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Physical Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,
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One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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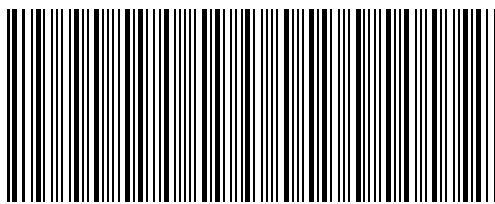
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ABOUT THE INSTITUTION

A nation's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discuss the innovative ideas and will promote to work in interdisciplinary mode.

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An Analysis of Stability of an Impulsive delay differential system

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ABSTRACT: First order linear impulsive delay differential equation with constant delays is investigated. The linear theory is the primary focus, for which theorems analogous to ordinary and impulsive differential equations are derived. Results explicitly connecting the asymptotic stability of impulsive differential equations to related impulse equations are proven. This work analyses a delay differential system under the impulsive effect, in which asymptotic behavior is analyzed with the aid of eigenvalue of the characteristic equation method. Numerical examples were presented to explain the theory.

Keywords: Impulsive, Delay, Differential systems, Asymptotic Stability

1. INTRODUCTION

In modern years differential calculus has fascinated the consideration of numerous investigators. It is an outstanding tool in modelling many phenomena in practical systems. Any physical system can be represented more accurately through a differential system. The differential equation involving an impulse effect, appear as a natural description of observed evolution phenomena of real world problems such as bursting rhythm models, optimal control models and frequency modulation etc. In everyday life, under particular conditions, an effect creates some changes instantaneously. When an instantaneous effects change the position or velocity of the moving body, such system is known as impulsive system. Also it has been found that study of impulsive delay is more appropriate to capture the real dynamical behaviour rather than differential calculus. It should be pointed out that delay system with impulse has gained the popularity due to its peculiar properties and recent progress of research in this area. For more details, one can see [1-5].

The impulsive delay differential equation is considered as:

$$u'(t) = au(t) + bu(t - \tau), t \geq 0, t \neq t_k, (1)$$

$$\Delta u(t_k) = \ell_k, k \in \mathbb{Z}^+ = \{1, 2, \dots\}, (2)$$

where I is the initial segment of natural numbers, a and b be the constant functions, and furthermore, ℓ_k for $k \in \mathbb{Z}^+$ are real constants and $\Delta u(t_k) = u(t_k^+) - u(t_k^-)$. The impulsive positive points t_k satisfy

$$0 < t_1 < \dots < t_k < t_{k+1} < \dots \text{ and } \lim_{k \rightarrow \infty} t_k = \infty,$$

Assume that the initial function ϕ is a given continuous real-valued function at the interval $[-\tau, 0]$, then an initial condition is imposed, that is, along with Equation(1):

$$u(t) = \phi(t), -\tau \leq t \leq 0. (3)$$

2. PRELIMINARIES

Basic definitions and lemmas are given in preliminary section.

Lemma 1: Suppose that λ_0 is a real root of the characteristic equation $\lambda = a + be^{-\lambda\tau}$ (4)

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and set $h_{\lambda_0} = a + be^{-\lambda_0\tau}$ for $t \geq -\tau$. (5)

Thus y is the single solution of the initial value problem

$$v'(t) = -b e^{-\lambda_0 t} [v(t) - v(t - \tau)]; v(t) = \phi(t) \exp[-h_{\lambda_0} t] \tag{6}$$

with impulsive condition $v(t_k) - v(t_k^-) = l_k \exp[-h_{\lambda_0} t_k], k \in Z^+$ (7)

and only if v is a solution of the following system $v(t) = \phi(t) \exp[-\lambda_0 t],$ if $-\tau \leq t \leq 0$ (8)

and with $n(t) = \max\{k \in Z^+ : t_k \leq t\}$ and $n(t) = 0$ if $t < t_1$. (9)

$$\text{and } v(t) = \phi(0) + e^{-\lambda_0 t} \int_{-\tau}^0 \bar{b}^-(s) \phi(s) \exp[-\lambda_0 s] ds - e^{-\lambda_0 t} \int_{t-\tau}^t \bar{b}^-(s) v(s) + \sum_{j=1}^{n(t)} l_j \exp[-\lambda_0 t], \text{ if } t \geq 0 \tag{10}$$

