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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

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Power Domination on Splitting and Degree Splitting Graph of Certain Graphs

Huldah Samuel [1] – K. Sathish Kumar [2] – J. Jayasudha [3]

ABSTRACT: A set $\boldsymbol{S}$ of vertices is defined to be a power dominating set of a graph $\boldsymbol{G}$ if every vertex and every edge in the system is monitored by the set $\boldsymbol{S}$ (following a set of rules for power system monitoring). The minimum cardinality of a power dominating set of a graph $\boldsymbol{G}$ is the power domination number $\boldsymbol{\gamma_p(G)}$. When operations on graphs are performed new kinds of graphs result from the initial graphs considered. The splitting and degree splitting are such operations, having some applications as well. In this paper, we investigate the power domination number $\boldsymbol{\gamma_p}$ of the splitting and degree splitting graphs of certain classes of graphs.

Keywords: Domination, Power Domination, Splitting graphs, Degree splitting graphs.

1. INTRODUCTION

Let $G = (V, E)$ be a finite, undirected and simple graph, with the number of vertices $|V(G)| = n$. A subset $S \subseteq V$ is a dominating set of G [2, 3] if every vertex in $V - S$ has at least one neighbour in S. A dominating set S of G is called a minimum dominating set, if S consists of a minimum number of vertices among all dominating sets of G. Based on the concept of domination in graphs, Haynes et al. [2] developed the concept of power domination while formulating in graph theoretical terms, a problem related to electric power system. There has been a number of studies on the power domination number for common graph classes [3] and also on the relationship between domination number and power domination number.

A subset $S \subseteq V$is a power dominating set [2] of G if all the vertices of V can be observed recursively by the following rules: i) all vertices in the neighbour set $N[S]$ are observed initially and ii) if an observed vertex u has all its neighbours observed except for one non-observed neighbour v, then v is observed (by u).

Given a graph G, while the domination number $\gamma(G)$ represents the number of vertices in a minimum dominating set of G, the power domination number $\gamma_p(G)$ is the minimum number of vertices required for a power dominating set of G. Let $P_n, C_n, W_{1,n}, K_n, K_{m,n}, K_{1,n}, B(n, n)$, and F_n denote respectively, the path, cycle, wheel, complete

Huldah Samuel*[1], Department of Mathematics, Madras Christian College, Tambaram, Tamil Nadu, India. E-mail: huldahsamuel@mcc.edu.in
K. Sathish Kumar*[2], Department of Mathematics, Madras Christian College, Tambaram, Tamil Nadu, India. E-mail: sathishkumar@mcc.edu.in
J. Jayasudha*[3], Department of Mathematics, Nallamuthu Gounder Mahalingam College, Coimbatore, Tamil Nadu, India. E-mail: jayasudha@ngmc.org

graph, complete bipartite graph, star graph, bistar graph and friendship graphs of order n [1, 7, 9, 10]. For undefined terminology and notation, we refer the reader to [1]. In this paper, we investigate power domination number of some splitting and degree splitting graphs.

2. MAIN RESULTS

2.1 Splitting Graphs

The splitting graph $S(G)$ was introduced by Sampathkumar and Walikar [6].

Definition 1. Let G be a (x, y) graph . The splitting graph S(G) of G is obtained as follows. For each vertex v of a graph G, take a new vertex v' and join v' to all the vertices of G that are adjacent to v. Observe that S(G) is a $(2x, 3y)$ graph.

Theorem 2 [3] *For any graph G, $1 \leq \gamma_p(G) \leq \gamma(G)$.*

Theorem 3 *For a connected graph G of order $n \geq 2$,*

$$\gamma_p(G) \leq \gamma_p(S(G)) \leq \gamma(G).$$

Proof. Let G be a connected graph with $V(G) = \{v_1, v_2, \ldots, v_n\}$. By the definition of $S(G)$ of a graph G, new vertices $v'_1, v'_2, \cdots, v'_n$ are introduced and for all $i, (1 \leq i \leq n)$ the vertex v'_i is adjacent to the neighbours of v_i in G. This newly constructed graph $S(G)$ consists of $V \cup V'(= 2n)$ vertices. If $D_{S(G)} = X \cup X'$ is a γ_p-set for $S(G)$ such that $X \subseteq V$ and $X' \subseteq V'$, then $D_G = X \cup Y$ is a power dominating set for G such that $Y \subseteq V$. So $\gamma_p(G) \leq \gamma_p(S(G))$.

