VOLUME XI
ISBN No.: 978-93-94004-00-9
Physical Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

Pollachi-642001

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One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

International Conference on Emerging Trends in Science and Technology (ETIST 2021) Jointly Organized by Department of Biological Science, Physical Science and Computational Science Nallamuthu Gounder Mahalingam College, Affiliated to Bharathiar University, Tamilnadu, India.
Published by NGMC - November 2021

$N\alpha I$-CONNECTED SPACES

V. Inthumathi[1], **R. Abinprakash**[2]

Abstract: Our main motive of this research work is to study the concepts of $N\alpha I$-connectedness,NPI-connectedness, NSI-connectedness, $N\beta I$-connectedness and study the interrlationship between these concepts in nano ideal topological spaces. Also we verify the concepts of $N\alpha I$-connectedness and nano connectedness are equivalent.

Keywords : Nano ideal topology, Nano connected, $N\alpha I$-connected, NPI-connected, NSI-connected, $N\beta I$-connected spaces. **2010 Subject classification:** 54A05, 54A10, 54B05

1 Introduction

The concept of connectedness is a crucial property in Topological spaces.The notion of α-connectedness, Pre connectedness, Semi connectedness and β-connectedness are introduced and studied by Jafari et.al. [9], Popa [16], Piptone [15] and Popa et.al. [17] respectively. The concept of ideal topological space was first introduced by Kuratowski [11]. Later Hamlett and Jankovic [4] investicated the further properties of ideal topological spaces. In 2008, the concept of connectedness in ideal topological spaces was first introduced by Ekici and Noiri [1] . Recently many authors introduced and investicated the new types connectedness concepts [2, 14] in ideal topological spaces. The contemporary researchers pulling this concept into varies branches of Topological spaces.

Lellis Thivagar and Carmel Richard[12] introduced the notion of Nano topological spaces. In 2016, Lellis Thivagar and Sudha devi [13] defined the notion of nano ideal topological spaces and discussed some of its properties.

The notion of nano connectedness in nano topological spaces was introduced by Krishnaprakash et.al. [10] . In this research work we introduce the concepts of $N\alpha I$-connectedness,NPI-connectedness, NSI-connectedness and $N\beta I$-connectedness in nano ideal topological spaces. We also characterize and interrelate with these connectedness.

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2 Preliminaries

Definition 2.1. *[12] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let* $X \subseteq U$.

1. *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by* $L_R(X)$.
   $L_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}$, *where R(x) denotes the equivalence class determined by x.*
2. *The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by* $U_R(X)$.
   $U_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \cap X \neq \phi\}$
3. *The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by* $B_R(X)$.
   $B_R(X) = U_R(X) - L_R(X)$

Definition 2.2. *[12] Let U be the universe, R be an equivalence relation on U and* $\tau_R(X) = \{\phi, U, L_R(X), U_R(X), B_R(X)\}$ *where* $X \subseteq U$. *Then* $\tau_R(X)$ *satisfies the following axioms:*

1. ϕ *and U are in* $\tau_R(X)$.
2. *The union of the elements of any subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.
3. *The intersection of the elements of any finite subcollection of* $\tau_R(X)$ *is in* $\tau_R(X)$.

That is, $\tau_R(X)$ *is a topology on U called the nano topology(NT) on U with respect to X and* $(U, \tau_R(X))$ *as the nano topological space (NTS). The elements of* $\tau_R(X)$ *are called as nano open (N-O)sets.*

Definition 2.3. *[12] Let* $(U, \tau_R(X))$ *be a NTS, the set* $B = \{\phi, U, L_R(X), B_R(X)\}$ *is the basis for* $\tau_R(X)$.

Definition 2.4. *[4] An ideal I on a topological space is a non-empty collection of subsets of X which satisfies*
1. $A \in I$ *and* $B \subseteq A$ *implies* $B \in I$.
2. $A \in I$ *and* $B \in I$ *implies* $A \cup B \in I$.

