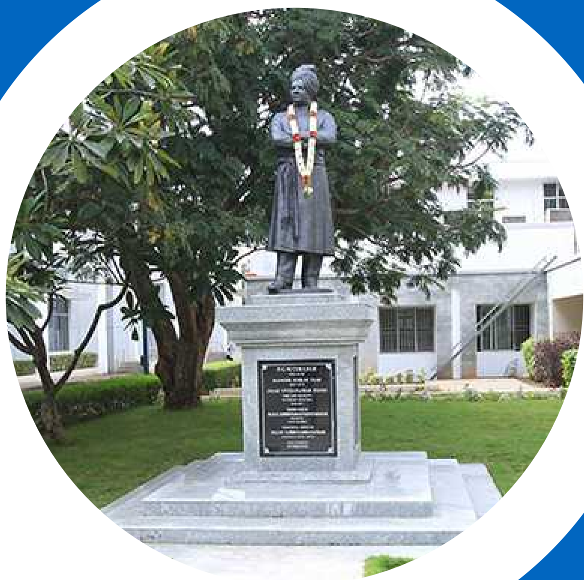

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27th October 2021

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Department of Biological Science, Physical Science and Computational Science

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A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

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Published by NGMC - November 2021

Decompositions of Nano continuous functions in Nano ideal topological spaces

V. Inthumathi[1], **R. Abinprakash**[2]

Abstract - In this paper, we introduce the notions of $NIg\beta$-open sets, $N\beta_{\mathcal{I}}$-sets, $N\mathrm{w}AB_I$-sets and their respective continuous functions to obtain a new decompositions of nano continuous functions in Nano ideal topological spaces. Also we introduce and study $NI\alpha g$-continuous functions,$NIgS$-continuous functions, $NIgP$-continuous functions,$NIg\beta$-continuous functions in nano ideal topological spaces. Finally we study the Nano βI-compactness in Nano ideal topological spaces.

Keywords *$NIg\beta I$-open sets, $N\beta_{\mathcal{I}}$-sets , $N\mathrm{w}AB_I$-sets, $NIg\beta$-continuous functions, $N\beta I$-compactness.*
2010 Subject classification: *54A05, 54A10, 54B05*

1 Introduction

The idea of ideal concept was first introduced by Kuratowski[11]. Jankovic and Hemlett[10] extended the further properties of ideal topological spaces. Hatir et.al [5] introduced the notions of βI-open sets and obtained the decomposition of continuty in ideal topological spaces. Recently many authors[2, 3, 4, 5] introduced new sets and some new decomposition concepts in ideal topological spaces. Lellis Thivagar and Carmel Richard [12] introduced the notions of nano topological spaces . Latter Lellis Thivagar and Suthadevi [2] introduced the notions of nano ideal topological spaces and investigated the properties of some weaker forms of nano open sets. In 2018, Rajasekaran et.al introduced the concepts of βI-open sets and investigated some of its properties in Nano ideal topological spaces.

In this paper, we introduce the notions of $NIg\beta$-open sets, $N\beta_{\mathcal{I}}$-sets , $N\mathrm{w}AB_I$-sets and their respective continuous functions to obtain a new decompositions of nano continuous functions in Nano ideal topological spaces. Also we introduce and study $NI\alpha g$-continuous functions,$NIgS$-continuous functions, $NIgP$-continuous functions,$NIg\beta$-continuous functions in nano ideal topological spaces. Finally we introduce and study the Nano βI-compactness and given comparison between Nano βI-compactness and some other nano compactness in Nano ideal topological spaces.

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2 Preliminaries

Definition 2.1. *[12] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let* $X \subseteq U$.

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by* $L_R(X)$.
   That is, $L_R(X) = \bigcup\limits_{x \in U} \{R(x) : R(x) \subseteq X\}$, *where R(x) denotes the equivalence class determined by x.*

2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by* $U_R(X)$. *That is,* $U_R(X) = \bigcup\limits_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$

3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by* $B_R(X)$. *That is,* $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2. *[12] Let U be the universe, R be an equivalence relation on U and* $\tau_R(X) = \{\phi, U, L_R(X), U_R(X),$ *where* $X \subseteq U$. *Then* $\tau_R(X)$ *satisfies the following axioms:*

1. ϕ *and U are in* $\tau_R(X)$.

