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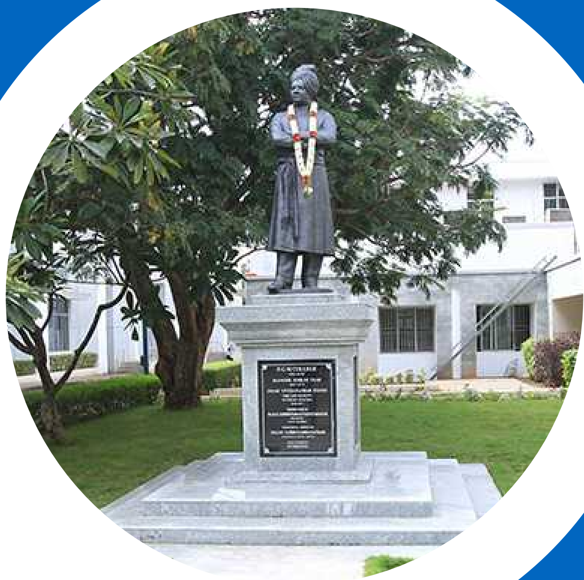

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EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

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GENERARLIZED PYTHAGOREAN FUZZY CLOSED SETS

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Abstract: In this paper, we introduce the concept of Generalized Pythagorean Fuzzy Closed Sets in Pythagorean fuzzy topological spaces and some of their properties investigated. Also we introduced the operations Generalized Pythagorean Fuzzy interior,Generalized Pythagorean Fuzzy closure on Pythagorean fuzzy topological spaces.

Keywords : Generalized Pythagorean closed sets, Generalized Pythagorean open sets, Generalized Pythagorean Fuzzy Closure, Generalized Pythagorean Fuzzy Interior. **2010 Subject classification:** 54A05, 54A10

1 Introduction

In 1965, fuzzy set theory first introdced by Zadeh. Fzzy set theory was characterized by a membership function which assigns to each target a membership value ranging between 0 and 1. In 1968, the concept of fuzzy topological space was introduced by Chang. Also generalized some basic notions of topology such as open set, closed set, continuity andcompactness to fuzzy topological spaces. Atanassov introduced the concept of intuitionistic fuzzy sets. An introduction to intuitionistic fuzzy topological spaces was given by Coker in 1997. Yager proposed another class of non-standard fuzzy sets, called Pythagorean fuzzy sets. The concept and notions of Pythagorean fuzzy topological spaces was introduced by Murat Olgun, Mehmet Unver and Seyhmus Yardimici. In 2020, Naeem et. al. studied Pythagorean m-polar fuzzy topology with TOPSIS approach in exploring most effectual method fr curring from COVID-19. Taha Yasin Ozturk and Adem Yolcu introduced some operations sucha as Pythagorean fuzzy interior, closure boundary on Pythagorean fuzzy topological spaces. Also Pythagorean fuzzy open(closed) functions and Pythagorean fuzzy homeomorphism are introdced and their basic properties are investigated in 2020.

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2 Preliminaries

Definition 2.1. *Let X be the non empty universe of discourse. A fuzzy set A in X,* $A = \{(x, \mu_A(x)) : x \in X\}$ *where* $\mu_A : X \to [0,1]$ *is the membership function of the fuzzy set A;* $\mu_A(x) \in [0,1]$ *is the membership of* $x \in X$ *in A*

Definition 2.2. *Let X be the non empty universe of discourse. An Intuitionistic fuzzy set(IFS) A in X is given by A=*$\{x, \mu_A(x), \nu_A(x) : x \in X\}$ *where the functions* $\mu_A(x) \in [0,1]$ *and* $\nu_A(x) \in [0,1]$ *denote the degree of membership and degree of non membership of each element* $x \in X$ *to the set A, respectively, and* $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ *for each* $x \in X$*. The degree of indeterminacy* $I_A = 1 - (\mu_A(x) + \nu_A(x))$ *for each* $x \in X$*.*

Definition 2.3. *Let* (X, T) *be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set A in* (X, T) *is said to be generalized intuitionistic fuzzy closed (in shortly GIF -closed) if* $IFcl(A) \subseteq G$ *whenever* $A \subseteq G$ *and G is intuitionistic fuzzy open. The complement of a GIF-closed set is GIF-open.*

Definition 2.4. *Let X be the non empty universe of discourse. A Pythagorean Fuzzy Set(PFS) P in X is given by P=*$\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ *where the functions* $\mu_P(x) \in [0,1]$ *and* $\nu_P(x) \in [0,1]$ *denote the degree of membership and degree of non membership of each element* $x \in X$ *to the set P, respectively, and* $0 \leq \mu_P^2(x) + \nu_P^2(x) \leq 1$ *for each* $x \in X$*. The degree of indeterminacy* $I_P = \sqrt{1 - (\mu_P^2(x) + \nu_P^2(x))}$ *for each* $x \in X$*.*

Definition 2.5. *Let* $P_1 = \{\langle x, \mu_{P_1}(x), \nu_{P_1}(x)\rangle : x \in X\}\}$ *and* $P_2 = \{\langle x, \mu_{P_2}(x), \nu_{P_2}(x)\rangle : x \in X\}\}$ *be two pythagorean fuzzy sets over X. Then,*