Definition 2.5. *[13] A NTS* $(U, \tau_R(X))$ *with an ideal I on U is called a Nano Ideal Topological space(NITS) and denoted as* $(U, \tau_R(X), I)$.

Definition 2.6. *[13] Let*$(U, \tau_R(X), I)$ *be a NITS. A set operator* $A^{*N} : P(U) \to P(U)$ *is called the nano local function of I on U with respect to I on* $\tau_R(X)$ *is defined as* $A^{*N} = \{x \in U : U \cap A \notin I\ ; \text{for every } U \in \tau_R(X)\}$ *and is denoted by* A^{*N}, *where nano closure operator is defined as* $NCl^*(A) = A \cup A^{*N}$.

Definition 2.7. *A subset A of a NITS $(U, \tau_R(X), I)$ is called,*

1. *$N\alpha I$ - open [13] if $A \subseteq Nint(NCl^*(Nint(A)))$*
2. *NSI- open [13] if $A \subseteq NCl^*(Nint(A))$.*
3. *NPI -open [6] if $A \subseteq Nint(NCl^*(A))$.*
4. *$N\beta I$ - open [5] if $A \subseteq NCl^*(Nint(NCl^*(A)))$.*

The family of all $N\alpha I$- open (resp., NSI- open, NPI-open,$N\beta I$-open) sets of a NITS is denoted by $N\alpha IO(U, X)$ (resp. $NSIO(U, X)$, $NPIO(U, X)$,$N\beta IO(U, X)$).
A subset A of a NITS $(U, \tau_R(X), I)$ is said to be $N\alpha I$-closed (resp.,NSI-closed, NPI-closed,$N\beta I$-closed), if its complement is $N\alpha I$-open (resp., NSI- open ,NPI-open,$N\beta I$- open).

Definition 2.8. *[7] A mapping $\psi : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is called $N\alpha I$-continuous (resp., NSI-continuous, NPI-continuous) if $\psi^{-1}(B)$ is $N\alpha I$-open (resp., NSI-open, NPI-open) set in $(U, \tau_R(X), I)$ for every N-O set B in $(V, \tau_{R'}(Y))$.*

Definition 2.9. *[8] A mapping $\psi : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y))$ is called $N\beta I$-continuous if $\psi^{-1}(B)$ is $N\beta I$-open set in $(U, \tau_R(X), I)$ for every N-O set B in $(V, \tau_{R'}(Y))$.*

Definition 2.10. *[7] A mapping $\psi : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is called $N\alpha I$-irresolute (resp., NSI-irresolute, NPI -irresolute) if $\psi^{-1}(B)$ is $N\alpha I$-open (resp., NSI-open, NPI-open) set in $(U, \tau_R(X), I)$ for every $N\alpha I$-open (resp., NSI-open, NPI-open) set B in $(V, \tau_{R'}(Y), J)$*

Definition 2.11. *[18] Let $(U, \tau_R(X))$ be a NTS. If V is a subset of $(U, \tau_R(X))$ and the collection $\tau_R(V, X) = \{V \cap B : B \in \tau_R(X)\}$ is a NT on V with respect to X, then $\tau_R(V, X)$ is called a nano subspace topology(NST).*

Definition 2.12. *[10] A NTS $(U, \tau_R(X))$ is called nano connected if $(U, \tau_R(X))$ cannot be expressed as a disjoint union of two non empty N-O sets. A subset of $(U, \tau_R(X))$ is nano connected as a subspace. A subset is said to be nano disconnected if and only if it is not nano connected.*

3 $N\alpha I$-connected spaces

Definition 3.1. *Two non empty subsets A and B of $(U, \tau_R(X), I)$ is called nano αI-separated(briefly. $N\alpha I$-separated)sets if $A \cap N\alpha ICl(B) = \phi$ and $B \cap N\alpha ICl(A) = \phi$.*
Any two $N\alpha I$-separated sets are disjoint.