2. *The union of elements of any subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

3. *The intersection of elements of any finite subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

That is, $\tau_R(X)$ *is a topology on U called the nano topology on U with respect to X. We call* $(U, \tau_R(X))$ *as the nano topological space (briefly. NTS). The elements of* $\tau_R(X)$ *are called as nano open sets(NO-sets).*

Definition 2.3. *[12] Let* $(U, \tau_R(X))$ *be a NTS, the set* $\mathcal{B} = \{U, L_R(X), B_R(X)\}$ *is the basis for* $\tau_R(X)$.

Definition 2.4. *[11] An ideal I on a topological space is a non-empty collection of subsets of X which satisfies*
1. $A \in I$ *and* $B \subseteq A$ *implies* $B \in I$.
2. $A \in I$ *and* $B \in I$ *implies* $A \cup B \in I$.

Definition 2.5. *[13] A NTS* $(U, \tau_R(X))$ *with an ideal I on U is called a nano ideal topological space(briefly. NITS) and denoted as* $(U, \tau_R(X), I)$.

Definition 2.6. *[13] Let*$(U, \tau_R(X), I)$ *be a NITS. A set operator* $(A)^{*N} : P(U) \rightarrow P(U)$ *is called the nano local function of I on U with respect to I on* $\tau_R(X)$ *is defined as* $(A)^{*N} = \{x \in U : U \cap A \notin I$ *; for every* $U \in \tau_R(X)\}$ *and is denoted by* $(A)^{*N}$, *where nano closure operator is defined as* $NCl^*(A) = A \cup (A)^{*N}$.

Definition 2.7. *[14] A subset A of a NITS* $(U, \tau_R(X), I)$ *is called nano I-open(briefly. NI-open) if* $A \subseteq Nint((A)^{*N})$.$A \subseteq U$ *called nano I-closed (briefly. NI-closed),if its complement is nano I-open.*

Definition 2.8. *A subset A of a NITS* $(U, \tau_R(X), I)$ *is said to be*

1. $N\alpha I$ *- open [13] if* $A \subseteq Nint(NCl^*(Nint(A)))$.

2. *NSI - open [13] if* $A \subseteq NCl^*(NInt(A))$.

3. *NPI-open [7] if* $A \subseteq Nint(NCl^*(A))$.

4. $N\beta I$ *- open [6] if* $A \subseteq NCl^*(NInt(NCl^*(A)))$.

The family of all $N\alpha I$*-open (resp. NSI-open, NPI-open,*$N\beta I$*-open) sets of a NITS is denoted by* $N\alpha IO$ *(resp.* $NSIO(U,X)$*,* $NPIO(U,X)$*,*$N\beta IO(U,X)$*).*
A subset A of a NITS $(U, \tau_R(X), I)$ *is said to be* $N\alpha I$*-closed (resp.,NSI-closed, NPI-closed,* $N\beta I$*-closed), if its complement is* $N\alpha I$*-open (resp. NSI-open, NPI-open ,* $N\beta I$*-open).*

Definition 2.9. *[1] Let* $A \subseteq U$ *of a NTS* $(U, \tau_R(X))$ *is called Nano generalized closed if* $NCl(A) \subseteq G$ *whenever* $A \subseteq G$ *and G is a NO-set. The family of all Nano generalized closed sets is denoted by* $NgC(U,X)$*.*

Definition 2.10. *[9] A subset A of a NITS* $(U, \tau_R(X), I)$ *is called Nano ideal* α*-generalized closed if* $N\alpha ICl(A) \subseteq G$ *whenever* $A \subseteq G$*, G is a NO-set. The family of all Nano ideal* α*-generalized closed sets is denoted by* $NI\alpha gC(U,X)$*.*

Definition 2.11. *[9] A subset A of a NITS* $(U, \tau_R(X), I)$ *is called Nano ideal generalized semi closed if* $NSICl(A) \subseteq G$ *whenever A*$\subseteq$ *G, G is a NO-set. The family of all Nano ideal generalized semi closed sets is denoted by* $NIgSC(U,X)$*.*

