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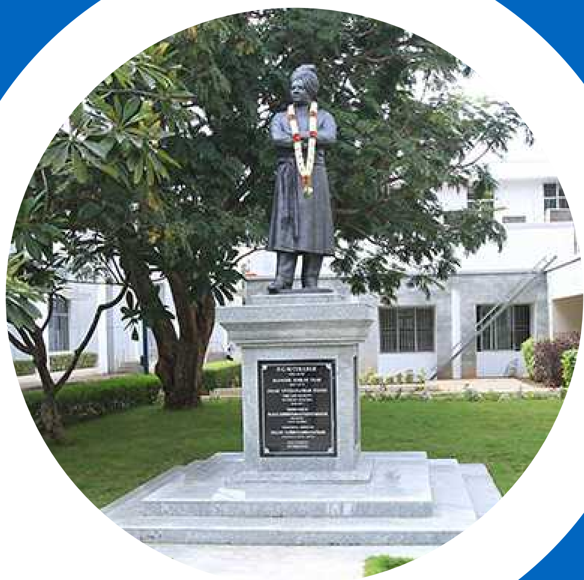

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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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Proceeding of the

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A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

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Fuzzy parameterized vague soft set theory and its applications

Yaya Li[1], **V. Inthumathi**[2], **Chang Wang** [3]

Abstract - A vague soft set is a combination of a vague set and a soft set. In this paper, we first define fuzzy parameterized vague soft sets (fpvs-sets) and study their operations. We then introduce fpvs-aggregation operator to form fpvs-decision making method that allows constructing more efficient decision processes. Finally, we give an example which shows that this method successfully works.

Keywords *vague set; soft set; vague soft set; fuzzy parameterized vague soft set; decision making.*
2010 Subject classification: *03E72; 28E10*

1 Introduction

Researchers in economics, engineering, environment science, the social science, medical science, business, management, and many other fields deal daily with the complexities of modeling uncertain data. Classical methods are not always successful, because the uncertainties appearing in these domains may be of various types. Probability theory, fuzzy set theory [1], intuitionistic fuzzy set theory [2], vague set theory [3], interval mathematics [4], and other mathematical tools are well know and often useful approaches to describing uncertainty. However, all of these theories have their own difficulties which have been pointed out in [5]. Molodtsov suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tools of these theories. To overcome these difficulties, Molodtsov [5] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Since then, many researches have investigated soft sets and have established some significant conclusions. For example, Maji et al. [6] first introduced the concept of fuzzy soft sets by combining fuzzy sets and soft sets. Majumdar and Samanta [7] further generalized the concept of fuzzy soft sets and some of their properties were studied, and relations on generalized fuzzy soft sets were also discussed by them. Yang et al. [8] introduced the concept of interval-valued fuzzy soft set, which is a combination of interval-valued fuzzy sets and soft sets. Xiao et al. [9] introduced the notion of exclusive disjunctive soft sets and gave an application of these new sets. Maji et al. [10, 11] initiated the notion of intuitionistic fuzzy soft sets by integrating the intuitionistic fuzzy sets with soft sets. By combing the vague set and the soft set, Xu et al. [12] introduced the notion of vague soft sets, derived its basic properties and illustrated its potential applications. Jiang et al. [13] constructed a new soft set model called interval-valued intuitionistic fuzzy soft sets by integrating the interval-valued intuitionistic fuzzy sets and soft sets.

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By combing the related works and the soft sets from parametrization point of view, fuzzy parameterized soft set theory [14], fuzzy parameterized fuzzy soft set theory [15], fuzzy parameterized interval-valued fuzzy soft set theory [16], intuitionistic fuzzy parameterized soft set theory [17], interval valued intuitionistic fuzzy parameterized soft set theory [18], multi Q-fuzzy parameterized soft set theory [19] and the decision making methods based on these theories have also been introduced by some scholars.

Since vague sets are equivalent to intuitionistic fuzzy sets [20], so vague soft sets are equivalent to intuitionistic fuzzy soft sets. Some scholars have studied intuitionistic fuzzy soft sets from different aspects. For example, Gunduz and Bayramov [21] introduced the concept of an intuitionistic fuzzy soft module and some operations on intuitionistic fuzzy soft sets were given, they also studied some of its basic properties. Jiang et al. [22] presented an adjustable approach to intuitionistic fuzzy soft sets based decision making by using level soft sets of intuitionistic fuzzy soft sets and gave some illustrative examples, the weighted intuitionistic fuzzy soft sets were introduced and its application to decision making was investigated. Zhang [23] proposed a novel approach to intuitionistic fuzzy soft set based decision making problems using rough set theory. Wang and Qu [24] introduced the definitions of entropy, similarity measure and distance measure of vague soft sets, the relations between these measures were discussed in detail. However, there has been rather little work completed for fuzzy parameterized vague soft set theory. The purpose of this paper is to further extend the concept of vague soft set theory proposed by Xu et al. in [12]. In this paper, we will present the definition of fuzzy parameterized vague soft set and introduce a decision making method based on fpvs-sets. An example is provided illustrates the effectiveness of the method which is more practical.

The rest of this paper is organized as follows. Section 2 recalls some basic concepts of vague sets, soft sets and vague soft sets et al. In Section 3, we introduce the definition of fuzzy parameterized vague soft sets and study some of their properties. In section 4, we define fpvs-aggregation operator to form fpvs-decision making method and give an example which shows that the method can be successfully applied to problems that contain uncertainties. Concluding remarks and open questions for further investigation are provided in Section 5.

2 Preliminaries

In this section, we will recall several definitions and results which are necessary for our paper. They are stated as follows:

Definition 2.1. [1] Let $U = \{u_1, u_2, ..., u_n\}$ be a universe. Then a fuzzy set X over U is a function defined as follows:

$$X = \{\frac{\mu_X(x)}{x} : x \in U\}$$

where $\mu_X : U \to [0, 1]$. Here, μ_X called membership function of X, and the value $\mu_X(x)$ is called the grade of membership of $x \in U$. The value represents the degree of x belonging to the fuzzy set X.

Definition 2.2. [3] A vague set X in the universe $U = \{u_1, u_2, ..., u_n\}$ can be expressed by the following notion, $X = \{(u_i, [t_X(u_i), 1 - f_X(u_i)])|u_i \in U\}$, i.e $X(u_i) = [t_X(u_i), 1 - f_X(u_i)]$ and the condition $0 \leq t_X(u_i) \leq 1 - f_X(u_i)$ should hold for any $u_i \in U$, where $t_X(u_i)$ is called the membership degree (true membership) of element u_i to the vague set X, while $f_X(u_i)$ is the degree of nonmembership (false membership) of the element u_i to the vague set X.