Definition 3.2. *A NITS $(U, \tau_R(X), I)$ is called nano αI-connected(briefly. $N\alpha I$-connected) space if U can not be expressed as a union of two $N\alpha I$-separated sets. A subset of $(U, \tau_R(X), I)$ is $N\alpha I$-connected as a subspace.*

Definition 3.3. *A NITS $(U, \tau_R(X), I)$ is said to be nano αI-disconnected if it is not $N\alpha I$-connected.*

Definition 3.4. *Two non empty subsets A and B of* $(U, \tau_R(X), I)$ *is called nano pre I-separated(briefly.NPI-separated)sets if* $A \cap NPICl(B) = \phi$ *and* $B \cap NPICl(A) = \phi$.
Any two NPI*-separated sets are disjoint.*

Definition 3.5. *Two non empty subsets A and B of* $(U, \tau_R(X), I)$ *is called nano semi I-separated(briefly.* NSI*-separated)sets if* $A \cap NSICl(B) = \phi$ *and* $B \cap NSICl(A) = \phi$.
Any two NSI*-separated sets are disjoint.*

Definition 3.6. *Two non empty subsets A and B of* $(U, \tau_R(X), I)$ *is called nano* βI*-separated(briefly.* $N\beta I$*-separated)sets if* $A \cap N\beta ICl(B) = \phi$ *and* $B \cap N\beta ICl(A) = \phi$.
Any two $N\beta I$*-separated sets are disjoint.*

Definition 3.7. *A NITS* $(U, \tau_R(X), I)$ *is called nano pre I-connected(resp., nano semi I-connected, nano* β *I-connected)(briefly.* NPI*-connected, NSI-connected,* $N\beta I$*-connected) space if U can not be expressed as a union of two* NPI*-separated sets(resp., NSI-separated sets,* $N\beta I$*-separated sets).*

Definition 3.8. *A NITS* $(U, \tau_R(X), I)$ *is said to be nano pre I-disconnected(resp., nano semi I-disconnected, nano* βI*-disconnected) if and only if it is not NPI-connected (resp., NSI-connected, N* βI*-connected).*

Theorem 3.9. *A NITS* $(U, \tau_R(X), I)$ *is* $N\alpha I$*-connected if and only if U cannot be written as a disjoint union of* $N\alpha I$*-open sets of* $(U, \tau_R(X), I)$.

Proof : Let $(U, \tau_R(X), I)$ be a $N\alpha I$-connected space. Contrarily assume that $U = A \cup B$, where A and B are non empty disjoint $N\alpha I$-open sets in $(U, \tau_R(X), I)$. Since A and B are $N\alpha I$-closed sets in $(U, \tau_R(X), I)$, $A \cap N\alpha ICl(B) = \phi = N\alpha ICl(A) \cap B$. Which is contradiction to the fact. Hence U cannot be written as a disjoint union of $N\alpha I$-open sets of $(U, \tau_R(X), I)$.
Conversely, Suppose that $U = A \cup B$, where A and B are non empty disjoint $N\alpha I$-open sets in $(U, \tau_R(X), I)$ and $A \cap N\alpha ICl(B) = \phi = N\alpha ICl(A) \cap B$.Which implies that A and B are $N\alpha I$-separated sets. Thus U is not $N\alpha I$-connected.

Theorem 3.10. *Let C be a* $N\alpha I$*-connected subset of* $(U, \tau_R(X), I)$*. If* $U = A \cup B$*, where A and B are* $N\alpha I$*-separated sets then either* $C \subset A$ *or* $C \subset B$.

Proof : Let $U = A \cup B$, where A and B are $N\alpha I$-separated sets of $(U, \tau_R(X), I)$. Since $C \subset U = A \cup B$. Now $C = C \cap (A \cup B)$=$(C \cap A) \cup (C \cap B)$.We know that C is $N\alpha I$-connected, then either $(C \cap A) = \phi$ or $(C \cap B) = \phi$. Thus either $C \subset A$ or $C \subset B$.