Definition 2.12. *[9] A subset A of a NITS* $(U, \tau_R(X), I)$ *is called Nano ideal generalized pre closed set if* $NPICl(A) \subseteq G$ *whenever A*$\subseteq$ *G, G is a NO-set. The family of all Nano ideal generalized pre closed sets is denoted by* $NIgPC(U,X)$*.*

Definition 2.13. *[8] A mapping* $\psi : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ *is called* $N\alpha I$*-continuous if* $\psi^{-1}(B)$ *is* $N\alpha I$*-open set in* $(U, \tau_R(X), I)$ *for every NO-set B in* $(V, \tau_{R'}(Y))$*.*

Definition 2.14. *[16] Let V be a subset of a NITS* $(U, \tau_R(X))$ *. If the collection* $\tau_R(V,X) = \{V \cap B : B \in \tau_R(X)\}$ *is a nano topology on V with respect to X, then* $\tau_R(V,X)$ *is called a nano subspace topology.*

Definition 2.15. *[15] A collection* $\{S_i : i \in \nabla\}$ *of* $N\alpha$ *I-open sets(resp. NPI-open,NSI-open)in a NITS* $(U, \tau_R(X), I)$ *is called* $N\alpha I$*-open(resp. NPI-open, NSI-open) cover of a subset S of U, if* $S \subseteq \bigcup_{i\in\nabla} S_i$ *holds.*

Definition 2.16. *[15] A NITS* $(U, \tau_R(X), I)$ *is called Nano* α *I-compact(briefly.* $N\alpha$ *I-compact)(resp. Nano pre I-compact(briefly. NPI-compact), Nano semi I-compact(briefly. NSI-compact)) space, if every* $N\alpha I$*-open(resp. NPI-open, NSI-open)cover has a finite* $N\alpha I$*-open(resp. NPI-open, NSI-open) subcover .*

Definition 2.17. *[15] A subset S of a NITS* $(U, \tau_R(X), I)$ *is called* $N\alpha I$*-compact(resp. NPI-compact, NSI-compact) relative to U, if for every collection* $\{S_i : i \in \nabla\}$ *of* $N\alpha I$*-open (resp. NPI-open, NSI-open) subsets of U, such that* $S \subseteq \bigcup_{i\in\nabla} S_i$ *there exists a finite subset* ∇_0 *of* ∇*,* $S \subseteq \bigcup_{i\in\nabla_0} S_i$*.*

3 Nano ideal generalised β- closed sets

Definition 3.1. *A subset A of NITS $(U, \tau_R(X), I)$ is called Nano ideal generalized β-closed (briefly. $NIg\beta$-closed)if $N\beta ICl(A) \subseteq G$ whenever $A \subseteq G$ where G is a NO-set.*

Theorem 3.2. *Every NC-set in $(U, \tau_R(X), I)$ is a $NIg\beta$-closed set.*

Proof: Let A be a NC-set in $(U, \tau_R(X), I)$. Suppose that $A \subseteq G$ where G is a NO- set in $(U, \tau_R(X), I)$. Since every NC- set is a $N\beta I$- closed set, $N\beta ICl(A) \subseteq NCl(A) = A \subseteq G$. Thus A is a $NIg\beta$-closed set.

Theorem 3.3. *Every $N\beta I$-closed set in $(U, \tau_R(X), I)$ is $NIg\beta$-closed.*

Proof: Suppose that $A \subseteq G$ where G is a NO- set in $(U, \tau_R(X), I)$. Since A is $N\beta I$-closed, $N\beta ICl(A) = A \subseteq G$. Hence A is $NIg\beta$-closed.

Remark 3.4. *The following examples shows*

1. *The intersection of two $NIg\beta$-closed sets $(U, \tau_R(X), I)$ need not be a $NIg\beta$-closed set in $(U, \tau_R(X), I)$.*
2. *The union of two $NIg\beta$-closed sets in $(U, \tau_R(X), I)$ need not be a $NIg\beta$-closed set in $(U, \tau_R(X), I)$*
3. *Converse of the theorem 3.2 need not be true.*
4. *Converse of the Theorem 3.3 need not be true.*