The upper bound is consequence of the following Theorem 5

Theorem 4 [4] *If a graph G has no isolated vertices, then $\gamma(G) \leq n/2$.*

Theorem 5 *For a path graph P_n of order $n \geq 2$,*

$$\gamma_p(S(P_n)) = 1.$$

Proof. Consider a path P_n with n vertices and $n-1$ edges. Let the vertices be denoted by $v_1, v_2, v_3, \ldots, v_n$. By the construction of splitting graph $S(P_n)$ of the path graph, new vertices $v'_1, v'_2, \cdots, v'_n$ are introduced and for all $i, (1 \leq i \leq n)$ the vertex v'_i is adjacent to the neighbours of v_i in P_n. We claim that $D = \{v_2\}$ is a γ_p-set for $S(P_n)$ because $\{v_1, v_3, v'_1, v'_3\}$ are dominated initially and other vertices of $S(P_n)$ are power dominated by v_2. Hence $\gamma_p(S(P_n)) = 1$.

Remark 6 *From Theorem 4, we note that for a path P_n, $n \geq 4$, the domination number $\gamma(P_n) \geq 2$ while $\gamma_p(DS(P_n))$ is 1.*

2.2 Degree Splitting Graphs

Ponraj and Somasundaram [5] introduced the concept of degree splitting graph $DS(G)$ of a graph G.

Definition 7 Given a graph $G = (V, E)$ with $V = S_1 \cup S_2 \cup \ldots S_t \cup T$ where t is an integer ≥ 1, each $S_i, 1 \leq i \leq t$, is a set of at least two vertices of G of the same degree and $T = V - \cup S_i$, then the degree splitting graph DS(G) of the graph G is defined as a graph which is obtained from G by adding vertices $w_1, w_2 \ldots, w_t$ and joining w_i, for each $i, 1 \leq i \leq t$, to each vertex of S_i. Note that if $V(G) = \bigcup_{i=1}^{t} S_i$ then $T = \phi$.

Theorem 8 *If G be a connected graph, then*

$$\gamma_p(DS(G)) \leq \gamma_p(G)$$

Proof. Let G be a connected graph with $V(G) = \{v_1, v_2, \dots, v_n\}$. Let $D = \{v_i: 1 \leq i < n\}$ be the minimum power dominating set of G. Then $\gamma_p(G) = |D|$. By the definition of $DS(G)$, we have $V(DS(G)) = S_1 \cup S_2 \cup, \cdots, \cup S_t \cup T$, where t is an integer ≥ 1 and T is as in the definition of $DS(G)$. Let w_i be the set of vertices of all the corresponding sets of S_i; $1 \leq i \leq t$.

If $T = \emptyset$, then $w_i; 1 \leq i \leq t$ is maximal independent set in $DS(G)$. Since every maximal independent set w_i is a minimum dominating set it is also a minimum power dominating set. Therefore, $D' = \{w_i; 1 \leq i \leq t\}$ is the minimum power dominating set of $DS(G)$.

If $T \neq \emptyset$. There is at least one vertex in G which is not in $S_i; 1 \leq i \leq t$. Since G is an induced subgraph of $DS(G)$ to power dominate all the vertices of $DS(G)$, we require at least $|w_i \cup T|$ vertices. Therefore, $D' = \{w_i; i \leq i \leq t\} \cup T$, becomes a minimum power dominating set of $DS(G)$ and also noted that $|D'| \leq |D|$. Hence, $\gamma_p(DS(G)) \leq \gamma_p(G)$.

Theorem 9 *For any connected graph G of order $n \geq 2$,*

$$\gamma_p(DS(G)) \leq \gamma_p(G) \leq \gamma_p(S(G)) \leq \gamma(G).$$

Proof. From Theorem 3 and Theorem 8, we get the required inequality.