Proposition 3.11. *If A is* $N\alpha I$*-connected subset of a* $N\alpha I$*-connected space* $(U, \tau_R(X), I)$ *such that* $U - A = B \cup C$*, B and C are* $N\alpha I$*-separated sets then* $A \cup B$ *and* $A \cup C$ *are* $N\alpha I$*-connected.*

Proof : Let $A \cup B$ is not $N\alpha I$-connected, there exist $N\alpha I$-separated sets G and H , $A \cup B = G \cup H$ and $A \subset G \cup H$. Since A is $N\alpha I$-connected, by Theorem 3.10 we have $A \subset G$ or $A \subset H$. Suppose $A \subset G \implies A \cup B \subseteq G \cup B$, Also $A \cup B = G \cup H \subseteq G \cup B \implies H \subseteq B$.
Since B and C are $N\alpha I$-separated sets, H and C are $N\alpha I$-separated sets. Thus H is $N\alpha I$-separated from G as well as C. Now we have to prove $N\alpha ICl(H) \cap (G \cup C) = N\alpha ICl(G \cup C) \cap H = \phi$. Thus, $N\alpha ICl(H) \cap (G \cup C) = (N\alpha ICl(H) \cap G) \cup (N\alpha ICl(H) \cap C) = \phi$. Also $N\alpha ICl(G \cup C) \cap (H) = \{N\alpha ICl(G) \cup N\alpha ICl(C)\} \cap H = \{N\alpha ICl(G) \cap H\} \cup \{N\alpha ICl(C) \cap H\} = \phi$. Hence H and $(G \cup C)$ are

$N\alpha I$-separated sets. Since $U-A=B\cup C \implies U=A\cup(B\cup C)=(A\cup B)\cup C=(G\cup H)\cup C=H\cup(G\cup C)$
.
Thus U is not a $N\alpha I$-Connected. Which is contradiction to the hypothisis. Hence $A\cup B$ is $N\alpha I$-Connected. Simillerly, We can prove $A\cup C$ is $N\alpha I$-Connected.

Theorem 3.12. *A non empty proper subset A of a NITS $(U,\tau_R(X),I)$ is both $N\alpha I$-open and $N\alpha I$-closed if and only if A is both N-O set and N-C set in $(U,\tau_R(X),I)$.*

Proof : Let A be a both $N\alpha I$-open and $N\alpha I$-closed in $(U,\tau_R(X),I)$. Now, $NCl^*(Nint(A))=A$ and $Nint(NCl^*(A))=A$.Hence A is N-O set and N-C set.
Conversly suppose that A is both N-O set and N-C set. Since every N-O set is $N\alpha I$-open. Hence A is both $N\alpha I$-open and $N\alpha I$-closed in $(U,\tau_R(X),I)$.

Theorem 3.13. *If $(U,\tau_R(X),I)$ be a NITS then the following are equivalent.*

1. *$(U,\tau_R(X),I)$ is $N\alpha I$-connected.*
2. *U and ϕ are the only both $N\alpha I$-open and $N\alpha I$-closed subsets of $(U,\tau_R(X),I)$.*
3. *Each $N\alpha I$-continuous mapping of $(U,\tau_R(X),I)$ into a nano discrete space $(V,\tau'_R(Y))$ with atleast two points is a constant mapping.*