Example 3.5. *Let $U = \{a, b, c, d\}$ be the Universe set with $U/R = \{\{a\}, \{b\}, \{c\}, \{d\}\}$ and $X = \{a, d\}$. The Nano topology $\tau_{R(X)} = \{\phi, \{a, d\}, U\}$ with Ideal $I = \{\phi, \{a\}\}$. Now $\{a, b, d\}$ and $\{a, c, d\}$ are $NIg\beta$-closed sets in $(U, \tau_R(X), I)$. Now $\{a, b, d\} \cap \{a, c, d\} = \{a, d\}$ is not a $NIg\beta$-closed set.*

Example 3.6. *Let $U = \{a, b, c, d\}$ be the Universe set with $U/R = \{\{d\}, \{c\}, \{a, b\}\}$ and $X = \{a, d\}$. The Nano topology $\tau_{R(X)} = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, U\}$ with Ideal $I = \{\phi, \{a\}\}$. Now $\{b\}$ is a $NIg\beta$-closed set but not NC-set and $N\beta I$-closed set. Also $\{c\}$ is a $NIg\beta$-closed set. Now $\{b\} \cup \{c\} = \{b, c\}$ is not a $NIg\beta$-closed set*

Theorem 3.7. *In a NITS, every $NI\alpha g$-closed set is a $NIg\beta$- closed set.*

Proof : Let G be a NO-set and $A \subset G$ be a $NI\alpha g$-closed set in $(U, \tau_R(X), I)$. Since every $N\alpha I$- closed set is a $N\beta I$-closed set, $N\beta ICl(A) \subseteq N\alpha ICl(A) \subseteq G$ where G is NO- set in $(U, \tau_R(X), I)$. Thus A is a $NIg\beta$-closed set.

Theorem 3.8. *In a NITS*
(1)Every $NIgP$-closed set is a $NIg\beta$- closed set.
(2) Every $NIgS$-closed set is a $NIg\beta$- closed set.

Proof : (1).Since every NPI- closed set is a $N\beta I$-closed set, $N\beta ICl(A) \subseteq NPICl(A)$. Thus A is a $NIgP$- closed set, $N\beta ICl(A) \subseteq NPICl(A) \subseteq G$ whenever $A \subseteq G$, G is NO-set. Hence A is a $NIg\beta$-closed set.
(2).Since every NSI- closed set is a $N\beta I$-closed set, $N\beta ICl(A) \subseteq NSICl(A)$. Thus A is a $NIgS$- closed set, $N\beta ICl(A) \subseteq NSICl(A) \subseteq G$ whenever $A \subseteq G$,G is NO-set. Hence A is a $NIg\beta$- closed set.

Definition 3.9. *A subset of a NITS $(U, \tau_R(X), I)$ is called Nano $\beta_{\mathcal{I}}$-set if $A = B \cap C$ where B is a NO-set and C is a $N\beta I$-closed set in $(U, \tau_R(X), I)$. The family of all Nano$\beta_{\mathcal{I}}$-sets are denoted by $N\beta_{\mathcal{I}}(U, X)$.*

Theorem 3.10. *In a NITS,*
(1). Every NO- set is a $N\beta_{\mathcal{I}}$-set.
(2). Every $N\beta I$-closed set is a $N\beta_{\mathcal{I}}$-set.

Proof: Proof is obvious.

Example 3.11. *Converse of the above theorem need not be true as shown in this example.*
Let $U = \{a, b, c, d\}$ be the Universe set with $U/R = \{\{d\}, \{c\}, \{a, b\}\}$ and $X = \{a, d\}$. The Nano topology $\tau_{R(X)} = \{\phi, \{d\}, \{a, b\}, \{a, b, d\}, U\}$ with Ideal $I = \{\phi, \{a\}\}$. Now
(1). $\{a, d\}$ is a $N\beta_{\mathcal{I}}$-set but not a NO- set.
(2). $\{a, b, d\}$ is a $N\beta_{\mathcal{I}}$-set but not a $N\beta I$- closed set.

Theorem 3.12. *Let A be a subset ofa a NITS $(U, \tau_R(X), I)$. Then, A is $N\beta_{\mathcal{I}}$-set if and only if $A = B \cap N\beta ICl(A)$ for a NO- set B in $(U, \tau_R(X), I)$*

Proof: Let A be a $N\beta_{\mathcal{I}}$-set. Then $A = B \cap C$ where B is a NO- set and C is a $N\beta I$-closed set.
Therefore $A \subseteq B \cap N\beta ICl(A) \subseteq B \cap N\beta ICl(C) = B \cap C = A$.
Thus $A = B \cap N\beta ICl(A)$.
Since $N\beta ICl(A)$ is $N\beta I$-closed, converse part of the Theorem is obvious.