Theorem 10 *If G is a regular graph, then*

$$\gamma_p(DS(G)) = \gamma(DS(G)) = 1$$

Proof. If G is a regular graph, then by definition, in the degree splitting graph $DS(G)$, only one vertex w_1 is newly introduced and is adjacent to all the vertices of G. We claim that $D' = \{w_1\}$ is a γ_p-set for $DS(G)$ because all vertices in a graph $DS(G)$ are dominated and power dominated by w_1.

Theorem 11 *For the complete bipartite graph $K_{m,n}, m \neq n,\ m, n \geq 3$,*

$$\gamma_p(DS(K_{m,n})) = 2.$$

Proof. Let $V_1 = \{v_1, v_2, \cdots, v_m\}$ and $V_2 = \{u_1, u_2, \cdots, u_n\}$ be the partition of $V(K_{m,n})$. The complete bipartite graph $K_{m,n}$ contains two types of vertices - vertices of degree n and vertices of degree m. Thus $V(K_{m,n}) = S_1 \cup S_2$, where $S_1 = \{v_i; 1 \leq i \leq m\}$ and $S_2 = \{u_j; 1 \leq j \leq n\}$. For obtaining $DS(K_{m,n})$ from $K_{m,n}$, we add w_1 and w_2 corresponding to S_1 and S_2 respectively. We claim that $D' = \{w_1, w_2\}$ is a minimum power dominating set for $DS(K_{m,n})$ because the vertices in $v_i (1 \leq i \leq m)$ dominate and hence power dominate by the vertex w_1 while the vertices of $u_j, (1 \leq j \leq n)$ dominate and hence power dominate the vertex w_2. Thus $\gamma_p(DS(K_{m,n})) = 2$.

On the other hand, while computing γ_p-set of $K_{m,n}$, note that every vertex of V_1 power dominates every vertex of V_2 and vice versa. Therefore, we could choose one vertex from V_1 and another vertex in V_2 for γ_p-set. Thus $\gamma_p(DS(K_{m,n})) = \gamma_p(K_{m,n})$.

Corollary 12 $\gamma_p(DS(K_{n,n})) = 1,\ m = n \geq 3$.

Proof. In a complete bipartite graph $K_{n,n}$ with a bipartition $V_1 \cup V_2$ of its vertex set, we construct $DS(K_{n,n})$ by introducing a new vertex w and join this to all the vertices of $K_{n,n}$ as all the vertices of $K_{n,n}$ have the same degree n. We observe that the vertex w dominates and hence power dominates all the vertices of $DS(K_{n,n})$.

Theorem 13 *If G is a k-regular bipartite graph with $k \geq 3$, then*

$$\gamma_p(DS(G)) < \gamma_p(G).$$

Proof. Let G be a k-regular bipartite graph with bipartition (V_1, V_2). Since G is k-regular, $k|V_1| = |E| = k|V_2|$ and so, since $k \geq 3$, $|V_1| = |V_2|$. In order to construct $DS(G)$ from G, we add w corresponding to V_1. Thus $V(DS(G)) = S_1 \cup \{w\}$. We claim that $D' = \{w\}$ is a γ_p-set for $DS(G)$, because w dominates and hence power dominates all the vertices of $DS(G)$. Hence $\gamma_p(DS(G)) = 1$.

On the other hand, we observe that any single vertex v_i of G cannot power dominate the entire k-regular bipartite graph G. Hence $\gamma_p(G) \geq 2$. Thus $\gamma_p(DS(G)) < \gamma_p(G)$.

Definition 14 [8] A binary tree is a tree in which there is exactly one vertex of degree two, namely the root vertex v_0 and each of the remaining vertices is of degree one or three. A complete binary tree is a binary tree in which all leaves are on the same level or all leaves have same distance to the root vertex v_0. We denote a complete binary tree with diameter 2k by BT(k), where $\mathrm{k} \geq 1$.

A graph $BT(k)$ can be constructed recursively from two copies of $BT(k-1)$ by joining their root vertices to a new vertex v_0.