Proof :(1) $\implies$ (2). Let as assume that A be both $N\alpha I$-open and $N\alpha I$-closed subset of $(U,\tau_R(X),I)$. Also, we have $U-A$ is both $N\alpha I$-open and $N\alpha I$-closed subset of $(U,\tau_R(X),I)$. Thus $U=A\cup(U-A)$, where A and $U-A$ are the disjoint non empty $N\alpha I$-open subsets of U. Which is a contradiction to $(U,\tau_R(X),I)$ is $N\alpha I$-connected. Hence $A=\phi$ or $A=U$.
(2) $\implies$ (1). Conversly suppose that $U=A\cup B$,where A and B are disjoint non empty $N\alpha I$-open sets of $(U,\tau_R(X),I)$. Let $B=U-A$, A be a both $N\alpha I$-open and $N\alpha I$-closed set. Since $A=\phi$ or $A=U$, thus $(U,\tau_R(X),I)$ is a $N\alpha I$-connected space.
(2) $\implies$ (3). Let $\psi:(U,\tau_R(X),I)\to(V,\tau'_R(Y))$ be a $N\alpha I$-continuous mapping. Now $(U,\tau_R(X),I)$ is covered by $N\alpha I$-open and $N\alpha I$-closed covering $\{\psi^{-1}(v):v\in V\}$. By assumption $\{\psi^{-1}(v)=\phi$ or U $\forall v\in V\}$. If $\psi^{-1}(v)=\phi$ then ψ fails to be a mapping. Then there exsits only one point $v\in V$ such that $\psi^{-1}(v)\neq\phi$ and hence $\psi^{-1}(v)=U$. This shows that ψ is constant mapping.
(3) $\implies$ (2). Let A be a $N\alpha I$-open and $N\alpha I$-closed set in U. Let $\psi:(U,\tau_R(X),I)\to(V,\tau'_R(Y))$ be a $N\alpha I$-continuous mapping defined by $\psi(A)=\{x_i\}$ and $\psi(U-A)=\{x_{i+1}\}$. By our assumption ψ is constant mapping. Thus $A=U$.

Theorem 3.14. *Let $(U,\tau_R(X),I)$ be a NITS and A and B be $N\alpha I$-connected subsets of $(U,\tau_R(X),I)$. If $A\cap B\neq\phi$ then $A\cup B$ is $N\alpha I$-connected.*

Proof : If $A\cup B$ is not $N\alpha I$-connected then there exist two disjoint non empty $N\alpha I$-open sets C and D such that $A\cup B=C\cup D$. Since A is a $N\alpha I$-connected subset of U. By Theorem 3.10, we have $A\subseteq C$ and $B\subseteq D$ or $A\subseteq D$ and $B\subseteq C$. Suppose that $A\subseteq C$ and $B\subseteq D$. Since $A\cap B\neq\phi$, $x_i\in A\cap B \implies x_i\in A$and $x_i\in B$. Thus $x_i\in C$ and $x_i\in D$. Which is contradiction to the fact C and D are disjoint . Hence $A\cup B$ is a $N\alpha I$-connected.

Theorem 3.15. *Let $(U,\tau_R(X),I)$ be a NITS and $\{A_i\}$be the family of $N\alpha I$-connected subspaces of $(U,\tau_R(X),I)$,If $\bigcap_{\forall i} A_i\neq\phi$then $\bigcup_{\forall i} A_i$ is a $N\alpha I$-connected space.*

It's proof is similar to proof of Theorem 3.14.

4 Properties of $N\alpha I$-connected spaces

Definition 4.1. *A mapping $\psi : (U, \tau_R(X), I) \to (V, \tau_{R'}(Y), J)$ is called $N\beta I$-irresolute if $\psi^{-1}(B)$ is $N\beta I$-open set in $(U, \tau_R(X), I)$ for every $N\beta I$-open set B in $(V, \tau_{R'}(Y), J)$.*

Theorem 4.2. *Every NPI-connected space $(U, \tau_R(X), I)$ is nano connected.*

Proof : Let $(U, \tau_R(X), I)$ be a NPI-connected space.Contrarily assume that $(U, \tau_R(X), I)$ is not nano connected, there exists a separation $U = A \cup B$, where A and B be two disjoint non empty N-O sets. We know that every N-O set is NPI-open. Which is contradiction to the fact, $(U, \tau_R(X), I)$ be a NPI-connected. Hence $(U, \tau_R(X), I)$ is connected.