Corollary 1. Suppose that λ_0 is a real root of the characteristic Equation (4) and set (5). Thus, v is the single solution of the initial value problem (1–3) if and only if the function v defined by $v(t) = u(t) \exp[-h_{\lambda_0} t]$ for $t \geq -\tau$

is the solution of the integral Equation (10) which gives the initial condition

$$v(t) = \phi(t) \exp[-h_{\lambda_0} t], t \in [-\tau, 0]. \tag{11}$$

Theorem 1. Assume that Lemma 1 is valid and that the root λ_0 satisfies

$$\mu(\lambda_0) = \sum_{j=1}^{\infty} |\ell_j| \exp[-h_{\lambda_0} t_j] + b\tau e^{-\lambda_0\tau} < 1 \tag{12}$$

Thus, the solution u of Equation (1–3) fulfills

$$\lim_{t \rightarrow \infty} \{u(t) \exp[-h_{\lambda_0} t]\} = \frac{L(\lambda_0, \phi)}{1 + \beta(\lambda_0)} \tag{13}$$

Where

$$L(\lambda_0; \phi) = \phi(0) + e^{-\lambda_0\tau} \int_{-\tau}^0 \bar{b}^-(s) \phi(s) \exp[-h_{\lambda_0} s] ds \tag{14}$$

$$\text{And } \beta(\lambda_0) = B \tau e^{-\lambda_0\tau} \tag{15}$$

Note: It is guaranteed by the property (12) that $0 < 1 + \beta(\lambda_0) < 2$.

Proof. By Equation (12), we have $|\beta(\lambda_0)| \leq \mu(\lambda_0) < 1$. Thus, this yields that $0 < 1 < \beta(\lambda_0) < 2$.

Assume that u a solution of Equations (1–3). Identify the function v using Equation (6). Afterwards, u will be the solution of Equations (1–3), and v the solution of the integral Equation (10) yielding the initial condition of Equation (8). Therefore, by Equation (14), using Equation (10), we obtain

$$v(t) = L(\lambda_0; \phi) + \sum_{j=1}^{n(t)} \ell_j \exp[-h_{\lambda_0} t_j] - e^{-\lambda_0 t} \int_{t-\tau}^t \bar{b}^-(s) v(s) ds. \tag{16}$$

Now, for $t \geq -\tau$ we construct

$$w(t) = v(t) - \frac{L(\lambda_0; \phi)}{1 + \beta(\lambda_0)}$$

Hence, from the Equation (16), it is reduced to the equation as below:

$$w(t) = \sum_{j=1}^{n(t)} \ell_j \exp[-h_{\lambda_0} t_j] - e^{-\lambda_0 t} \int_{t-\tau}^t \bar{b}^-(s) z(s) ds, \text{ for } t \geq 0. \tag{17}$$

Moreover, Equation (8) is defined as for $t \in [-\tau, 0]$

$$w(t) = \phi(t) \exp[-h_{\lambda_0} t] - \frac{L(\lambda_0; \phi)}{1 + \beta(\lambda_0)} \tag{18}$$

Using y and z , we should prove the equality (13), that is,

$$\lim_{t \rightarrow \infty} w(t) = 0 \quad (19)$$

$$\text{Put } W(\lambda_0; \phi) = \max \left\{ 1, \max_{t \in [-\tau, 0]} \left| \phi(t) \exp[-h_{\lambda_0} t] \int^t -\frac{L(\lambda_0; \phi)}{1 + \beta(\lambda_0)} \right| \right\}$$

Thus, by Equation (18) we obtain

$$|w(t)| \leq W(\lambda_0; \phi) \text{ for } -\tau \leq t \leq 0 \quad (20)$$

Now, the following inequality will be proved

$$|z(t)| \leq W(\lambda_0; \phi) \text{ for } t \geq -\tau \quad (21)$$

the contrary, assume that a point is found where $\tilde{t} > 0$ such that $|z(\tilde{t})| > W(\lambda_0; \phi)$. Let

$$t^* = \inf \{ \tilde{t} : |z(\tilde{t})| > W(\lambda_0; \phi) \}.$$

According to the continuity from right, either $|z(t^*)| = W(\lambda_0; \phi)$ without impulsive point at t^* , or $|w(t^*)| \geq W(\lambda_0; \phi)$ with a jump at t^* . In both cases, by the right continuity, we obtain $|w(t)| \leq W(\lambda_0; \phi)$ for $-\tau \leq t < t^*$, where $|w(t^*)| = w(\lambda_0; \phi)$ provided that this satisfies at a non-impulsive point. Therefore, considering Equation (12), by the integral representation of $w(t)$, which all solutions to Equation (17), we obtain