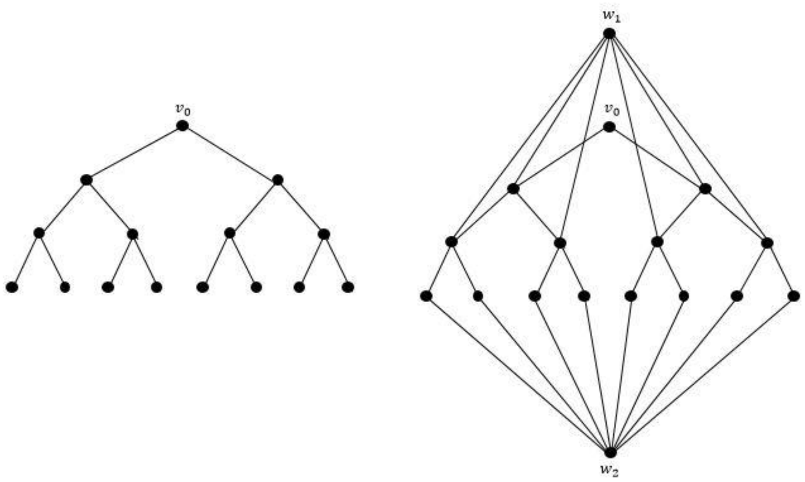

Figure 1: Complete binary tree BT(3) and its Degree splitting graph DS(BT(3))

Theorem 15 *Let $BT(k)$ be a complete binary tree with height $k \geq 3$. Then $\gamma_p(DS(BT(k)) = 2$*

Proof. Let $BT(k)$ be a complete binary tree with diameter $2k$, where $k \geq 1$. Each of the vertices $v_1, v_2, \cdots, v_n$ of $BT(K)$, other than the root vertex v_o is of degree one or three. The vertex set of $BT(k)$ is $V = S_1 \cup S_2 \cup T$ where $T = \{v_0\}$, $S_1 = \{v_i | deg(v_i) = 3\}$ and $S_2 = \{v_i | deg(v_i) = 1\}$. The degree splitting graph $DS(BT(k)$ is constructed by introducing the new vertices w_1 and w_2 corresponding to S_1 and S_2 respectively. Let $D' = \{w_1, w_2\}$ is γ_p-set for $DS(BT(k))$ because each vertex in S_1 and S_2 are dominated by w_1 and w_2 respectively, while the root vertex v_0 is power dominated by S_2. Thus $\gamma_p(DS(BT(k)) = 2$.

Remark 16 *Note that for a complete binary tree $BT(k)$ with height $k \geq 3$, the power domination number $\gamma_p(BT(K)) = 2^{n-1}$.*

Definition 17 [9] The bistar $\mathrm{B}_{\mathrm{n,n}}$ is the graph obtained by joining the center vertices of two copies of $\mathrm{K}_{1,\mathrm{n}}$ by an edge.

Theorem 18 *For $n \geq 2$, $\gamma_p(DS(B_{n,n})) = 1$.*

Proof. Let $B_{n,n}$ be a bistar graph with $V = S_1 \cup S_2$ where $S_1 = \{u, v\}$ with u and v adjacent and $S_2 = \{u_i, v_i | 1 \leq i \leq n\}$. Here for $1 \leq i \leq n$, u_i and v_i, are pendant vertices with all u_i joined to u and all v_i joined to v. In order to obtain the degree splitting graph of $B_{n,n}$, we introduce new vertices w_1 and w_2

corresponding to S_1 and S_2. The vertex w_1 is joined to u and v while w_2 is joined to all the remaining vertices of $B(n,n)$. We claim that $D' = \{w_1\}$ is a γ_p-set for $DS(B_{n,n})$, because $V(DS(B_{n,n}))$ are power dominated by w_1. Hence $\gamma_p(DS(B_{n,n})) = 1$. On the other hand, while computing γ_p-set of $B_{n,n}$, add the vertices w_1 and w_2 to D. Hence $\gamma_p(B(n,n)) = 2$

2.3 Graphs with $\gamma_p(DS(G)) = \gamma_p(G)$.

For any graph G, we have $\gamma_p(DS(G)) \leq \gamma_p(G)$. In this section we investigate graphs for which $\gamma_p(DS(G)) = \gamma_p(G)$. In particular, for a path, cycle and complete graph with $\gamma_p(DS(G)) = \gamma_p(G)$.

It follows from Theorem 11 that for a complete bipartite graph $K_{m,n}$, $m \neq n$, of order $m, n \geq 3$, $\gamma_p(DS(K_{m,n})) = \gamma_p(K_{m,n})$. In the following theorems we characterize the graphs with $\gamma_p(DS(G)) = \gamma_p(G)$.