Corollary 4.3. *In a NITS $(U, \tau_R(X), I)$,the following are hold.*

1. *Every NSI-connected space is nano connected.*
2. *Every $N\beta I$-connected space is nano connected.*

Proof : It follows from the fact that every N-O set is a NSI-open set and $N\beta I$-open set.

Corollary 4.4. *Every NPI-connected(resp.NSI-connected) space $(U, \tau_R(X), I)$ is $N\alpha I$-connected.*

Proof : Proof follows from the fact that every $N\alpha I$-open set is NPI-open set and NSI-open) set.

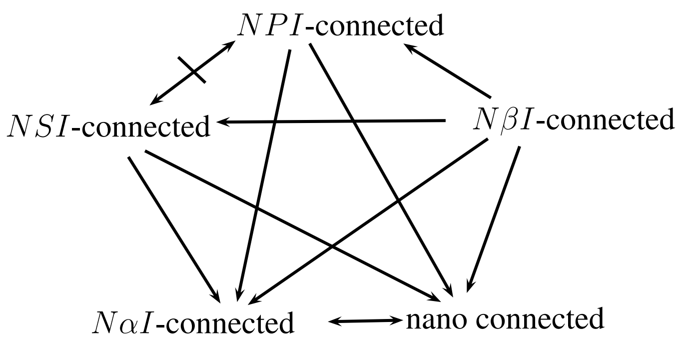
Corollary 4.5. *Every $N\beta I$-connected space $(U, \tau_R(X), I)$ is*

1. *$N\alpha I$-connected.*
2. *NSI-connected.*
3. *NPI-connected.*

Proof : It is obvious from the fact that every $N\alpha I$-open(resp. NSI-open, NPI-open) set is $N\beta I$-open.

Remark 4.6. *From Theorem 3.12, we have $N\alpha I$-connected space and nano connected space are equivalent in $(U, \tau_R(X), I)$.*

Remark 4.7. *The notions of NPI-connected spaces and NSI-connected spaces are independent.*



The above figure illustrates the relations of nano connectedness.

Remark 4.8. *The following Examples show the converse of above implications are not true.*

Example 4.9. *Let* $U = \{a_1, a_2, a_3, a_4\}$ *be the universe with* $U/R = \{\{a_1\}, \{a_4\}, \{a_2, a_3\}\}$ *and* $X = \{a_1, a_4\}$. *Let* $\tau_R(X) = \{\phi, \{a_1, a_4\}, U\}$*with* $I = \{\phi, \{a_1\}\}$. *Now* $(U, \tau_R(X), I)$ *is* NSI*-connected but not* NPI*-connected and* $N\beta I$*-connected.*

Example 4.10. *Let* $U = \{a_1, a_2, a_3, a_4\}$ *be the universe with* $U/R = \{\{a_4\}, \{a_3\}, \{a_1, a_2\}\}$. *Let* $X = \{a_1, a_4\} \subseteq U$ *and* $\tau_R(X) = \{\phi, \{a_4\}, \{a_1, a_2\}, \{a_1, a_2, a_4\}, U\}$*be a nano topology with ideal* $I = \{\phi, \{a_1\}\}$. *Now* $(U, \tau_R(X), I)$ *is* NPI*-connected but not* NSI*-connected and* $N\beta I$*-connected.*

Example 4.11. *Let* $U = \{a_1, a_2, a_3, a_4\}$ *be the universe with* $U/R = \{\{a_1\}, \{a_3\}, \{a_2, a_4\}\}$. *Let* $X = \{a_1, a_2\} \subseteq U$ *and* $\tau_R(X) = \{\phi, \{a_1\}, \{a_2, a_4\}, \{a_1, a_2, a_4\}, U\}$ *be a nano topology with ideal* $I = \{\phi, \{a_1\}\}$. *Now* $(U, \tau_R(X), I)$ *is nano connected but not* NPI*-connected,* NSI*-connected and* $N\beta I$*-connected.*