Definition 2.3. [3] Let A, B be two vague sets in the universe $U = \{u_1, u_2, ..., u_n\}$, then the union, intersection and complement of vague sets are defined as follows:
$A \cup B = \{\ (u_i, [max(t_A(u_i), t_B(u_i)), max(1 - f_A(u_i), 1 - f_B(u_i))])|u_i \in U\}$,
$A \cap B = \{\ (u_i, [min(t_A(u_i), t_B(u_i)), min(1 - f_A(u_i), 1 - f_B(u_i))])|u_i \in U\}$,
$A^c = \{\ (u_i, [f_A(u_i), 1 - t_A(u_i)])|u_i \in U\}$.

Definition 2.4. [3] Let A, B be two vague sets in the universe $U = \{u_1, u_2, ..., u_n\}$. If $\forall u_i \in U, t_A(u_i) \leq t_B(u_i), 1 - f_A(u_i) \leq 1 - f_B(u_i)$, then A is called a vague subset of B, denoted by $A \subseteq B$, where $i = 1, 2, 3, ..., n$.

Definition 2.5. [5] Let U be an initial universe set, $P(U)$ be the power set of U, E be the set of all parameters and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a function f_A representing a mapping $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$. Here, f_A is called approximate function of the soft set F_A and the value $f_A(x)$ is a set called x-element of the soft set for all $x \in E$. It is worth noting that the sets $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. Thus, a soft set F_A over U can be represented by the set of ordered pairs

$$F_A = \{(x, f_A(x)), x \in E, f_A(x) \in P(U)\}.$$

Note that, the set of all soft sets over U will be denoted by $S(U)$.

Definition 2.6. [12] Let U be an initial universe set, $V(U)$ be the set of all vague sets over U, E be the set of all parameters and $A \subseteq E$. Then, a vague soft set(VS-set) Γ_A over U is a set defined by a function γ_A representing a mapping $\gamma_A : E \to V(U)$ such that $\gamma_A(x) = \emptyset$ if $x \notin A$. Thus, a soft set Γ_A over U can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)), x \in E, \gamma_A(x) \in V(U)\}.$$

The value $\gamma_A(x)$ is a vague set over U. That is

$$\gamma_A(x) = \{(u, [t_{A(x)}(u), 1 - f_{A(x)}(u)]), x \in E, u \in U\}$$

where $t_{A(x)}(u)$ and $f_{A(x)}(u)$ are the membership and non-membership degrees of u to the parameter x, respectively. Note that, the set of all vague soft sets over U is denoted by $VS(U)$.

Definition 2.7. [12] Let Γ_A and Γ_B be two vague soft sets over a universe U. If $A \subseteq B$ and $\forall x \in A, \gamma_A(x)$ is a vague subset of $\gamma_B(x)$, then Γ_A is called a vague soft subset of Γ_B . This relation is denoted by $\Gamma_A \widetilde{\subseteq} \Gamma_B$.

Definition 2.8. [12] Two vague soft sets Γ_A and Γ_B over a universe U are said to be vague soft equal, if Γ_A is a vague soft subset of Γ_B and Γ_B is a vague soft subset of Γ_A. This relation is denoted by $\Gamma_A = \Gamma_B$.

Definition 2.9. [12] A vague soft set Γ_A over U is said to be a null vague soft set denoted by $\Gamma_\emptyset$, if $\gamma_A(x) = \emptyset$ for all $x \in E$, that is $\gamma_A(x) = \{(u, [0, 0]), x \in E, u \in U\}$.

Definition 2.10. [12] A vague soft set Γ_A over U is said to be a A-universal vague soft set denoted by $\Gamma_{\widetilde{A}}$, if $\gamma_A(x) = \{(u, [1, 1]), x \in E, u \in U\}$.

If $A = E$, then the A-universal vague soft set is called universal vague soft set and denoted by $\Gamma_{\widetilde{E}}$.

Definition 2.11. [12] Let $E = \{e_1, e_2, ..., e_n\}$ be a parameter set. The not set of E denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, ..., \neg e_n\}$, where $\neg e_i = not\ e_i$.

Definition 2.12. [12] The complement of vague soft set Γ_A is denoted by Γ_A^c and is defined by $\Gamma_A^c = \{(x, \gamma_{A^c}(x)), x \in E\}$, where $\gamma_{A^c}(x) = \gamma_A^c(x)$ is the complement of vague set $\gamma_A(x)$, defined by

$$\gamma_A^c(x) = \{(u, [f_{A(x)}(u), 1 - t_{A(x)}(u)]), x \in E, u \in U\}.$$

Clearly $(\Gamma_A^c)^c = \Gamma_A$.

Definition 2.13. [12] Let Γ_A and Γ_B be two vague soft sets over a universe U, the union of two Γ_A and Γ_B, denoted by $\Gamma_A \widetilde{\cup} \Gamma_B$, and is defined by

$$\Gamma_A \widetilde{\cup} \Gamma_B = \{(x, \gamma_A(x) \cup \gamma_B(x)), x \in E\}$$

where $\gamma_A(x) \cup \gamma_B(x) = \{(u, max(t_{A(x)}(u), t_{B(x)}(u)), max(1 - f_{A(x)}(u), 1 - f_{B(x)}(u)), x \in E, u \in U\}$.

Definition 2.14. [12] Let Γ_A and Γ_B be two vague soft sets over a universe U, the intersection of two Γ_A and Γ_B, denoted by $\Gamma_A \widetilde{\cap} \Gamma_B$, and is defined by

$$\Gamma_A \widetilde{\cap} \Gamma_B = \{(x, \gamma_A(x) \cap \gamma_B(x)), x \in E\}$$

where $\gamma_A(x) \cap \gamma_B(x) = \{(u, min(t_{A(x)}(u), t_{B(x)}(u)), min(1 - f_{A(x)}(u), 1 - f_{B(x)}(u)), x \in E, u \in U\}$.

Definition 2.15. [14] Let U be an initial universe, $P(U)$ be the power set of U, E be a set of all parameters and X be a fuzzy set over E. Then a FP-soft set (f_X, E) on the universe U is defined as follows:

$$(f_X, E) = \{(u_X(x)/x, f_X(x)), x \in E\}$$

where $u_X : E \to [0, 1]$ and $f_X : E \to P(U)$ such that $f_X(x) = \emptyset$ if $u_X(x) = 0$.