1. *the pythagorean fuzzy complement of* P_1 *is defined by*

$$ {P_1}^c = \{\langle x, \nu_{P_1}(x), \mu_{P_1}(x)\rangle : x \in X\}\} $$

2. *the pythagorean fuzzy intersection of* P_1 *and* P_2 *is defined by*

$$ P_1 \cap P_2 = \{\langle x, min\{\mu_{P_1}(x), \mu_{P_2}(x)\}, max\{\nu_{P_1}(x), \nu_{P_2}(x)\}\langle x \in X\}\}, $$

3. *the pythagorean fuzzy union of* P_1 *and* P_2 *is defined by*

$$ P_1 \cup P_2 = \{\langle x, max\{\mu_{P_1}(x), \mu_{P_2}(x)\}, min\{\nu_{P_1}(x), \nu_{P_2}(x)\}\rangle : x \in X\}, $$

4. *we say* P_1 *is a pythagorean fuzzy subset of* P_2 *and we write* $P_1 \subseteq P_2$ *if* $\mu_{P_1}(x) \leq \mu_{P_2}(x)$ *and* $\nu_{P_1}(x) \geq \nu_{P_2}(x)$ *for each* $x \in X$*,*

5. $0_X = \{\langle x, 0, 1\rangle\}, x \in X\}$ *and* $1_X = \{\langle x, 1, 0\rangle : x \in X\}$*.*

Definition 2.6. *Let $(X, \tau)_P$ be an Pythagorean Fuzzy topological space and P=$\{\langle x, \mu_P(x), \nu_P(x)\rangle : x \in X\}$ be a Pythagorean fuzzy set over X. Then the Pythagorean fuzzy interior, Pythagorean fuzzy closure and Pythagorean fuzzy boundary of P are defined by;*
a. $int(P) = \bigcup\{G : G \text{ is a } PFOS \text{ in } X \text{ and } G \subseteq P\}$
b.$cl(P) = \bigcap\{K : K \text{ is a } PFCS \text{ in } X \text{ and } P \subseteq K\}$
c. $Fr(P) = cl(P) \cap cl(P^c)$

Remark 2.7. *It is clear that,*
a. $int(P)$ is the biggest Pythagorean fuzzy open set contained in P,
b. $cl(P)$ is the smallest Pythagorean fuzzy closed set containing P.

Remark 2.8. *From the definition Pythagorean fuzzy union and intersection, it is obvious that pythagorean fuzzy interor, closure and boundary is a pythagorean fuzzy set.*

3 Generalized Pythagorean fuzzy closed sets

Definition 3.1. *Let (X, τ) be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set A in (X, τ) is said to be generalized intuitionistic fuzzy closed (GIFC) if $IFcl(A) \subseteq G$ whenever $A \subseteq G$ and G is IFO*

The complement of GIFC set is GIFO set.

Definition 3.2. *Let $(X, \tau)_P$ be an Pythagorean Fuzzy topological space. An Pythagorean Fuzzy set A in $(X, \tau)_P$ is said to be generalized Pythagorean fuzzy closed (shortly GPFC) if $PFcl(A) \subseteq P$ whenever $A \subseteq P$ and P is PFO.*

The complement of GPFC is GPFO

Definition 3.3. *Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A be a PFS in X. Then Generalized Pythagorean fuzzy closure and Generalized Pythagorean Fuzzy interior of A are defined by*
(1) $GPFcl(A) = \bigcap\{G : G \text{ is } GPF \text{ closed set in } X \text{ and } A \subseteq G\}$
(2) $GPFint(A) = \bigcup\{G : G \text{ is } GPF \text{ open set in } X \text{ and } A \supseteq G\}$

Proposition 3.4. *Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A and B be any two Pythagorean fuzzy sets in $(X, \tau)_P$. Then the Generalized Pythagorean Fuzzy closure operator satisfy the following properties.*

1. $A \subseteq GPFcl(A)$
2. $GPFcl(GPFcl(A)) = GPFcl(A)$
3. $A \subseteq B \Rightarrow GPFcl(A) \subseteq GPFcl(B)$
4. $GPFcl(A \cup B) = GPFcl(A) \cup GPFcl(B)$
5. $GPFcl(1_X) = 1_X$; $GPFcl(0_X) = 0_X$.

