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NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

Pollachi-642001



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PROCEEDING

One day International Conference EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on "Emerging Trends in Science and Technology (ETIST-2021)" is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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S. No.	Article ID	Title of the Article	Page No.
1	P3049T	Fuzzy parameterized vague soft set theory and its applications - Yaya Li, Velusamy Inthumathi, Chang Wang	1-14
2	P3050T	Intuitionistic fuzzy soft commutative ideals of BCK-algebras - Nana Liu, Velusamy Inthumathi, Chang Wang	15-37
3	P3051T	Intuitionistic fuzzy soft positive implicative ideals of BCK-algebras - Nana Liu, Velusamy Inthumathi, Chang Wang	38-56
4	P3052T	Vague Soft Fundamental Groups - M. Pavithra, Saeid Jafari, V. Inthumathi	57-70
5	P3053T	Nano Generalized pre c-Homeomorphism in Nano Topologicalspaces - P.Padmavathi and R.Nithyakala	71-76
6	P3054D	Third order nonlinear difference equations with a superlinearneutral term - S.Kaleeswari, Ercan Tunc	77-88
7	P3055OR	Usance of Mx/G(a,b)/1 Queue Model for a Real Life Problem - B.Lavanya, R.Vennila, V.Chitra	89-99
8	P3056T	Solving Intuitinistic Fuzzy Multi-Criteria Decision Making forProblems a Centroid Based Approach	100-109
9	P3057T	- M. Suresh, K. Arun Prakash and R. Santhi Magnitude Based Ordering of Triangular Neutrosophic Numbers K. Radhika, K. Arunprakash and P. Santhi	110-118
10	P3058D	Solution of Linear Fuzzy Volterra Integro- Differential Equationusing Generalized Differentiability	119-143
11	P3059D	- S. Indrakumar, K. Kanagarajan, R. Santhi An Analysis of Stability of an Impulsive delay differential system -	144-149
12	P3060T	S. Priyadharsinil E. Kungumaraj and R. Santhi The Knight's Path Analysis to reach the Aimed Destination by using the Knight's Fuzzy Matrix	150-155
13	P3061T	- K. Sugapriya, B. Amudhambigai A new conception of continuous functions in binary topologicalspaces	156-160
14	P3063T	-P. Sathishmohan, K. Lavanya, V. Kajendran and M. Amsaveni The Study of Plithogenic Intuitnistic fuzzy sets and its applicationin Insurance Sector	161-165
15	P3064T	- S.P. Priyadharshini and F. Nirmala Irudayam Contra *αω continuous functions in topological spaces	166-175
		- K.Baby, M.Amsaveni, C.Varshana Stability analysis of heterogeneous bulk service queueing model -	
16	P3065OR	R. Sree Parimala	176-182
17	P3067T	Generarlized pythagorean fuzzy closedsets - T.Rameshkumar, S. Maragathavalli and R. Santhi	183-188
18	P3068T	Generalized anti fuzzy implicative ideals of near-rings - M. Himaya Jaleela Begum, P. Ayesha Parveen and J.Jayasudha	189-193
19	P3069T	Horizontal trapezoidal intuitionistic fuzzy numbers in stressDetection of cylindrical shells - J.Akila Padmasree, R. Parvathi and R.Santhi	194-201
20	P3070MH	Role of mathematics in history with special reference to pallavaweights and measure -S. Kaleeswari and K. Mangayarkarasi	202-207
21	P3071G	Feature selection and classification from the graph using neuralnetwork based constructive learning approach -A. Sangeethadevi, A. Kalaiyani and A. shanmugapriya	208-221
22	P3072T	Properties of fuzzy beta rarely continuous functions -M. Saraswathi, J.Jayasudha	222-224
23	P3073OR	Computational approach for transient behaviour of M/M(a,b)/1bulk service queueing system with starting failure	225-238
24	P3001T	-Snahon, with the galapath Subramanian and Gopal sekar b- $H\beta$ -open sets in HGTS -V. Chitra and R. Ramesh	239-245
25	P3034G	The geodetic number in comb product of graphs - Dr. S. Sivasankar, M. Gnanasekar	246-251

GENERARLIZED PYTHAGOREAN FUZZY CLOSED SETS

T. Ramesh kumar¹, S. Maragathavalli², R. Santhi³

Abstract: In this paper, we introduce the concept of Generalized Pythagorean Fuzzy Closed Sets in Pythagorean fuzzy topological spaces and some of their properties investigated. Also we introduced the operations Generalized Pythagorean Fuzzy interior, Generalized Pythagorean Fuzzy closure on Pythagorean fuzzy topological spaces.