Here f_X called approximate function and u_X called membership function of FP-soft sets.

Definition 2.16. [15] Let U be an initial universe, $F(U)$ be the set of all fuzzy sets over U, E be a set of all parameters and X be a fuzzy set over E with the membership function $u_X : E \to [0, 1]$ and $\gamma_X(x)$ be a fuzzy set over U for all $x \in E$. Then, a $fpfs-$set Γ_X over U is a set defined by a function γ_X representing a mapping $\gamma_X : E \to F(U)$ such that $\gamma_X(x) = \emptyset$ if $u_X(x) = 0$.

Here, γ_X is called fuzzy approximate function of the $fpfs-$set for all $x \in E$. Thus, a $fpfs-$set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(u_X(x)/x, \gamma_X(x)), x \in E, \gamma_X(x) \in F(U), u_X(x) \in [0, 1]\}.$$

It must be noted that the sets of all $fpfs-$sets over U will be denoted by $FPFS(U)$.

Definition 2.17. [17] Let U be an initial universe, $P(U)$ be the power set of U, E be a set of all parameters and X be an intuitionistic fuzzy set over E. An intuitionistic FP-soft sets Γ_X over U is defined as follows:

$$\Gamma_X = \{((x, \alpha_X(x), \beta_X(x)), f_X(x)), x \in E\}$$

where $\alpha_X : E \to [0, 1], \beta_X : E \to [0, 1]$ and $f_X : E \to P(U)$ with the property $f_X(x) = \emptyset$ if $\alpha_X(x) = 0, \beta_X(x) = 1$.

Here the function α_X and β_X called membership function and non-membership of intuitionistic FP-soft set, respectively. The value $\alpha_X(x)$ and $\beta_X(x)$ is the degree of importance and unimportance of the parameter x.

3 Fuzzy parameterized vague soft sets

In this section, we define fuzzy parameterized vague soft sets (fpvs-sets)and their operations.

Definition 3.1. Let U be an initial universe, E be the set of all parameters and X be a fuzzy set over E with membership function $\mu_X : E \to [0,1]$, let $\gamma_X(x)$ be a vague set over U for all $x \in E$. Then, a fuzzy parameterized vague soft set Γ_X over U is a set defined by a function γ_X representing a mapping

$$\gamma_X : E \to V(U)$$

such that $\gamma_X(x) = \emptyset$ if $\mu_X(x) = 0$. Here, γ_X is called vague approximate function of the fpvs-set Γ_X, and $\gamma_X(x)$ is a vague set called x-element of the fpvs-set for all $x \in E$. Thus, a fpvs-set Γ_X over U can be represented by the set of ordered pairs

$$\Gamma_X = \{(\frac{\mu_X(x)}{x}, \gamma_X(x)) : x \in E, \gamma_X(x) \in V(U)\}.$$

It must be noted that the set of all fpvs-sets over U will be denoted by $FPVS(U)$.

Definition 3.2. Let $\Gamma_X \in FPVS(U)$. If $\gamma_X(x) = \emptyset$ for all $x \in E$, then Γ_X is called a X-empty fpvs-set, denoted by $\Gamma_{\emptyset_X}$. If $X = \emptyset$, then the X-empty fpvs-set $\Gamma_{\emptyset_X}$ is called an empty fpvs-set, denoted by $\Gamma_{\emptyset}$.

Definition 3.3. Let $\Gamma_X \in FPVS(U)$. If $\mu_X(x) = 1$ and $\gamma_X(x) = U$ for all $x \in E$, then Γ_X is called a X-universal fpvs-set, denoted by $\Gamma_{\widetilde{X}}$. If $X = E$, then the X-universal fpvs-set $\Gamma_{\widetilde{X}}$ is called a universal fpvs-set, denoted by $\Gamma_{\widetilde{E}}$.

Example 3.1. *Assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universe set and $E = \{x_1, x_2, x_3\}$ is a set of parameters. If $X = \{\frac{0.3}{x_1}, \frac{0.5}{x_2}, \frac{1}{x_3}\}$ and $\eta_X(x)$ is defined as follows:*

$\eta_X(x_1) = \{\frac{[0.4,0.5]}{u_1}, \frac{[0.3,0.9]}{u_2}, \frac{[0.2,0.7]}{u_4}\}, \eta_X(x_2) = \emptyset, \eta_X(x_3) = U,$

then a fpvs-set Γ_X is written by

$\Gamma_X = \{(\frac{0.3}{x_1}, \{\frac{[0.4,0.5]}{u_1}, \frac{[0.3,0.9]}{u_2}, \frac{[0.2,0.7]}{u_4}\}), (\frac{0.5}{x_2}, \emptyset), (\frac{1}{x_3}, U)\}.$

If $Y = \{\frac{0.1}{x_1}, \frac{0.7}{x_3}\}$ and $\gamma_Y(x_1) = \emptyset, \gamma_Y(x_3) = \emptyset$, then the fpvs-set Γ_Y is a Y-empty fuzzy parameterized vague soft set, i.e. $\Gamma_Y = \Gamma_{\emptyset_Y}$.

If $L = \{\frac{0}{x_1}, \frac{0}{x_2}, \frac{0}{x_3}\}$, then the fpvs-set Γ_L is an empty fuzzy parameterized vague soft set.

If $Z = \{\frac{1}{x_2}, \frac{1}{x_3}\}$ and $\gamma_Z(x_2) = U, \gamma_Z(x_3) = U$, then the fpvs-set Γ_Z is a Z-universal fpvs-set, i.e. $\Gamma_Z = \Gamma_{\widetilde{Z}}$.

If $M = \{\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}\}$ and $\gamma_M(x_1) = U, \gamma_M(x_2) = U, \gamma_M(x_3) = U$, then the fpvs-set Γ_M is a universal fuzzy parameterized vague soft set, i.e. $\Gamma_M = \Gamma_{\widetilde{E}}$.

Definition 3.4. Let $\Gamma_X, \Gamma_Y \in FPVS(U)$. Then Γ_X is a fuzzy parameterized vague soft subset of Γ_Y, denoted by $\Gamma_X \widetilde{\subseteq} \Gamma_Y$, if and only if $\mu_X(x) \leq \mu_Y(x)$ and $\gamma_X(x) \subseteq \gamma_Y(x)$ for all $x \in E$.

