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NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

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One day International Conference EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

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Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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The International conference on "Emerging Trends in Science and Technology (ETIST-2021)" is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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A New Conception of Continuous Functions in Binary

Topological Spaces

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ABSTRACT: The paper proposes the introducing of strongly binary semi generalized continuous functions in binary topological spaces. It also provides appropriate examples for the clear perception of abstract concepts.

Keywords: b-sg-continuous, strongly b-sg-continuous.

1. INTRODUCTION

Binary topology from X to Y was introduced by Nithyanantha Jothi and Thangavelu [1] in 2011. They founded and investigated the concepts of binary closed, binary closure, binary interior and binary continuity, base and sub base of a binary topological space as well. The experts [2] postulated the idea of generalized binary closed sets in 2014. Nithyanantha Jothi [4] introduced binary semi open sets in binary topological spaces and obtained certain basic results. Recently, Sathishmohan et.al, [5] introduced and studied and brought into emergence the concepts of binary generalized semi closed sets and binary semi generalized closed sets in binary topological spaces. Consequently they [6] conceptualized binary generalized semi (binary semi generalized)-continuous functions in binary topological spaces. This paper proposes the introducing of strongly binary semi generalized continuous functions in binary topological spaces. It also provides appropriate examples are provided to illustrate the behavior of this new class of function.

2. PRELIMINARIES

Definition 2.1. Let X and Y be any two nonempty sets. A binary topology [1] from X to Y is a binary structure $\mathcal{M} \subseteq \mathcal{P}(X) \times \mathcal{P}(Y)$ that satisfies the axioms.

- (1) (ϕ, ϕ) and $(X, Y) \in \mathcal{M}$.
- (2) $(A_1 \cap A_2, B_1 \cap B_2) \in \mathcal{M}$ whenever $(A_1, B_1) \in \mathcal{M}$ and $(A_2, B_2) \in \mathcal{M}$.

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(3) If $\{(A_{\alpha}, B_{\alpha}) : \alpha \in \Delta\}$ is a family of members of M then $\left(\begin{array}{cc} 0 & A_{\alpha}, \\ \alpha \in \Delta A_{\alpha}, & \alpha \in \Delta \end{array}\right) \in \mathcal{M}$.

Definition 2.2. [1] If M is a binary topology from X to Y then the triplet (X, Y, M) is called a binary topological space and the members of M are called the binary open subsets of the binary topological space (X, Y, M) . The elements of X×Y are called the binary points of the binary topological space (X,Y,M). If Y=X then M is called a binary topology on X in which case we write (X, M) as a binary topological space.

Definition 2.3. [1] Let X and Y be any two nonempty sets and let (A,B) and (C,D) $\in \mathcal{P}(X) \times \mathcal{P}(Y)$. We say that $(A,B) \subseteq (C,D)$ if $A \subseteq C$ and $B \subseteq D$.

Definition 2.4. [1] Let (X, Y, M) be a binary topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary closed in (X, Y, \mathcal{M}) if $(X/A, Y/B) \in \mathcal{M}$.

Proposition 2.5. [1] Let (X, Y, M) be a binary topological space and $(A, B) \subseteq (X, Y)$. Let $(A, B) \subseteq (X, Y)$. Let $(A,B)^{1^*} = \bigcap \{A_{\alpha}:(A_{\alpha},B_{\alpha}) \text{ is binary closed and } (A,B) \subseteq (A_{\alpha},B_{\alpha}) \}$ and $(A,B)^{2^*} = \bigcap \{B_{\alpha}:(A_{\alpha},B_{\alpha}) \text{ is binary closed and } (A,B) \neq 0 \}$ $(A,B) \subseteq (A_{\alpha},B_{\alpha})\}.$ Then $((A,B)^{1*}, (A,B)^{2*})$ is binary closed and $(A,B) \subseteq ((A,B)^{1*}, (A,B)^{2*})$

Definition 2.6. [1] The ordered pair $((A,B)^{1*},(A,B)^{2*})$ is called the binary closure of (A,B) , denoted by b-cl (A,B) in the binary space (X, Y, M) where $(A, B) \subseteq (X, Y)$.