$$\begin{aligned} |w(t^*)| &= \left| \sum_{j=1}^{n(t^*)} \ell_j \exp[-h_{\lambda_0} t_j] - e^{-\lambda_0 \tau} \int_{t^*-\tau}^{t^*} \bar{b}(s) w(s) ds \right| \\ &\leq \sum_{j=1}^{n(t^*)} |\ell_j| \exp[-h_{\lambda_0} t_j] + \sum_{i \in I} e^{-\lambda_0 \tau} \int_{t^*-\tau}^{t^*} |\bar{b}(s)| |w(s)| ds \\ &\leq \left\{ \sum_{j=1}^{n(t^*)} |\ell_j| \exp[-h_{\lambda_0} t_j] + \sum_{i \in I} e^{-\lambda_0 \tau} \int_{t^*-\tau}^{t^*} |\bar{b}(s)| ds \right\} W(\lambda_0; \phi) \\ &\leq \mu(\lambda_0) W(\lambda_0; \phi) < W(\lambda_0; \phi) \end{aligned}$$

which contradicts with the definition of t^* because we showed $|w(t^*)| < W(\lambda_0; \phi)$, and we suppose $|w(t^*)| = W(\lambda_0; \phi)$ where t^* is continuous, or $|w(t^*)| \geq W(\lambda_0; \phi)$ where t^* is discontinuous. Hence, the inequality (21) holds.

Next, by Equation (21), considering Equation (17) we obtain for $t \geq 0$,

$$\begin{aligned} |w(t)| &= \left| \sum_{j=1}^{n(t)} \ell_j \exp[-h_{\lambda_0} t_j] - e^{-\lambda_0 \tau} \int_{t-\tau}^t \bar{b}(s) w(s) ds \right| \\ &\leq \sum_{j=1}^{n(t)} |\ell_j| \exp[-h_{\lambda_0} t_j] + e^{-\lambda_0 \tau} \int_{t-\tau}^t |\bar{b}(s)| |w(s)| ds \\ &\leq \left\{ \sum_{j=1}^{n(t)} |\ell_j| \exp[-h_{\lambda_0} t_j] + e^{-\lambda_0 \tau} |B_1| \tau \right\} W(\lambda_0; \phi) \end{aligned}$$

$$\leq \mu(\lambda_0) W(\lambda_0; \phi),$$

In other words, we have

$$|w(t)| \leq \mu(\lambda_0) W(\lambda_0; \phi) \text{ for } t \geq 0. \quad (22)$$

By Equations (12), (21) and (22), using an easy induction, from Equation (17) it can be proved that

$$|w(t)| \leq [\mu(\lambda_0)]^n W(\lambda_0; \phi) \text{ for } t \geq n\tau - \tau (n = 0, 1, \dots) \quad (23)$$

Due to (12), we obtain $\lim_{n \rightarrow \infty} [\mu(\lambda_0)]^n = 0$. Thus, from Equation (23) we obtain

$$\lim_{t \rightarrow \infty} w(t) = \lim_{t \rightarrow \infty} \left\{ u(t) \exp[-h_{\lambda_0} t] - \frac{L(\lambda_0; \phi)}{1 + \beta(\lambda_0)} \right\} = 0$$

that is, Equation (13) satisfies. Theorem 1 has been already proven.

Corollary 2. Assume that

$$a(t) + \sum_{i \in I} b_i(t) = 0 \text{ for } t \in [0, \infty) \quad (24)$$

$$\text{And } |B_1| \tau + \sum_{j=1}^{\infty} |\ell_j| < 1 \quad (25)$$

Thus, the solution u of Equations(1–3) satisfies for any $\phi \in ([-\tau, 0], \mathbb{R})$,

$$\lim_{t \rightarrow \infty} u(t) = \frac{\phi(0) + \int_{-\tau}^0 \bar{b}^-(s) \phi(s) ds}{1 + B_1 \tau}$$

Note: It is guaranteed by Equation (25) that $2 > 1 + B_1 \tau > 0$.

3. MAIN RESULT - STABILITY CRITERION

Theorem 2. Assume that Theorem 1 is satisfied and Let λ_0 be a real root of Equation (4) satisfying Equation (5) and set $R(\lambda_0; \phi) = \max \left\{ 1, \max_{-\tau \leq t \leq 0} |\phi(t)|, \max_{-\tau \leq t \leq 0} [e^{-\lambda_0 t} |\phi(t)|] \right\}$.