Remark 3.1. $\Gamma_X \widetilde{\subseteq} \Gamma_Y$ *does not imply that every element of Γ_X is an element of Γ_Y as in the definition of classical subset.*

Example 3.2. *Assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set of objects and $E = \{x_1, x_2, x_3\}$ is a set of all parameters. If $X = \{\frac{0.3}{x_1}\}$ and $Y = \{\frac{0.4}{x_1}, \frac{0.8}{x_3}\}$, and*

$$\Gamma_X = \{(\frac{0.3}{x_1}, \{\frac{[0.2, 0.6]}{u_2}, \frac{[0.4, 0.7]}{u_4}\})\},$$

$$\Gamma_Y = \{(\frac{0.4}{x_1}, \{\frac{[0.3, 0.5]}{u_1}, \frac{[0.3, 0.8]}{u_2}, \frac{[0.6, 0.9]}{u_4}\}), (\frac{0.8}{x_3}, \{\frac{[0.2, 0.5]}{u_1}, \frac{[0.4, 0.7]}{u_3})\},$$

then for all $x \in E$, $\mu_X(x) \leq \mu_Y(x)$ and $\gamma_X(x) \subseteq \gamma_Y(x)$ is valid. Hence $\Gamma_X \widetilde{\subseteq} \Gamma_Y$. It is clear that $(\frac{0.3}{x_1}, \{\frac{[0.2,0.6]}{u_2}, \frac{[0.4,0.7]}{u_4}\}) \in \Gamma_X$ but $(\frac{0.3}{x_1}, \{\frac{[0.2,0.6]}{u_2}, \frac{[0.4,0.7]}{u_4}\}) \notin \Gamma_Y$.

Proposition 3.1. *Let $\Gamma_X, \Gamma_Y, \Gamma_Z \in FPVS(U)$. Then,*
(1) $\Gamma_X \widetilde{\subseteq} \Gamma_{\widetilde{E}}$
(2) $\Gamma_{\emptyset_X} \widetilde{\subseteq} \Gamma_X$
(3) $\Gamma_{\emptyset} \widetilde{\subseteq} \Gamma_X$
(4) $\Gamma_X \widetilde{\subseteq} \Gamma_X$
(5) $\Gamma_X \widetilde{\subseteq} \Gamma_Y$ and $\Gamma_Y \widetilde{\subseteq} \Gamma_Z \Rightarrow \Gamma_X \widetilde{\subseteq} \Gamma_Z$

Proof. The above properties of $\widetilde{\subseteq}$ trivially follow from the above definitions. □

Definition 3.5. Let $\Gamma_X, \Gamma_Y \in FPVS(U)$. Then Γ_X and Γ_Y are fuzzy parameterized vague soft equal, written by $\Gamma_X = \Gamma_Y$, if and only if $\mu_X(x) = \mu_Y(x)$ and $\gamma_X(x) = \gamma_Y(x)$ for all $x \in E$.

Proposition 3.2. *Let $\Gamma_X, \Gamma_Y, \Gamma_Z \in FPVS(U)$. Then,*
(1) $\Gamma_X = \Gamma_Y$ and $\Gamma_Y = \Gamma_Z \Rightarrow \Gamma_X = \Gamma_Z$
(2) $\Gamma_X \widetilde{\subseteq} \Gamma_Y$ and $\Gamma_Y \widetilde{\subseteq} \Gamma_X \Rightarrow \Gamma_X = \Gamma_Y$

Proof. The above properties of $=$ and $\widetilde{\subseteq}$ trivially follow from the above definitions 3.4 and 3.5. □

Definition 3.6. Let $\Gamma_X \in FPVS(U)$. Then complement of Γ_X, denoted by Γ_X^c, is a fuzzy parameterized vague soft set defined by

$$\Gamma_X^c = \{(\frac{1 - \mu_X(x)}{x}, \gamma_X^c(x)) : x \in E\}$$

where $\gamma_X^c(x)$ is complement of the vague set $\gamma_X(x)$, that is $\gamma_X^c(x) = \gamma_{X^c}(x)$ for every $x \in E$.

Proposition 3.3. *Let $\Gamma_X \in FPVS(U)$. Then,*
(1) $\Gamma_{\widetilde{E}}^c = \Gamma_{\emptyset}$
(2) $(\Gamma_X^c)^c = \Gamma_X$
(3) $\Gamma_{\emptyset}^c = \Gamma_{\widetilde{E}}$

Proof. Let $\Gamma_{\widetilde{E}} = \{(\frac{1}{x}, U), \forall x \in E\}$.
Then, from Definition 3.6, we have:
$\Gamma_{\widetilde{E}}^c = \{(\frac{0}{x}, \emptyset), \forall x \in E\} = \Gamma_{\emptyset}$.
Similarity (2) and (3) easily can be made. □

Definition 3.7. Let $\Gamma_X, \Gamma_Y \in FPVS(U)$. Then union of Γ_X and Γ_Y, denoted by $\Gamma_X \widetilde{\cup} \Gamma_Y$, is a fpvs-set defined by

$$\Gamma_X \widetilde{\cup} \Gamma_Y = \{(\frac{max(\mu_X(x), \mu_Y(x))}{x}, \gamma_{X\widetilde{\cup}Y}(x)) : x \in E\}$$

where $\gamma_{X\widetilde{\cup}Y}(x) = \gamma_X(x) \cup \gamma_Y(x)$.

Proposition 3.4. *Let* $\Gamma_X, \Gamma_Y, \Gamma_Z \in FPVS(U)$*. Then,*
(1) $\Gamma_X \widetilde{\cup} \Gamma_X = \Gamma_X$
(2) $\Gamma_X \widetilde{\cup} \Gamma_\emptyset = \Gamma_X$
(3) $\Gamma_X \widetilde{\cup} \Gamma_{\widetilde{E}} = \Gamma_{\widetilde{E}}$
(4) $\Gamma_X \widetilde{\cup} \Gamma_Y = \Gamma_Y \widetilde{\cup} \Gamma_X$
(5) $(\Gamma_X \widetilde{\cup} \Gamma_Y) \widetilde{\cup} \Gamma_Z = \Gamma_X \widetilde{\cup} (\Gamma_Y \widetilde{\cup} \Gamma_Z)$

Proof. The proofs can be easily obtained from Definition 3.7. □

Definition 3.8. Let $\Gamma_X, \Gamma_Y \in FPVS(U)$. Then intersection of Γ_X and Γ_Y, denoted by $\Gamma_X \widetilde{\cap} \Gamma_Y$, is a fpvs-set defined by

$$\Gamma_X \widetilde{\cap} \Gamma_Y = \{(\frac{min(\mu_X(x), \mu_Y(x))}{x}, \gamma_{X\widetilde{\cap}Y}(x)) : x \in E\}$$

where $\gamma_{X\widetilde{\cap}Y}(x) = \gamma_X(x) \cap \gamma_Y(x)$.