Definition 2.7. [1] (i) $(A,B)^{1^\circ} = \bigcup \{A_\alpha : (A_\alpha,B_\alpha) \text{ is binary open and } (A_\alpha,B_\alpha) \subseteq (A,B)\}.$

(ii) $(A,B)^{2^\circ} = \cup \{B_\alpha : (A_\alpha, B_\alpha)$ is binary open and $(A_\alpha, B_\alpha) \subseteq (A,B)\}.$

Definition 2.8. [1] Let (X, Y, M) be a binary topological space and $(A, B) \subseteq (X, Y)$. The ordered pair $((A,B)^{1\degree},(A,B)^{2\degree})$ is called the binary interior of (A,B) , denoted by b-int(A,B).

Definition 2.9. [1] Let (X, Y, M) be a binary topological space. Let $(A, B) \subseteq (X, Y)$. Define $\mathcal{M}_{(A,B)}$ ={A∩U,B∩V):(U,V) $\in \mathcal{M}$ }. Then $\mathcal{M}_{(A,B)}$ is a binary topology from A to B. The binary topological space $(A,B,\mathcal{M}_{(A,B)})$ is called a binary subspace of (X,Y,\mathcal{M}) .

Definition 2.10. A subset (A,B) of a binary topological space (X,Y,\mathcal{M}) is called

(1) binary semi-closed [4], if b-int(b-cl(A,B)) \subseteq (A,B).

- (2) binary g-closed [2], if b-cl(A) \subseteq (U,V) whenever (A,B) \subseteq (U,V) and (U,V) is binary open.
- (3) binary gs-closed [5], if b-scl(A) \subseteq (U,V) whenever (A,B) \subseteq (U,V) and (U,V) is binary open.
- (4) binary g-closed [5], if b-scl(A) \subseteq (U,V) whenever (A,B) \subseteq (U,V) and (U,V) is binary semi open.

Definition 2.11. [3], Let $f : Z \to X \times Y$ be a function. Let $A \subseteq X$ and $B \subseteq Y$, we define $f^{-1}(A,B) = \{z \in Z : f(z)$ $=(x, y) \in (A, B)$.

Definition 2.12. Let (Z, τ) be a topological space and (X, Y, M) be a binary topological space. Then the map f: $Z \rightarrow X \times Y$ is called

- (1) binary continuous [3], if $f^{-1}(A,B)$ is open in Z for every binary open set (A,B) in (X, Y, M) .
- (2) binary semi continuous [4], if $f^{-1}(A,B)$ is semi open in Z for every binary open set (A,B) in (X,Y,M) .
- (3) generalized binary continuous [3], if $f^{-1}(A,B)$ is generalized open in Z for every binary open set (A,B) in (X,Y,M) .
- (4) binary generalized semi continuous(briefly, b-gs-continuous)[6], if f^{-1} (A,B) is generalized semi open in Z for every binary open set (A,B) in (X,Y,M) .
- (5) binary semi generalized continuous(briefly, b-sg-continuous)[6], if f^{-1} (A,B) is semi generalized open in Z for every binary open set (A,B) in (X,Y,M) .
- (6) strongly binary continuous [4], if $f^{-1}(A,B)$ is clopen in Z for every binary set (A,B) in (X, Y, M) .
- (7) strongly binary semi continuous [4], if $f^{-1}(A,B)$ is semi clopen in Z for every binary set (A,B) in (X,Y,M) .
- (8) strongly generalized binary continuous(briefly, strongly b-g-continuous) [6], if f^{-1} (A,B) is generalized clopen in Z for every binary set (A,B) in (X,Y,\mathcal{M}) .
- (9) strongly binary generalized semi continuous(briefly, strongly b-gs-continuous)[6], if f^{-1} (A,B) is generalized semi clopen in Z for every binary set (A,B) in (X,Y,\mathcal{M}) .

3. STRONGLY BINARY SEMI GENERALIZED CONTINUOUS FUNCTIONS

This part is dedicated to present another class of binary functions known as strongly binary semi generalized continuous functions and to concentrate on some of their characterizations.