Definition 3.13. *A subset A of a NITS $(U, \tau_R(X), I)$ is called Nano $\beta^* I$-open (briefly $N\beta^* I$-open)if $A \subseteq NCl(Nint^*(NCl(A)))$. The complement of Nano $\beta^* I$-open set is called a Nano $\beta^* I$-closed set and it is denoted by $N\beta^* I$-closed.*

Theorem 3.14. *Every $N\beta I$-open set in $(U, \tau_R(X), I)$ is a $N\beta^* I$-open set.*

Proof: Let A be a $N\beta I$-open set in $(U, \tau_R(X), I)$, then $A \subseteq NCl^*(Nint(NCl^*(A)))$. Since $\tau_R^*(X)$ is finer then $\tau_R(X)$, we have $A \subseteq NCl^*(Nint(NCl^*(A))) \subseteq NCl(Nint^*(NCl(A)))$. Thus A is $N\beta^* I$-open.

Example 3.15. *In Example 3.11, $N\beta^* I$-open sets are$\{\phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U\}$.Now $\{a, c\}$ is $N\beta^* I$-open set but not $N\beta I$-open set.*

4 Decompositions of Nano open sets and Nano βI- closed sets

Definition 4.1. *A subset A of $(U, \tau_R(X), I)$ is called Nano βI- Clopen(briefly $N\beta I$-Clopen) if A is both $N\beta I$-open and $N\beta I$-closed.*

Definition 4.2. *A subset A of $(U, \tau_R(X), I)$ is called Nano weak AB_I-set(briefly $N\mathtt{w}AB_I$-set) if $A = B \cap C$,where B is a NO-set and C is a $N\beta I$-Clopen set.*

Remark 4.3. *(1). Every NO-set is a $N\mathtt{w}AB_I$-set.*
(2) Every $N\beta I$-Clopen set is a $N\mathtt{w}AB_I$-set. But converse need not be true as shown in the following Example.

Example 4.4. *In Example 3.5,*
(1)The set $\{a, d\}$ is a $N\mathtt{w}AB_I$-set but not a $N\beta I$-Clopen set.
(2)The $\{a, b\}$ is a $N\mathtt{w}AB_I$-set but not a NO-set.

Theorem 4.5. *Let $A \subseteq U$ of $(U, \tau_R(X), I)$ is a $N\beta I$-closed set if and only if A is $NIg\beta$-closed and $N\beta_{\mathcal{I}}$-set.*

Proof: Necessary part follows from Theorem 3.3 and Theorem 3.10.
Conversely, suppose that A is $N\beta_{\mathcal{I}}$-set, then $A = B \cap N\beta ICl(A)$ where B is a NO -set. Now $A \subseteq B$ and since A is $NIg\beta$-closed, $N\beta ICl(A) \subseteq B$. Hence A is $N\beta I$-closed.

Theorem 4.6. *For a subset A of a space $(U, \tau_R(X), I)$ the following are equivalent.*

1. *A is NO- set,*
2. *A is $N\alpha I$-open and $N\mathtt{w}AB_I$-set,*
3. *A is $N\alpha I$-open and $N\beta_{\mathcal{I}}$-set.*

Proof : (1) $\implies$ (2). Since every NO- set is $N\alpha I$- open and by Remark 4.3, the proof is immediate .
(2) $\implies$ (3). Since every $N\beta I$-Clopen set is $N\beta I$-closed, proof is obvious.
(3) $\implies$ (1). Let A be a $N\beta_{\mathcal{I}}$-set then we have $A = B \cap C$, where B is a NO-set and C is a $N\beta I$-closed set. By Theorem 3.14, we have C is $N\beta^* I$-closed, $Nint(NCl^*(Nint(C))) \subseteq C$. Which implies that $Nint(NCl^*(Nint(C))) \subseteq Nint(C)$. Since A is $N\alpha I$-open and $A \subseteq C$, then $A \subseteq Nint(NCl^*(Nint(A))) \subseteq Nint(NCl^*(Nint(C))) \subseteq Nint(C)$. Thus $A \subseteq B \cap Nint(C) = Nint(B \cap C) = Nint(A)$. Hence A is a NO- set.