Proof. (i), (ii), (iii) and (v) can be esily obtained by the definition of the GPFclosure.
(iv) From $GPFcl(A) \subseteq GPFcl(A \cup B)$. We obtain $GPFcl(A) \cup GPFcl(B) \subseteq GPFcl(A \cup B)$. On the other hand, from the facts $A \subseteq GPFcl(A)$ and $B \subseteq GPFcl(B) \implies A \cup B \subseteq GPFcl(A) \cup GPFcl(B)$ and $GPFcl(A) \cup GPFcl(B) \in GPFCS$. We have $GPFcl(A \cup B) \subseteq GPFcl(A) \cup GPFcl(B)$.
Thus, proof of the axioms (iv) is obtained from these two inequalities. □

Proposition 3.5. *Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A and B be any two Pythagorean fuzzy sets in $(X, \tau)_P$. Then the Generalized Pythagorean Fuzzy interior operator satisfy the following properties.*

1. $GPFint(A) \subseteq A$
2. $GPFint(GPFint(A)) = GPFint(A)$
3. $A \subseteq B \Rightarrow GPFint(A) \subseteq GPFint(B)$
4. $GPFint(A \cap B) = GPFint(A) \cap GPFint(B)$
5. $GPFint(1_X) = 1_X$; $GPFint(0_X)) = 0_X$

Proof. (i), (ii), (iii) and (v) can be easily obtained from the definiton of the Generalized Pythagorean Fuzzy interior .
(iv) From $GPFint(A \cap B) \subseteq GPFint(A)$ and $GPFint(A \cap B) \subseteq GPFint(B)$.

We obtain $GPFint(A \cap B) \subseteq GPFint(A) \cap GPFint(B)$. On the other hand, from the facts $GPFint(A) \subseteq A$ and $GPFint(B) \subseteq B \implies GPFint(A) \cap GPFint(B) \subseteq A \cap B$ and $GPFint(A) \cap GPFint(B) \in \tau_P$. We have $GPFint(A) \cap GPFint(B) \subseteq GPFint(A \cap B)$. Thus, proof of the axioms (iv) is obtained from these two inequalities. □

Proposition 3.6. *Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A and B be any two Pythagorean fuzzy sets in $(X, \tau)_P$. Then the following properties hold.*

1. $1 - GPFcl(A) = GPFint(1 - A)$
2. $1 - GPFint(A) = GPFcl(1 - A)$

Proposition 3.7. *If A and B are GPF-closed sets, then $A \cup B$ is a GPF-closed set.*

Remark 3.8. *The intersection of two GPF-closed sets need not be GPF-closed set.*

Proposition 3.9. *Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. If B is GPF-closed and $B \subseteq A \subseteq PFcl(B)$ then A is GPF-closed.*

Proposition 3.10. *In an Pythagorean fuzzy topological space $(X, \tau)_P$, $\tau_P = T_P$ (The family of all Pythagorean fuzzy closed Sets) iff every Pythagorean fuzzy closed set of $(X, \tau)_P$ is a GPF closed set.*

Proof. Suppose that evey Pythagorean fuzzy set A of $(X, \tau)_P$ is GPF closed. Let $A \in \tau_P$. Since $A \subseteq A$ and A is GPF-closed, $PFcl(A) \subseteq A$. But $A \subseteq PFcl(A)$. Hence, $PFcl(A) = A$. Thus, $A \in \tau_P$. Therefore, $\tau_P \subseteq T_P$. If $B \in T$, then $1_X - B \in \tau_P \subseteq T_P$ and hence $B \in \tau_P$.
That is $T_P \subseteq \tau_P$. Therefore $\tau_P = T_P$
Conversely, Suppose that A be a Pythagorean Fuzzy set in $(X, \tau)_P$. Let B be a Pythagorean fuzzy open set in $(X, \tau)_P$ such that $A \subseteq B$. By hypohtesis, B is Pythagorean fuzzy closed set. By the definition of Pythagorean fuzzy closure $PFcl(A) \subseteq B$). Therefore A is GPF-closed. □

Proposition 3.11. *If $GPFint(A) \subseteq B \subseteq A$ and if A is GPF-open then B is also GPF-open.*

Proposition 3.12. *Let $(X, \tau)_P$ be a Pythagorean fuzzy topological space. A Pythagorean fuzzy set A is GPF-open iff $B \subseteq GPFint(A)$, whenever B is Pythagorean fuzzy closed and $B \subseteq A$.*

Proof. Let A be a GPF-open set and B be a Pythagorean fuzzy closed set, such that $B \subseteq A$. Now, $B \subseteq A \implies 1_X - A \subseteq 1_X - B$ and $1_X - A$ is a GPF-closed set $\implies PFcl(1_X - A) \subseteq 1_X - B$. That is, $B = 1_X - (1_X - B) \subseteq 1_X - PFcl(1_X - A)$. But $1_X - PFcl(1_X - A) = PFint(A)$. Thus, $B \subseteq PFint(A)$. Conversely, suppose that A be a Pythagorean fuzzy set, such that $B \subseteq PFint(A)$ whenever B is Pythagorean fuzzy closed and $B \subseteq A$. Let $1_X - A \subseteq B$ whenever B is Pythagorean fuzzy-open. Now, $1_X - A \subseteq B \implies 1_X - B \subseteq A$. Hence by assumption, $1_X - B \subseteq PFint(A)$. That is, $1_X - PFint(A) \subseteq B$. But $1_X - PFint(A) = PFcl(1_X - A)$. Hence, $PFcl(1_X - A) \subset B$. That is, $1_X - A$ is GPF-closed. Therefore, A is GPF-open. □

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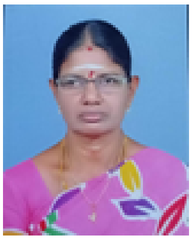
Biography



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