Definition 3.1. Let (Z, τ) be a topological space and (X, Y, M) be a binary topological space. Then the map f : Z \rightarrow X × Y is called strongly binary semi generalized continuous(briefly, strongly b-sg-continuous), if $f^{-1}(A,B)$ is semi generalized clopen in Z for every binary set (A,B) in (X,Y,M) .

Theorem 3.2. A function $f: Z \rightarrow X \times Y$ the following hold

(1) Every strongly binary continuous is binary semi generalized continuous.

(2) Every strongly binary continuous is strongly binary semi generalized continuous.

(3) Every strongly binary semi continuous is strongly binary semi generalized continuous.

(4) Every strongly binary semi generalized continuous is binary semi generalized continuous.

Proof: Let (A,B) be a binary set in (X, Y, M) . Since f is strongly binary continuous, we have $f^{-1}(A, B)$ is both open and closed in Z then $f^{-1}(A,B)$ is sg-open in Z. Hence f is binary semi generalized continuous. Proof of (2) to (4) is obvious.

The converse of the above theorems need not be true as seen from the subsequent example.

Example 3.3. Let $X = \{\alpha, \beta\}$, $Y = \{\alpha, \beta, \gamma\}$ and $Z = \{a,b,c\}$. Clearly $\mathcal{M} = \{(\emptyset, \emptyset), (\{\beta\}\{\beta, \gamma\})\}$ is a binary topology form X to Y and $\tau = \{ \emptyset, Z, \{a\}, \{b,c\} \}$ is a topology on Z. Then closed subset in Z are $\emptyset, Z, \{b,c\}, \{a\}.$ Hence the clopen sets in Z are \emptyset ,{a},{b,c},Z and the semi clopen sets in Z are \emptyset ,{a},{b,c},Z. Now sg-open subset in Z are \emptyset , Z , $\{a\}$, $\{b\}$, $\{c\}$, $\{a,b\}$, $\{a,c\}$, $\{b,c\}$. Thus the sg-clopen sets are \emptyset ,Z,{a},{b},{c},{a,b},{a,c},{b,c}. Define f : Z \rightarrow X \times Y by f(b)=({ $\{\beta\}$,{ $\{\beta,\gamma\}$) and f(a)=({ $\alpha\}$,{ $\gamma\}$)=f(c), clearly f is b-sg-continuous. For, $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\emptyset, \{\alpha\}) = \emptyset$, $f^{-1}(\emptyset, \{\beta\}) = \emptyset$, $f^{-1}(\emptyset, \{\gamma\}) = \emptyset$, $f^{-1}(\emptyset, \{\alpha, \beta\}) = \emptyset$, $f^{-1}(\emptyset,\{\alpha,\gamma\})=\emptyset$, $f^{-1}(\emptyset, \{\beta,\gamma\})=\emptyset$, $f^{-1}(\emptyset,\gamma)=\emptyset$, $f^{-1}(\{\alpha\},\emptyset)=\emptyset$, $f^{-1}(\{\alpha\},\{\alpha\})=\emptyset$, $f^{-1}(\{\alpha\},\{\beta\})=\emptyset$, $f^{-1}(\{\alpha\},\{\gamma\})=\{a,c\},\ f^{-1}(\{\alpha\},\{\alpha,\beta\})=\emptyset,\ f^{-1}(\{\alpha\},\{\alpha,\gamma\})=\{a,c\},\ f^{-1}(\{\alpha\},\{\beta\gamma\})=\emptyset,\ f^{-1}(\{\alpha\},Y)=\{a,c\},\$ $f^{-1}(\{\beta\},\emptyset)=\emptyset$, $f^{-1}(\{\beta\},\{\alpha\})=\emptyset$, $f^{-1}(\{\beta\},\{\beta\})=\emptyset$, $f^{-1}(\{\beta\},\{\gamma\})=\emptyset$, $f^{-1}(\{\beta\},\{\alpha,\beta\})=\emptyset$.