Remark 4.7. *In a NITS $(U, \tau_R(X), I)$,*
(1)$NIg\beta$-closed sets and $N\beta_{\mathcal{I}}$-sets are independent.
(2) $N\alpha I$-open sets and $N\mathtt{w}AB_I$-sets are independent.
(3) $N\alpha I$-open sets and $N\beta_{\mathcal{I}}$-sets are independent.

Example 4.8. *In Example 3.11, the set$\{b\}$ is $NIg\beta$-closed but not $N\beta_{\mathcal{I}}$-set and $\{a, b, d\}$ is a $N\beta_{\mathcal{I}}$-set but not a $NIg\beta$-closed set.*

Example 4.9. *In Example 3.5,*
(1) The set$\{a, b\}$ is a $N\mathtt{w}AB_I$-set but not a $N\alpha I$- open set and $\{a, c, d\}$ is a $N\alpha I$- open set but not a $N\mathtt{w}AB_I$-set.
(2) The set $\{a, b\}$ is a $N\beta_{\mathcal{I}}$-set but not a $N\alpha I$- open set and $\{a, c, d\}$ is a $N\alpha I$- open set but not a $N\beta_{\mathcal{I}}$-set.

Definition 4.10. *A mapping $\psi : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is called Nano ideal α generalized -continuous (briefly. $NI\alpha g$-continuous)(resp. Nano ideal generalized semi-continuous, Nano ideal generalized pre-continuous, Nano ideal generalized β-continuous (briefly., $NIgS$-continuous,$NIgP$-continuous, $NIg\beta$-continuous))if $\psi^{-1}(V)$ is a $NI\alpha g$-open(resp. $NIgS$-open,$NIgP$-open, $NIg\beta$-open) set in $(U, \tau_R(X), I)$ for every NO - set V in $(V, \tau_{R'}(Y))$.*

Definition 4.11. *A mapping $\psi : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is called Nano $\beta_{\mathcal{I}}$-continuous (briefly. $N\beta_{\mathcal{I}}$-continuous)(resp. nano weak AB_I-continuous (briefly. $N\mathtt{w}AB_I$-continuous)) if $\psi^{-1}(V)$ $N\beta_I$-open set(resp. $N\mathtt{w}AB_I$-set) in $(U, \tau_R(X), I)$ for every NO- set V in $(V, \tau_{R'}(Y))$.*

Theorem 4.12. *If a mapping $\psi : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$, then the following are hold.*
(1) Every $NI\alpha g$-continuous is a $NIg\beta I$- continuous.
(2)Every $NIgP$-continuous is a $NIg\beta$- continuous.
(3) Every $NIgS$-continuous is a $NIg\beta$- continuous.

Proof : it's Proof is obvious from Theorem 3.7 and Theorem 3.8.

Theorem 4.13. *Let ψ be a mapping from$(U, \tau_R(X), I)$ to $(V, \tau_{R'}(Y))$, then the following statements are equivalent.*

1. *ψ is nano continuous*
2. *ψ is nano αI-continuous and $N\mathtt{w}AB_I$-continuous,*
3. *ψ is nano αI-continuous and $N\beta_{\mathcal{I}}$-continuous.*

Proof : Proof is immediate from Theorem 4.6.

5 Nano βI- Compact spaces

Definition 5.1. *A collection $\{S_i : i \in \nabla\}$ of $N\beta$ I-open sets in a NITS $(U, \tau_R(X), I)$ is called $N\beta I$-open cover of a subset S of U, if $S \subseteq \bigcup\limits_{i\in\nabla} S_i$ holds.*

Definition 5.2. *A NITS $(U, \tau_R(X), I)$ is called a Nano β I-compact(briefly. $N\beta$ I-compact) space, if every $N\beta I$-open cover of U has a finite $N\beta I$-open subcover .*

Definition 5.3. *A subset S of a NITS$(U, \tau_R(X), I)$ is called $N\beta I$-compact relative to U, if for every collection $\{S_i : i \in \nabla\}$ of $N\beta I$-open subsets of U, such that $S \subseteq \bigcup\limits_{i\in\nabla} S_i$ there exists a finite subset ∇_0 of ∇, $S \subseteq \bigcup\limits_{i\in\nabla_0} S_i$.*