Proposition 3.5. *Let* $\Gamma_X, \Gamma_Y, \Gamma_Z \in FPVS(U)$*. Then,*
(1) $\Gamma_X \widetilde{\cap} \Gamma_X = \Gamma_X$
(2) $\Gamma_X \widetilde{\cap} \Gamma_\emptyset = \Gamma_\emptyset$
(3) $\Gamma_X \widetilde{\cap} \Gamma_{\widetilde{E}} = \Gamma_X$
(4) $\Gamma_X \widetilde{\cap} \Gamma_Y = \Gamma_Y \widetilde{\cap} \Gamma_X$
(5) $(\Gamma_X \widetilde{\cap} \Gamma_Y) \widetilde{\cap} \Gamma_Z = \Gamma_X \widetilde{\cap} (\Gamma_Y \widetilde{\cap} \Gamma_Z)$

Proof. The proofs can be easily obtained from Definition 3.8. □

Remark 3.2. *Let* $\Gamma_X \in FPVS(U)$*, If* $\Gamma_X \neq \Gamma_\emptyset$ *or* $\Gamma_X \neq \Gamma_{\widetilde{E}}$*, then* $\Gamma_X \widetilde{\cup} \Gamma_X^c \neq \Gamma_{\widetilde{E}}$ *and* $\Gamma_X \widetilde{\cap} \Gamma_X^c \neq \Gamma_\emptyset$*.*

Example 3.3. *Assume that* $U = \{u_1, u_2, u_3, u_4\}$ *is a universal set of objects and* $E = \{x_1, x_2\}$ *is a set of all parameters. If* $X = \{\frac{0.3}{x_1}, \frac{0.5}{x_2}\}$*, and*

$$\Gamma_X = \{(\frac{0.3}{x_1}, \{\frac{[0.3, 0.5]}{u_1}, \frac{[0.5, 0.5]}{u_2}, \frac{[0.6, 0.9]}{u_4}\}), (\frac{0.5}{x_2}, \{\frac{[0.4, 0.6]}{u_1}, \frac{[0.3, 0.8]}{u_3}\})\},$$

then $X^c = \{\frac{0.7}{x_1}, \frac{0.5}{x_2}\}$*, and*

$$\Gamma_X^c = \{(\frac{0.7}{x_1}, \{\frac{[0.5, 0.7]}{u_1}, \frac{[0.5, 0.5]}{u_2}, \frac{[0.1, 0.4]}{u_4}\}), (\frac{0.5}{x_2}, \{\frac{[0.4, 0.6]}{u_1}, \frac{[0.2, 0.7]}{u_3}\})\},$$

Therefore,

$$\Gamma_X \widetilde{\cup} \Gamma_X^c = \{(\frac{0.7}{x_1}, \{\frac{[0.5, 0.7]}{u_1}, \frac{[0.5, 0.5]}{u_2}, \frac{[0.6, 0.9]}{u_4}\}), (\frac{0.5}{x_2}, \{\frac{[0.4, 0.6]}{u_1}, \frac{[0.3, 0.8]}{u_3}\})\} \neq \Gamma_{\widetilde{E}},$$

and

$$\Gamma_X \widetilde{\cap} \Gamma_X^c = \{(\frac{0.3}{x_1}, \{\frac{[0.3, 0.5]}{u_1}, \frac{[0.5, 0.5]}{u_2}, \frac{[0.1, 0.4]}{u_4}\}), (\frac{0.5}{x_2}, \{\frac{[0.4, 0.6]}{u_1}, \frac{[0.2, 0.7]}{u_3}\})\} \neq \Gamma_\emptyset.$$

Proposition 3.6. *Let* $\Gamma_X, \Gamma_Y \in FPVS(U)$*. Then, the following De Morgan's types of results are true:*
(1) $(\Gamma_X \widetilde{\cup} \Gamma_Y)^c = \Gamma_X^c \widetilde{\cap} \Gamma_Y^c$
(2) $(\Gamma_X \widetilde{\cap} \Gamma_Y)^c = \Gamma_X^c \widetilde{\cup} \Gamma_Y^c$

Proof. (1) For all $x \in E$,

$\gamma_{(X\widetilde{\cup}Y)^c}(x) = \gamma^c_{X\widetilde{\cup}Y}(x) = (\gamma_X(x) \cup \gamma_Y(x))^c = (\gamma_X(x))^c \cap (\gamma_Y(x))^c = \gamma^c_X(x) \cap \gamma^c_Y(x) = \gamma_{X^c}(x) \cap \gamma_{Y^c}(x) = \gamma_{X^c\widetilde{\cap}Y^c}(x)$.

and $\Gamma_X \widetilde{\cup} \Gamma_Y = \{(\frac{max(\mu_X(x),\mu_Y(x))}{x}, \gamma_{X\widetilde{\cup}Y}(x)) : x \in E\}$

$= \{(\frac{max(\mu_X(x),\mu_Y(x))}{x}, \gamma_{X\widetilde{\cup}Y}(x)) : x \in E\}$

$(\Gamma_X \widetilde{\cup} \Gamma_Y)^c = \{(\frac{1-max(\mu_X(x),\mu_Y(x))}{x}, \gamma_{(X\widetilde{\cup}Y)^c}(x)) : x \in E\}$

$= \{(\frac{min(1-\mu_X(x),1-\mu_Y(x))}{x}, \gamma_{X^c\widetilde{\cap}Y^c}(x)) : x \in E\}$

$= \{(\frac{1-\mu_X(x)}{x}, \gamma_{X^c}(x)) : x \in E\} \widetilde{\cap} \{(\frac{1-\mu_Y(x)}{x}, \gamma_{Y^c}(x)) : x \in E\} = \Gamma^c_X \widetilde{\cap} \Gamma^c_Y$

The proof of (2) can be made similarly. □

Proposition 3.7. *Let $\Gamma_X, \Gamma_Y, \Gamma_Z \in FPVS(U)$. Then,*

(1) $\Gamma_X \widetilde{\cup} (\Gamma_Y \widetilde{\cap} \Gamma_Z) = (\Gamma_X \widetilde{\cup} \Gamma_Y) \widetilde{\cap} (\Gamma_X \widetilde{\cup} \Gamma_Z)$