Thus the solution u of the system Equations (1) and (3) satisfies

$$|u(t)| \leq N(\lambda_0) R(\lambda_0; \phi) e^{\lambda_0 t} \text{ for } t \geq 0,$$

$$\text{If } N(\lambda_0) = \mu(\lambda_0) + (1 + \mu(\lambda_0)) \left(\frac{1 + |b|}{1 + b} \frac{|\tau|}{\tau} \frac{e^{-\lambda_0 \tau}}{e^{-\lambda_0 \tau}} \right)$$

Moreover, the trivial solution:

- (i) asymptotically stable if $\lambda_0 < 0$,
- (ii) stable if $\lambda_0 = 0$ or, equivalently, providing that the conditions (43) are met, and
- (iii) unstable if $\lambda_0 > 0$.

Proof.

Suppose that x is the solution of Equations (1–3) and v, w are defined as above, that is, for $t \geq -\tau$

$$v(t) = u(t) \exp[-h_{\lambda_0} t] \text{ and } w(t) = y(t) - \frac{L(\lambda_0; \phi)}{1 + \beta(\lambda_0)},$$

where $L(\lambda_0; \phi)$ is defined as in Equation(14). Therefore, we specify $W(\lambda_0; \phi)$ as in the proof of Theorem 1, that is,

$$W(\lambda_0; \phi) = \max \left\{ 1, \max_{t \in [-\tau, 0]} \left| \phi(t) \exp[-h_{\lambda_0} t] - \frac{L(\lambda_0; \phi)}{1 + \beta(\lambda_0)} \right| \right\}$$

Hence, as in Theorem 1, it can be also proved that w satisfies inequality. Thus, for $t \geq 0$ we get

$$|v(t)| \leq \mu(\lambda_0) W(\lambda_0; \phi) + \frac{|L(\lambda_0; \phi)|}{1 + \beta(\lambda_0)}. \quad (26)$$

we obtain

$$\begin{aligned} |L(\lambda_0; \phi)| &\leq |\phi(t)| + e^{-\lambda_0 t} \int_{-\tau}^0 |\bar{b}(s)| |\phi(s)| \left| \exp[-h_{\lambda_0} s] - \frac{L(\lambda_0; \phi)}{1 + \beta(\lambda_0)} \right| ds \\ &\leq \left(1 + e^{-\lambda_0 \tau} \int_{-\tau}^0 |\bar{b}(s)| ds \right) R(\lambda_0; \phi) \\ &= (1 + |B| \tau e^{-\lambda_0 \tau}) R(\lambda_0; \phi) \\ &= k(\lambda_0) R(\lambda_0; \phi) \end{aligned}$$

Furthermore, using the definition of $W(\lambda_0; \phi)$ we have;

$$\begin{aligned} W(\lambda_0; \phi) &\leq \max \left\{ 1, R(\lambda_0, \phi) + \frac{|L(\lambda_0; \phi)|}{1 + \beta(\lambda_0)} \right\} = R(\lambda_0; \phi) + \frac{|L(\lambda_0; \phi)|}{1 + \beta(\lambda_0)} \\ &\leq R(\lambda_0; \phi) + \frac{k(\lambda_0)R(\lambda_0; \phi)}{1 + \beta(\lambda_0)} = \left(1 + \frac{K(\lambda_0)}{1 + \beta(\lambda_0)} \right) R(\lambda_0; \phi). \end{aligned}$$

we reach for $t \geq 0$

$$\begin{aligned} |v(t)| &\leq \mu(\lambda_0) \left(1 + \frac{k(\lambda_0)}{1 + \beta(\lambda_0)} \right) R(\lambda_0; \phi) + \frac{k(\lambda_0)R(\lambda_0; \phi)}{1 + \beta(\lambda_0)} \\ &= \left\{ \mu(\lambda_0) \left(1 + \frac{k(\lambda_0)}{1 + \beta(\lambda_0)} \right) + \frac{k(\lambda_0)}{1 + \beta(\lambda_0)} \right\} R(\lambda_0; \phi) \\ &= N(\lambda_0)R(\lambda_0; \phi). \end{aligned}$$

Last of all, using the definition of v , we get

$$|u(t)| \leq N(\lambda_0)R(\lambda_0; \phi) \exp[h_{\lambda_0} t], \text{ for all } t \geq 0.$$

Therefore, the first part of this theorem has been proven. Now, we can start to establish a proof for the second part (stability criterion).