Theorem 5.4. *Let A be a $N\beta I$-closed subset of a $N\beta I$- compact space then A is $N\beta I$-compact relative to U.*

Proof : Let $(U, \tau_R(X), I)$ be a $N\beta I$- compact space and A be a $N\beta I$-closed subset of U. Now, A^c is $N\beta I$-open in U. Suppose that $C = \{S_i : i \in \nabla\}$ be the collection of $N\beta I$-open subsets of $(U, \tau_R(X), I)$ which covers A. Now, $C^* = C \cup A^c$ is a $N\beta I$- open cover of U. Thus $U = \bigcup\limits_{i\in\nabla} S_i \cup A^c$. Since $(U, \tau_R(X), I)$ is a $N\beta I$-compact space, then C^* has a finite subcover $\bigcup\limits_{i=1}^{n} S_i \cup A^c$ of U. Since A and A^c are disjoint then $A \subseteq \bigcup\limits_{i=1}^{n} S_i \in C$. Hence A is $N\beta I$-compact relative to U.

Theorem 5.5. *A NITS $(U, \tau_R(X), I)$ is $N\beta I$-compact space iff every family of $N\beta I$-closed sets of U having a finite intersection property has a non-empty intersection.*

Proof : Suppose that$(U, \tau_R(X), I)$ is a $N\beta I$-compact space. Let $\{S_i : i \in \nabla\}$ be a family of $N\beta I$-closed sets with finite intersection property. By contrary assume that $\bigcap\limits_{i\in\nabla} S_i = \phi$. Then , $U - \bigcap\limits_{i\in\nabla} S_i = U$. This implies $\bigcup\limits_{i\in\nabla} (U - S_i) = U$. Suppose that the collection $\{U - S_i : i \in \nabla\}$ is $N\beta I$-open cover of U. Since U is $N\beta I$-compact. Then $\{U - S_i : i \in \nabla\}$ has a finite subcover $\{U - S_i : i = 1, 2, \cdots n\}$. This implies $U = \bigcup\limits_{i=1}^{n} (U - S_i)$.

$\Rightarrow U = U - \bigcap\limits_{i=1}^{n} S_i$

$\Rightarrow U - U = U - [U - \bigcap\limits_{i=1}^{n} S_i]$

$\Rightarrow \bigcap\limits_{i=1}^{n} S_i = \phi$. But this is a contradiction to our assumption that $N\beta I$-closed sets of U having finite intersection property . This proves $\bigcap\limits_{i=1}^{n} S_i \neq \phi$.

Conversly, suppose every family of $N\beta I$-closed sets of U with finite intersection property has a non-empty intersection. Suppose U is not $N\beta I$-compact. Then there exists a $N\alpha I$-open cover of U has no finite subcover. Suppose $\{S_i : i \in \nabla\}$ is a $N\beta I$-open cover of U. There exists a finite subcollection $\{S_i : i = 1, 2, \cdots n\}$ of $\{S_i : i \in \nabla\}$such that $\bigcup\limits_{i=1}^{n} S_i \neq U$

$\Rightarrow U - \bigcup\limits_{i=1}^{n} S_i \neq U - U$. This implies $\bigcap\limits_{i=1}^{n} (U - S_i) \neq \phi$, for every nonempty finite subcollection of $\{S_i : i \in \nabla\}$. The set $\{U - S_i : i = 1, 2, \cdots, n\}$ is a collection of $N\beta I$-closed subsets of U that has the finite intersection property, $\bigcap\limits_{i\in\nabla} (U - S_i) \neq \phi$. That is $U - \bigcup\limits_{i\in\nabla} S_i \neq \phi$. This is a contradiction to our assumption $\{S_i : i \in \nabla\}$ is a$N\beta I$-open cover of U. Thus $(U, \tau_R(X), I)$ is $N\beta I$-compact space.