(2) $\Gamma_X \widetilde{\cap} (\Gamma_Y \widetilde{\cup} \Gamma_Z) = (\Gamma_X \widetilde{\cap} \Gamma_Y) \widetilde{\cup} (\Gamma_X \widetilde{\cap} \Gamma_Z)$

Proof. (1) For all $x \in E$,

$\mu_{X\widetilde{\cup}(Y\widetilde{\cap}Z)}(x) = max(\mu_X(x), \mu_{Y\widetilde{\cap}Z}(x)) = max(\mu_X(x), min(\mu_Y(x), \mu_Z(x)))$

$= min(max(\mu_X(x), \mu_Y(x)), max(\mu_X(x), \mu_Z(x))) = min(\mu_{X\widetilde{\cup}Y}(x), \mu_{X\widetilde{\cup}Z}(x)) = \mu_{(X\widetilde{\cup}Y)\widetilde{\cap}(X\widetilde{\cup}Z)}(x)$

and $\gamma_{X\widetilde{\cup}(Y\widetilde{\cap}Z)}(x) = \gamma_X(x) \cup \gamma_{Y\widetilde{\cap}Z}(x) = \gamma_X(x) \cup (\gamma_Y(x) \cap \gamma_Z(x))$

$= (\gamma_X(x) \cup \gamma_Y(x)) \cap (\gamma_X(x) \cup \gamma_Z(x)) = \gamma_{X\widetilde{\cup}Y}(x) \cap \gamma_{X\widetilde{\cup}Z}(x) = \gamma_{(X\widetilde{\cup}Y)\widetilde{\cap}(X\widetilde{\cup}Z)}(x)$.

The proof of (2) can be made similarly. □

Definition 3.9. Let $\Gamma_X, \Gamma_Y \in FPVS(U)$. Then OR-product of Γ_X and Γ_Y, denoted by $\Gamma_X \vee \Gamma_Y$, is defined by

$$(\Gamma_X \vee \Gamma_Y)(x, y) = \{(\frac{max(\mu_X(x), \mu_Y(y))}{(x, y)}, \gamma_X(x) \vee \gamma_Y(y)) : x \in E, y \in E\}$$

where $\gamma_X(x) \vee \gamma_Y(y) = \{(u, [max(t_{A(x)}(u), t_{A(y)}(u)), max(1 - f_{A(x)}(u), 1 - f_{A(y)}(u))]), x \in E, y \in E, u \in U\}$.

Definition 3.10. Let $\Gamma_X, \Gamma_Y \in FPVS(U)$. Then AND-product of Γ_X and Γ_Y, denoted by $\Gamma_X \wedge \Gamma_Y$, is defined by

$$(\Gamma_X \wedge \Gamma_Y)(x, y) = \{(\frac{min(\mu_X(x), \mu_Y(y))}{(x, y)}, \gamma_X(x) \wedge \gamma_Y(y)) : x \in E, y \in E\}$$

where $\gamma_X(x) \wedge \gamma_Y(y) = \{(u, [min(t_{A(x)}(u), t_{A(y)}(u)), min(1 - f_{A(x)}(u), 1 - f_{A(y)}(u))]), x \in E, y \in E, u \in U\}$.

Example 3.4. *Assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set of objects and $E = \{x_1, x_2\}$ is a set of all parameters. If $X = \{\frac{0.5}{x_1}, \frac{0.8}{x_2}\}$, $Y = \{\frac{0.3}{y_1}, \frac{0.6}{y_2}\}$,and*

$$\Gamma_X = \{(\frac{0.5}{x_1}, \{\frac{[0.3, 0.6]}{u_1}, \frac{[0.6, 0.8]}{u_2}, \frac{[0.5, 0.5]}{u_4}\}), (\frac{0.8}{x_2}, \{\frac{[0.1, 0.5]}{u_1}, \frac{[0.4, 0.5]}{u_2}, \frac{[0.3, 0.8]}{u_3}\})\},$$

and

$$\Gamma_Y = \{(\frac{0.3}{y_1}, \{\frac{[0.2, 0.7]}{u_1}, \frac{[0.5, 0.6]}{u_2}, \frac{[0.4, 0.8]}{u_3}\}), (\frac{0.6}{y_2}, \{\frac{[0.5, 0.6]}{u_1}, \frac{[0.3, 0.5]}{u_2}, \frac{[0.2, 0.7]}{u_3}, \frac{[0.5, 0.9]}{u_4}\})\}.$$

Therefore,

$$(\Gamma_X \vee \Gamma_Y)(x,y) = \{(\frac{0.5}{(x_1,y_1)}, \{\frac{[0.3,0.7]}{u_1}, \frac{[0.6,0.8]}{u_2}, \frac{[0.4,0.8]}{u_3}, \frac{[0.5,0.5]}{u_4}\}),$$

$$(\frac{0.6}{(x_1,y_2)}, \{\frac{[0.5,0.6]}{u_1}, \frac{[0.6,0.8]}{u_2}, \frac{[0.2,0.7]}{u_3}, \frac{[0.5,0.9]}{u_4}\}), (\frac{0.8}{(x_2,y_1)}, \{\frac{[0.2,0.7]}{u_1}, \frac{[0.5,0.6]}{u_2}, \frac{[0.4,0.8]}{u_3}\}),$$

$$(\frac{0.8}{(x_2,y_2)}, \{\frac{[0.5,0.6]}{u_1}, \frac{[0.4,0.6]}{u_2}, \frac{[0.3,0.8]}{u_3}, \frac{[0.5,0.9]}{u_4}\})\},$$

and

$$(\Gamma_X \wedge \Gamma_Y)(x,y) = \{(\frac{0.3}{(x_1,y_1)}, \{\frac{[0.2,0.6]}{u_1}, \frac{[0.5,0.6]}{u_2}\}), (\frac{0.5}{(x_1,y_2)}, \{\frac{[0.3,0.6]}{u_1}, \frac{[0.3,0.5]}{u_2}, \frac{[0.5,0.5]}{u_4}\}),$$

$$(\frac{0.3}{(x_2,y_1)}, \{\frac{[0.1,0.5]}{u_1}, \frac{[0.4,0.5]}{u_2}, \frac{[0.3,0.8]}{u_3}\}), (\frac{0.6}{(x_2,y_2)}, \{\frac{[0.1,0.5]}{u_1}, \frac{[0.3,0.5]}{u_2}, \frac{[0.2,0.7]}{u_3}\})\}.$$