 $(\{\beta\}, {\{\alpha,\gamma\}})=\emptyset$, $f^{-1}(\{\beta\}, {\{\beta\gamma\}})=\{b\}, f^{-1}(\{\beta\}, Y)=\{b\}, f^{-1}(X,\emptyset)=\emptyset, f^{-1}(X,{\{\alpha\}})=\emptyset, f^{-1}(X,{\{\beta\}})=\emptyset$, $f^{-1}(X,\{\gamma\})=\{a,c\}, f^{-1}(X,\{\alpha,\beta\})=\emptyset, f^{-1}(X,\{\alpha,\gamma\})=\{a,c\}, f^{-1}(X,\{\beta,\gamma\})=\{b\}, f^{-1}(X,Y)=Z,$

This gives inverse image of every binary sets in (X, Y, M) is semi generalized clopen in Z. Hence f is binary semi generalized continuous ,strongly binary semi generalized continuous, . But f is not strongly binary continuous, strongly binary semi continuous, since {b} is not a clopen, semi clopen in Z.

Theorem 3.4. A function f : $Z \rightarrow X \times Y$ from a topological spaces Z into binary topological spaces (X,Y) is strongly binary semi generalized continuous if and only if the inverse image of every binary set in (X, Y) is semi generalized clopen in Z.

Proof: Assume that f is strongly binary semi generalized continuous. Let (A,B) be any binary set in (X,Y) . Then $(A,B)^c$ is binary set in (X,Y) . Since f is strongly binary semi generalized continuous. $f^{-1}((A,B)^c)$ is semi generalized clopen in Z. But $f^{-1}((A,B)^c) = Z - f^{-1}(A,B)$ and so $f^{-1}(A,B)$ is semi generalized clopen in Z.

Conversely, assume that the inverse image of every binary set in (X,Y) is semi generalized clopen in Z. Then (A,B) ^c is binary set in (X,Y) . By assumption $f^{-1}((A,B)$ ^c) is semi generalized clopen in Z but $f^{-1}((A,B)$ ^c)=Z $f^{-1}(A,B)$ and so $f^{-1}(A,B)$ is semi generalized clopen in Z. Therefore, f is strongly binary semi generalized continuous.

Theorem 3.5. Let $f : Z \to X \times Y$ be a function, (Z, τ) be a topological space and (X, Y, M) be a binary topological spaces. Then the following are equivalent.

- (i) f is strongly b-sg-continuous
- (ii) for every $z \in Z$ and for every binary set (A,B) with $f(z) \in (A,B)$ there is a sg-clopen set $U\subseteq Z$ such that $f(U)\subseteq (A,B)$.

Proof

(1) \rightarrow (2) Suppose f : Z \rightarrow X \times Y is a strongly b-sg-continuous and (A,B) be a binary set with f(z)=(x,y) \in (A,B) such that $z \in f^{-1}(A, B)$. Since f is strongly b-sg-continuous, $z \in$

 $f^{-1}(A,B)$ is sg-clopen in Z. Let U= $z \in f^{-1}(A,B)$ then U is sg-clopen in Z and $z \in U$. Also $f(U)=\{f(u): u \in E\}$ U }⊆(A,B). This implies $f(U)$ ⊆(A,B).

(2)→(1) We assume that for all z ∈ Z and for every binary set (A,B) in (X,Y, M). Let $z \in f^{-1}(A,B)$ be a any arbitrary point. This implies $f(z) \in (A,B)$ therefore by (2) there is a sg-clopen set U in z with $z \in U$, $f(U) \subseteq (A,B)$, which implies $u \in f^{-1}(A,B)$ is a sg-clopen neighbourhood of z. Since z is arbitrary, it implies $f^{-1}(A,B)$ is a sgclopen neighbourhood of each of its points. This proves that $f^{-1}(A,B)$ is sg-clopen in Z that implies f is strongly b-sg-continuous.

4. CONCLUSION

In this paper, we had introduced and studied the concept of strongly binary semi generalized continuous in binary topological spaces and interrogate some of their characterizations.

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