Firstly, set

$$p(\lambda_0) = \sup_{t \geq 0} \{ \exp[h_{\lambda_0} t] \}$$

Obviously $p(\lambda_0)$ is a real constant such that $p(\lambda_0) \geq 1$. Furthermore, we set $p(\lambda_0) = p(\lambda_0)N(\lambda_0)$. Since $N(\lambda_0) > 1$, we also obtain $P(\lambda_0) > 1$. Let ϕ be any arbitrary function in $C([-\tau, 0], \mathbb{R})$ and x be the solution of Equations (1-3). Thus,

$$|u(t)| \leq p(\lambda_0)R(\lambda_0; \phi) \text{ for all } t \geq 0$$

When $\|\phi\| = \max_{-\tau \leq t \leq 0} |\phi(t)| \leq R(\lambda_0; \phi)$ and $p(\lambda_0) > 1$, it gives that

$$|x(t)| \leq P(\lambda_0)R(\lambda_0; \phi) \text{ for all } t \geq -\tau.$$

For any $\epsilon > 0$, choosing $\delta = \frac{\epsilon}{P(\lambda_0)}$ with $R(\lambda_0; \phi) < \delta$, we get that $\|\phi\| < \delta$. Hence,

$$|u(t)| \leq P(\lambda_0)R(\lambda_0; \phi) < P(\lambda_0)\delta = \epsilon$$

As a result, we obtain the stability of the trivial solution of Equations (1) and (2). In particular, let us consider the case that $\lambda_0 = 0$ and $h_{\lambda_0} = 0$ on the interval $[-\tau, \infty)$ as mentioned previously.

Next, thus, the trivial solution of Equations (1) and (2) is stable. Moreover, since $\lim_{t \rightarrow \infty} u(t) = 0$, it is guaranteed by the inequality that the trivial solution of Equations (1-2) is asymptotically stable.

Finally, we will prove that the trivial solution of Equations (1) and (2) is unstable. On the contrary, assume that it is stable. Hence, we can choose $\delta > 0$ such that for each $\phi \in C([-\tau, 0], \mathbb{R})$ with

$$|u(t)| < 1 \text{ for all } t \geq -\tau \tag{27}$$

Define

$$\phi_0(t) = \exp[h_{\lambda_0} t] \text{ for } t \in [-\tau, 0].$$

We see $\phi_0 \in C([-\tau, 0], \mathbb{R})$ and $\phi_0 \neq 0$. From Equation (14), we have

$$\begin{aligned} L(\lambda_0; \phi_0) &= \phi_0(0) + e^{-\lambda_0 \tau} \int_{-\tau}^0 \bar{b}_1(s) \phi_0(s) \exp[-h_{\lambda_0} s] ds \\ &= 1 + e^{-\lambda_0 \tau} \int_{-\tau}^0 \bar{b}_1(s) ds = 1 + B_1 \tau e^{-\lambda_0 \tau} \tag{28} \\ &= 1 + \beta(\lambda_0) > 0. \end{aligned}$$

Now, we take a number $\delta_0 > 0$ with $0 < \delta_0 < \delta$ and we define

$$\phi = \frac{\delta_0}{\|\phi_0\|} \phi_0.$$

Clearly, $\phi \in C([-\tau, 0], \mathbb{R})$ and $\|\phi\| = \delta_0 < \delta$. Therefore, the solution x of Equations (1–3) fulfills (34), that is, u is always bounded on $[-\tau, \infty)$. Thus, we may obtain

$$\lim_{t \rightarrow \infty} \{u(t) \exp[-h_{\lambda_0} t]\} = 0$$

Furthermore, since the operator $L(\lambda_0; \cdot)$ is linear and we obtain

$$\lim_{t \rightarrow \infty} \{u(t) \exp[-h_{\lambda_0} t]\} = \frac{L(\lambda_0; \phi)}{1 + \beta(\lambda_0)} = \frac{\left(\frac{\delta_0}{\|\phi_0\|}\right) L(\lambda_0; \phi)}{1 + \beta(\lambda_0)} = \frac{\delta_0}{\|\phi_0\|} > 0$$

We consequently reached a contradiction here that the trivial solution of Equations (1) and (2) is unstable.

4. CONCLUSION

In this work, asymptotic criterion is established for a impulsive delay differential system with constant co-efficient. The stability of the trivial solution is ascertained by converting the constructed equation into integral equations. These results were obtained using a suitable real root for the characteristic equation. Namely that, characteristic equation and real root plays an important role in establishing the results of the article.

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