Proposition 3.8. *Let $\Gamma_X, \Gamma_Y, \Gamma_Z \in FPVS(U)$. Then,*
(1) $\Gamma_X \wedge \Gamma_\emptyset = \Gamma_\emptyset$
(2) $\Gamma_X \wedge \Gamma_Y = \Gamma_Y \wedge \Gamma_X$
(3) $\Gamma_X \vee \Gamma_Y = \Gamma_Y \vee \Gamma_X$
(4) $(\Gamma_X \wedge \Gamma_Y) \wedge \Gamma_Z = \Gamma_X \wedge (\Gamma_Y \wedge \Gamma_Z)$
(5) $(\Gamma_X \vee \Gamma_Y) \vee \Gamma_Z = \Gamma_X \vee (\Gamma_Y \vee \Gamma_Z)$

Proof. The proofs can be easily obtained from Definition 3.9 and Definition 3.10. □

Proposition 3.9. *Let $\Gamma_X, \Gamma_Y \in FPVS(U)$. Then, the following De Morgan's types of results are true:*
(1) $(\Gamma_X \vee \Gamma_Y)^c = \Gamma_X^c \wedge \Gamma_Y^c$
(2) $(\Gamma_X \wedge \Gamma_Y)^c = \Gamma_X^c \vee \Gamma_Y^c$

Proof. (1) For all $x \in E, y \in E$,

$$(\Gamma_X \vee \Gamma_Y)^c = \{(\frac{1 - max(\mu_X(x), \mu_Y(y))}{(x,y)}, (\gamma_X(x) \vee \gamma_Y(y))^c) : x \in E, y \in E\}$$

$$= \{(\frac{min(1-\mu_X(x), 1-\mu_Y(y))}{(x,y)}, ([max(t_{A(x)}(u), t_{A(y)}(u)), max(1-f_{A(x)}(u), 1-f_{A(y)}(u))])^c) : x \in E, y \in E\}$$

$$= \{(\frac{min(1-\mu_X(x), 1-\mu_Y(y))}{(x,y)}, ([max(t_{A(x)}(u), t_{A(y)}(u)), 1 - min(f_{A(x)}(u), f_{A(y)}(u))])^c) : x \in E, y \in E\}$$

$$= \{(\frac{min(1-\mu_X(x), 1-\mu_Y(y))}{(x,y)}, ([min(f_{A(x)}(u), f_{A(y)}(u)), 1 - max(t_{A(x)}(u), t_{A(y)}(u))])) : x \in E, y \in E\}$$

$$= \{(\frac{min(1-\mu_X(x), 1-\mu_Y(y))}{(x,y)}, ([min(f_{A(x)}(u), f_{A(y)}(u)), min(1 - t_{A(x)}(u), 1 - t_{A(y)}(u))])) : x \in E, y \in E\}$$

$$= \{(\frac{1-\mu_X(x)}{x}, [f_{A(x)}(u), 1 - t_{A(x)}(u)] : x \in E, y \in E\} \wedge \{(\frac{1-\mu_Y(y)}{y}, [f_{A(y)}(u), 1 - t_{A(y)}(u)] : x \in E, y \in E\}$$

$= \Gamma_X^c \wedge \Gamma_Y^c$

The proof of (2) can be made similarly. □

Proposition 3.10. *Let $\Gamma_X, \Gamma_Y, \Gamma_Z \in FPVS(U)$. Then,*
(1) $\Gamma_X \vee (\Gamma_Y \wedge \Gamma_Z) = (\Gamma_X \vee \Gamma_Y) \wedge (\Gamma_X \vee \Gamma_Z)$
(2) $\Gamma_X \wedge (\Gamma_Y \vee \Gamma_Z) = (\Gamma_X \wedge \Gamma_Y) \vee (\Gamma_X \wedge \Gamma_Z)$

Proof. (1) For all $x \in E, y \in E, z \in E$,
$\mu_{X\vee(Y\wedge Z)}(x,y,z) = max(\mu_X(x), \mu_{Y\wedge Z}(y,z)) = max(\mu_X(x), min(\mu_Y(y), \mu_Z(z)))$
$= min(max(\mu_X(x), \mu_Y(y)), max(\mu_X(x), \mu_Z(z))) = min(\mu_{X\vee Y}(x,y), \mu_{X\vee Z}(x,z)) = \mu_{(X\vee Y)\wedge(X\vee Z)}(x,y,z)$.
Since $t_{X\vee(Y\wedge Z)}(x,y,z) = max(t_X(x), t_{Y\wedge Z}(y,z)) = max(t_X(x), min(t_Y(y), t_Z(z)))$
$= min(max(t_X(x), t_Y(y)), max(t_X(x), t_Z(z))) = min(t_{X\vee Y}(x,y), t_{X\vee Z}(x,z)) = t_{(X\vee Y)\wedge(X\vee Z)}(x,y,z)$,
$f_{X\vee(Y\wedge Z)}(x,y,z) = min(f_X(x), f_{Y\wedge Z}(y,z)) = min(f_X(x), max(f_Y(y), f_Z(z)))$
$= max(min(f_X(x), f_Y(y)), min(f_X(x), f_Z(z))) = max(f_{X\vee Y}(x,y), f_{X\vee Z}(x,z)) = f_{(X\vee Y)\wedge(X\vee Z)}(x,y,z)$,
so $\gamma_{X\vee(Y\wedge Z)}(x,y,z) = \gamma_X(x) \vee \gamma_{Y\wedge Z}(y,z) = \gamma_X(x) \vee (\gamma_Y(y) \wedge \gamma_Z(z))$
$= (\gamma_X(x) \vee \gamma_Y(y)) \wedge (\gamma_X(x) \vee \gamma_Z(z)) = \gamma_{X\vee Y}(x,y) \wedge \gamma_{X\vee Z}(x,z) = \gamma_{(X\vee Y)\wedge(X\vee Z)}(x,y,z)$.
Hence $\Gamma_X \vee (\Gamma_Y \wedge \Gamma_Z) = (\Gamma_X \vee \Gamma_Y) \wedge (\Gamma_X \vee \Gamma_Z)$.
The proof of (2) can be made similarly. □

4 fpvs-aggregation operator

In this section, we define an aggregate vague set of a fuzzy parameterized vague soft set, we also define fpvs-aggregation operator that produced an aggregate vague set from a fpvs-set and its fuzzy parameter set. Also we give an application of this operator in decision making problem.