Theorem 19 *Let $P_n, n \geq 2$ be a path on n vertices. Then $\gamma_p(DS(P_n)) = 1$*

Proof. Let $P_n, n \geq 2$ be a path with $V = S_1 \cup S_2$ where $S_1 = \{v_1, v_n\}$ and $S_2 = \{v_i | 2 \leq i \leq n-1\}$. The degree splitting graph $DS(P_n)$ of P_n is obtained by adding two new vertices w_1 and w_2 corresponding to S_1 and S_2 respectively. Thus $V(DS(P_n)) = V(P_n) \cup \{w_1, w_2\}$ and $E(DS(P_n)) = E(P_n) \cup \{w_1v_1, w_1v_n\} \cup \{w_2v_i | v_i, 2 \leq i \leq n-2.\}$ The number of vertices in $DS(P_n)$ is therefore $n+2$ and the number of edges is $2n-1$. We claim that $D' = \{w_2\}$ is a γ_p-set for $DS(G)$ because $\{v_i | 2 \leq i \leq n-1\}$ are observed in domination step and $\{v_1, v_n, w_1\}$ are observed in power domination step. Haynes et al. [2] have obtained a characterization of path graphs $P_n, n \geq 2$ with $\gamma_p(P_n) = 1$. Hence $\gamma_p(DS(P_n)) = \gamma_p(P_n) = 1$.

Theorem 20 (i) *Let $C_n, n \geq 3$ be a cycle on n vertices. Then $\gamma_p(DS(C_n)) = 1$.*

(ii) *Let K_n be a complete graph on n vertices. Then $\gamma_p(DS(K_n)) = 1$.*

Proof. (i) The cycle C_n is a two regular graph. Hence by Theorem 10, we have $\gamma_p(DS(C_n)) = 1$.

(ii) If K_n is a complete graph, then $DS(K_n) = K_{n+1}$. The graph K_{n+1} being a n-regular graph, and by Theorem 10, we have $\gamma_p(DS(K_n) = 1$. It is known [3] that $\gamma_p(K_n) = 1$ and hence $\gamma_p(DS(K_n)) = \gamma_p(K_n)$.

Theorem 21 *If $G \in \{W_{1,n}, K_{1,n}, F_n\}$ and n be an any arbitrary positive integer, then*

$$\gamma_p(DS(G)) = \gamma_p(G) = 1.$$

3. CONCLUSION

We have computed power domination number of the degree splitting graph of several standard graphs. And we have compared the power domination number of splitting and degree splitting graphs with the power domination for standard graphs and obtained interesting and useful results in this paper.

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BIOGRAPHY

Dr. (Mrs). Huldah Samuel is an Assistant Professor in the Department of Mathematics at Madras Christian College, Chennai. She obtained her Ph.D. from the University of Madras for her work on combinatorics on words in the area of Computer Mathematics. Her M.Phil. Dissertation "A study on chromatic number of graphs" was in the area of Graph theory. She has successfully guided five M.Phil. Candidates in various topics of graph theory on labelling and coloring of graphs. She has a teaching experience of nearly two decades. She has to her credit 10 research publication in proceedings/journals/Scopus indexed journals. She is a recipient of UGC funding for a minor research project titled "Combinatorics on words – its special reference to generalized parikh vectors", which she completed successfully between 2010 – 2012.

Dr. K. Sathish Kumar is an Assistant Professor in the Department of Mathematics at Madras Christian College, Chennai. He has been teaching in this institution for the past eight years. He obtained his Ph.D. from the University of Madras for his work on power dominator coloring of graphs in the area of applied Mathematics. His area of interests include the study of Graph theory, particularly in Domination and Coloring. He has published seven papers in UGC/SCI/ Scopus indexed journals.

Dr. J. Jayasudha is currently working as an Assistant Professor in the PG and Research Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi. She obtained her Ph.D. in Mathematics from Bharathiar University, Coimbatore. She has around 9 years of research experience and her research interest include Topology, Fuzzy Topology and Functional Analysis. She has guided nine M.Phil. Candidates and is currently the Ph.D. supervisor for two Ph.D. research scholars.