Theorem 4.12. *If* $\psi : (U, \tau_R(X), I) \to (V, \tau'_R(Y))$ *is a* $N\alpha I$*-continuous surjection mapping and* $(U, \tau_R(X), I)$ *is* $N\alpha I$*-connected then* $(V, \tau'_R(Y))$ *is nano connected.*

Proof : Let $(V, \tau'_R(Y))$ be not a nano connected space, then there exist a separation $V = A \cup B$, where A and B are nonempty disjoint N-O sets in V. Since ψ is $N\alpha I$-continuous surjection mapping then $U = \psi^{-1}(A) \cup \psi^{-1}(B)$, where $\psi^{-1}(A)$ and $\psi^{-1}(B)$ are nonempty disjoint $N\alpha I$-open sets in U. Which is contrdiction to the fact $(U, \tau_R(X), I)$ is $N\alpha I$-connected. Thus $(V, \tau'_R(Y))$ is nano connected.

Corollary 4.13. *If* $\psi : (U, \tau_R(X), I) \to (V, \tau'_R(Y))$ *is a* $N\alpha I$*-continuous surjection mapping and* $(V, \tau'_R(Y))$ *is nano connected then* $(U, \tau_R(X), I)$ *is* $N\alpha I$*-connected.*

Proof : Let $(U, \tau_R(X), I)$ be not a $N\alpha I$-connected space, then there exist a separation $U = \psi^{-1}(A) \cup \psi^{-1}(B)$, where $\psi^{-1}(A)$ and $\psi^{-1}(B)$ are nonempty disjoint $N\alpha I$-open sets in U. Since ψ is nano αI-continuous surjection mapping then $V = A \cup B$, where A and B are nonempty disjoint N-O sets in V. This contrdicts the fact $(V, \tau'_R(Y))$ is nano connected. Hence $(U, \tau_R(X), I)$ a $N\alpha I$-connected space.

Corollary 4.14. *If* $\psi : (U, \tau_R(X), I) \to (V, \tau'_R(Y))$ *is a NPI-continuous(resp.* $N\beta I$*-continuous, NSI-continuous) surjection mapping and* $(U, \tau_R(X), I)$ *is NPI-connected (resp.* $N\beta I$*-connected ,*NSI*-connected)then* $(V, \tau'_R(Y))$ *is nano connected.*

Proof : Proof is similar to proof of Theorem 4.12.

Theorem 4.15. *If* $\psi : (U, \tau_R(X), I) \to (V, \tau'_R(Y), J)$ *is a* $N\alpha I$*-irresolute surjection mapping and* $(U, \tau_R(X), I)$ *is* $N\alpha I$*-connected then* $(V, \tau'_R(Y), J)$ *is* $N\alpha I$*-connected.*

Proof : Let $(V, \tau'_R(Y), J)$ is not a nano αI-connected space, then there exist a separation $V = A \cup B$, where A and B are nonempty disjoint $N\alpha I$-open sets in V. Since $\psi$is $N\alpha I$-irresolute surjection mapping then $U = \psi^{-1}(A) \cup \psi^{-1}(B)$, where $\psi^{-1}(A)$ and $\psi^{-1}(B)$ are nonempty disjoint $N\alpha I$-open sets in U. Which is contrdiction to the fact $(U, \tau_R(Y), I)$ is $N\alpha I$-connected. Thus $(V, \tau'_R(Y), J)$ is $N\alpha I$-connected.

Corollary 4.16. *If* $\psi : (U, \tau_R(X), I) \to (V, \tau'_R(Y), J)$ *is a NPI-irresolute(resp.* $N\beta I$*-irresolute, NSI-irresolute) surjection mapping and* $(U, \tau_R(X), I)$ *is* NPI*-connected(resp.* $N\beta I$*-connected,*NSI*-connected) then* $(V, \tau'_R(Y), J)$ *is* NPI*-connected(resp.* $N\beta I$*-connected,*NSI*-connected).*

Proof : Proof is simillar to proof of Theorem 4.15

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