Definition 4.1. Let $\Gamma_X \in FPVS(U)$. Then a fpvs-aggregation operator, denoted by $FPVS_{agg}$, is defined by

$$FPVS_{agg} : V(E) \times FPVS(U) \to V(U), FPVS_{agg}(X, \Gamma_X) = \Gamma_X^*$$

where

$$\Gamma_X^* = \{\frac{[t_{\Gamma_X^*}(u), 1 - f_{\Gamma_X^*}(u)]}{u} : u \in U\}$$

which is a vague set over U. The value Γ^*_X is called aggregate vague set of the Γ_X. Here, the membership degree $t_{\Gamma^*_X}(u)$ and nonmembership degree $f_{\Gamma^*_X}(u)$ of u is defined as follows:

$$t_{\Gamma^*_X}(u) = \frac{1}{|E|}\sum_{x\in E}\mu_X(x)t_{\gamma_X(x)}(u)$$

and

$$f_{\Gamma^*_X}(u) = \frac{1}{|E|}\sum_{x\in E}\mu_X(x)f_{\gamma_X(x)}(u)$$

where $|E|$ is the cardinality of E and $t_{\gamma_X(x)}(u)$ is membership degree and $f_{\gamma_X(x)}(u)$ is nonmembership degree of $u \in U$ in the vague set $\gamma_X(x)$.

Now, we construct a fuzzy parameterized vague soft set decision making method by the following algorithm:

Step 1. Constructs a feasible fuzzy subset X over the parameters set E based on a decision maker (DM) which is expert.

Step 2. Constructs a fuzzy parameterized vague soft set Γ_X over the alternatives set U based on a DM.

Step 3. Computes the aggregate vague set Γ^*_X of fuzzy parameterized vague soft set Γ_X.

Step 4. Find $max(t) = max\{t_{\Gamma^*_X}(u) : u \in U\}$ and $max(1-f) = max\{1 - f_{\Gamma^*_X}(v) : v \in U\}$.

Step 5. Find $\alpha \in [0,1]$ such that $\frac{[max(t),\alpha]}{u} \in \Gamma^*_X$ and $\beta \in [0,1]$ such that $\frac{[\beta,max(1-f)]}{v} \in \Gamma^*_X$.

Step 6. Computes $\frac{max(t)}{max(t)+(1-\alpha)} = \alpha'$ and $\frac{\beta}{\beta+(1-max(1-f))} = \beta'$.

Step 7. If $\alpha' > \beta'$, the optimal decision is u, if $\alpha' < \beta'$, the optimal decision is v.

Example 4.1. *Suppose that a workplace wants to fill a position. There are five candidates who fill in a form in order to apply formally for the position. There is a decision maker(DM), that is from the department of human resources.*

He wants to interview the candidates, but it is very difficult to make it all of them. Therefore, by using the fpvs-set decision making method, the number of candidates are reduced to a suitable one. Assume that the set of candidates $U = \{u_1, u_2, u_3, u_4, u_5\}$ *which may be characterized by a set of parameters* $E = \{x_1, x_2, x_3, x_4\}$ *which is* $''x_1 = experience\,''$, $''x_2 = technical\ information\,''$, $''x_3 = good\ speaking\,''$, $''x_4 = young\ age\,''$. *Now, we can apply the method as follows:*

Step 1. Assume that DM constructs a feasible vague subset X *over the parameters set* E *as follows:*

$$X = \{\frac{0.8}{x_1}, \frac{0.9}{x_2}, \frac{0.5}{x_3}, \frac{0.6}{x_4}\}$$

Step 2. DM constructs a vague parameterized vague soft set Γ_X *over the alternatives set* U *as follows:*

$$\Gamma_X = \{(\frac{0.8}{x_1}, \{\frac{[0.6,0.7]}{u_1}, \frac{[0.5,0.8]}{u_2}, \frac{[0.7,0.8]}{u_3}, \frac{[0.4,0.6]}{u_4}, \frac{[0.6,0.8]}{u_5}\}),$$

$$(\frac{0.9}{x_2}, \{\frac{[0.4,0.6]}{u_1}, \frac{[0.3,0.6]}{u_2}, \frac{[0.6,0.8]}{u_3}, \frac{[0.4,0.7]}{u_4}\}),$$

$$(\frac{0.5}{x_3}, \{\frac{[0.6, 0.7]}{u_1}, \frac{[0.5, 0.9]}{u_3}, \frac{[0.6, 0.8]}{u_5}\}),$$

$$(\frac{0.6}{x_4}, \{\frac{[0.5, 0.8]}{u_1}, \frac{[0.6, 0.9]}{u_2}, \frac{[0.4, 0.6]}{u_4}, \frac{[0.7, 0.8]}{u_5}\})\}$$

Step 3. DM computes the aggregate vague set Γ^*_X *of vague parameterized vague soft set* Γ_X *as:*

$$\Gamma^*_X = \{\frac{[0.36, 0.7825]}{u_1}, \frac{[0.2575, 0.855]}{u_2}, \frac{[0.3375, 0.9025]}{u_3}, \frac{[0.23, 0.7925]}{u_4}, \frac{[0.3, 0.905]}{u_5}\}$$

Step 4. $max(t) = 0.36$ *and* $max(1 - f) = 0.905$*.*
Step 5. $\frac{[0.36,0.7825]}{u_1} \in \Gamma^*_X$, $\frac{[0.3,0.905]}{u_5} \in \Gamma^*_X$*.*
Step 6. $\alpha' = \frac{0.36}{0.36+(1-0.7825)} = 0.6234$, $\beta' = \frac{0.3}{0.3+(1-0.905)} = 0.7595$
Step 7. Since $\alpha' < \beta'$*, the optimal decision is* u_5*.*

Note that, although membership degree of u_1 is bigger than u_5, opportune element of U is u_5. This example show how the effect on decision making of non-membership degrees of elements.

5 Conclusion

In this paper, we first defined fuzzy parameterized vague soft set and their various operations. Then, we introduced the method of decision making on the fpvs-set theory. We also gave an example that demonstrated that the decision making method can successfully work. These conclusions can be extensively applied in many fields such as pattern recognition, image processing, approximate reasoning, and fuzzy control. For further study, we will study algebraic structure of fpvs-sets and extend our work to other decision models and applications for modeling vagueness and uncertainty.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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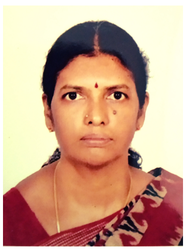
BIOGRAPHY



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