Theorem 5.6. *Every $N\beta I$-compact space is a $N\alpha I$- compact space.*

Proof : Suppose that the collection $\mathcal{C} = \{S_i : i \in \nabla\}$ is a $N\alpha I$-open covering of U. Since every $N\alpha I$-open set is a $N\beta I$-open set and $(U, \tau_R(X), I)$ is a $N\beta I$-compact space,, then $\mathcal{C}$ has a finite $N\alpha I$-open subcover $C_0 = \{S_i : i = 1, 2, \cdots n\}$ of U. Thus $(U, \tau_R(X), I)$ is $N\alpha I$- compact space.

Remark 5.7. *The converse of Theorem 5.6 need not be true as shown in the following example.*

Example 5.8. *Let $U = \{a_1, a_2, a_3, a_4\}$ be the universe with $U/R = \{\{a_1\}, \{a_3\}, \{a_2, a_4\}\}$. Let $X = \{a_1, a_2\} \subseteq U$ and $\tau_R(X) = \{\phi, \{a_1\}, \{a_2, a_4\}, \{a_1, a_2, a_4\}, U\}$ be a nano topology with Ideal $I = \{\phi, \{a_1\}\}$. Then $(U, \tau_R(X), I)$ is a $N\alpha I$- compact space.But it is not a $N\beta I$-compact space. Because $\{\{a_1\}, \{a_4\}\ \{a_2, a_3\}\}$ $N\beta I$- open cover of $(U, \tau_R(X), I)$ does not have any finite $N\beta I$-open subcover for $(U, \tau_R(X), I)$.*

Theorem 5.9. *Every $N\beta I$-compact space is NPI- compact space.*

Proof : Suppose that the collection $\mathcal{C} = \{S_i : i \in \nabla\}$ is a NPI-open covering of U. Since every NPI-open set is a $N\beta I$-open set and $(U, \tau_R(X), I)$ is a $N\beta I$-compact space,then $\mathcal{C}$ has a finite NPI-open subcover $C_0 = \{S_i : i = 1, 2, \cdots n\}$ of U. Thus $(U, \tau_R(X), I)$ is NPI- compact space.

Example 5.10. *Let $U = \{a_1, a_2, a_3, a_4\}$ be the universe with $U/R = \{\{a_1\}, \{a_2\}, \{a_3, a_4\}\}$. Let $X = \{a_2, a_3\} \subseteq U$ and $\tau_R(X) = \{\phi, \{a_2\}, \{a_3, a_4\}, \{a_2, a_3, a_4\}, U\}$ be a nano topology with Ideal $I = \{\phi, \{a_1\}\}$. Then $(U, \tau_R(X), I)$ is a NPI- compact space. It is not a $N\beta I$-compact space. Because $\{\{a_2\}, \{a_1, a_3\}, \{a_3, a_4\}\}$ $N\beta I$- open cover of $(U, \tau_R(X), I)$ does not have any finite $N\beta I$-open subcover for $(U, \tau_R(X), I)$.*

Theorem 5.11. *Every $N\beta I$-compact space is NSI- compact space.*

Proof : Suppose that the collection $\{S_i : i \in \nabla\}$ is a NSI-open covering of U. Since every NPI-open set is a $N\beta I$-open set and $(U, \tau_R(X), I)$ is a $N\beta I$-compact space, $\mathcal{C}$ has a finite NSI-open subcover $C_0 = \{S_i : i = 1, 2, \cdots n\}$ of U. Thus $(U, \tau_R(X), I)$ is NSI- compact space.

Example 5.12. *Let $U = \{a_1, a_2, a_3\}$ be the universe with $U/R = \{\{a_2\}, \{a_1, a_3\}\}$. Let $X = \{a_1, a_3\} \subseteq U$ and $\tau_R(X) = \{\phi, \{a_1, a_3\}, U\}$ be a nano topology with Ideal $I = \{\phi, \{a_1\}\}$. Then $(U, \tau_R(X), I)$ is a NSI-compact space. Now $\{\{a_1, a_3\}, \{a_2, a_3\}\}$ $N\beta I$- open cover of $(U, \tau_R(X), I)$ does not have any finite $N\beta I$-open subcover for $(U, \tau_R(X), I)$.Hence $(U, \tau_R(X), I)$ is not a $N\beta I$-compact space.*

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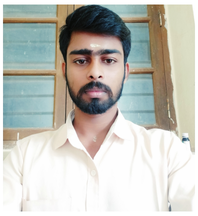

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