Keywords : Generalized Pythagorean closed sets, Generalized Pythagorean open sets, Generalized Pythagorean Fuzzy Closure, Generalized Pythagorean Fuzzy Interior. **2010 Subject classification:** 54A05, 54A10

1 Introduction

In 1965, fuzzy set theory first introduced by Zadeh. Fzzy set theory was characterized by a membership function which assigns to each target a membership value ranging between 0 and 1. In 1968, the concept of fuzzy topological space was introduced by Chang. Also generalized some basic notions of topology such as open set, closed set, continuity and compactness to fuzzy topological spaces. Atanassov introduced the concept of intuitionistic fuzzy sets. An introduction to intuitionistic fuzzy topological spaces was given by Coker in 1997. Yager proposed another class of non-standard fuzzy sets, called Pythagorean fuzzy sets. The concept and notions of Pythagorean fuzzy topological spaces was introduced by Murat Olgun, Mehmet Unver and Seyhmus Yardimici. In 2020, Naeem et. al. studied Pythagorean m-polar fuzzy topology with TOPSIS approach in exploring most effectual method fr curring from COVID-19. Taha Yasin Ozturk and Adem Yolcu introduced some operations such as Pythagorean fuzzy interior, closure boundary on Pythagorean fuzzy topological spaces. Also Pythagorean fuzzy interior, closure boundary on fuzzy topological spaces. Also Pythagorean fuzzy open(closed) functions and Pythagorean fuzzy homeomorphism are introdeed and their basic properties are investigated in 2020.

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2 Preliminaries

Definition 2.1. Let X be the non empty universe of discourse. A fuzzy set A in X, $A = \{(x, \mu_A(x)) : x \in X\}$ where $\mu_A : X \to [0, 1]$ is the membership function of the fuzzy set A; $\mu_A(x) \in [0, 1]$ is the membership of $x \in X$ in A

Definition 2.2. Let X be the non empty universe of discourse. An Intuitionistic fuzzy set(IFS) A in X is given by $A = \{x, \mu_A(x), \nu_A(x) : x \in X\}$ where the functions $\mu_A(x) \in [0,1]$ and $\nu_A(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. The degree of indeterminacy $I_A = 1 - (\mu_A(x) + \nu_A(x))$ for each $x \in X$.

Definition 2.3. Let (X,T) be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set A in (X,T) is said to be generalized intuitionistic fuzzy closed (in shortly GIF -closed) if $IFcl(A) \subseteq G$ whenever $A \subseteq G$ and G is intuitionistic fuzzy open. The complement of a GIF-closed set is GIF-open.

Definition 2.4. Let X be the non empty universe of discourse. A Pythagorean Fuzzy Set(PFS) P in X is given by $P = \{\langle x, \mu_P(x), \nu_P(x) \rangle : x \in X\}$ where the functions $\mu_P(x) \in [0,1]$ and $\nu_P(x) \in [0,1]$ denote the degree of membership and degree of non membership of each element $x \in X$ to the set P, respectively, and $0 \leq \mu_P^2(x) + \nu_P^2(x) \leq 1$ for each $x \in X$. The degree of indeterminacy $I_P = \sqrt{1 - (\mu_P^2(x) + \nu_P^2(x))}$ for each $x \in X$.

Definition 2.5. Let $P_1 = \{\langle x, \mu_{P_1}(x), \nu_{P_1}(x) \rangle : x \in X\}\}$ and $P_2 = \{\langle x, \mu_{P_2}(x), \nu_{P_2}(x) \rangle : x \in X\}\}$ be two pythagorean fuzzy sets over X. Then,

1. the pythagorean fuzzy complement of P_1 is defined by

$$P_1^{c} = \{ \langle x, \nu_{P_1}(x), \mu_{P_1}(x) \rangle : x \in X \} \}$$

2. the pythagorean fuzzy intersection of P_1 and P_2 is defined by

$$P_1 \cap P_2 = \{ \langle x, \min\{\mu_{P_1}(x), \mu_{P_2}(x)\}, \max\{\nu_{P_1}(x), \nu_{P_2}(x)\} \langle x \in X\} \},\$$

3. the pythagorean fuzzy union of P_1 and P_2 is defined by

$$P_1 \cup P_2 = \{ \langle x, max\{\mu_{P_1}(x), \mu_{P_2}(x)\}, min\{\nu_{P_1}(x), \nu_{P_2}(x)\} \rangle : x \in X \},\$$

- 4. we say P_1 is a pythagorean fuzzy subset of P_2 and we write $P_1 \subseteq P_2$ if $\mu_{P_1}(x) \leq \mu_{P_2}(x)$ and $\nu_{P_1}(x) \geq \nu_{P_2}(x)$ for each $x \in X$,
- 5. $0_X = \{ \langle x, 0, 1 \rangle \}, x \in X \}$ and $1_X = \{ \langle x, 1, 0 \rangle : x \in X \}.$

Definition 2.6. Let $(X, \tau)_P$ be an Pythagorean Fuzzy topological space and $P = \{\langle x, \mu_P(x), \nu_P(x) \rangle : x \in X\}$ be a Pythagorean fuzzy set over X. Then the Pythagorean fuzzy interior, Pythagorean fuzzy closure and Pythagorean fuzzy boundary of P are defined by; a. $int(P) = \bigcup \{G : G \text{ is a PFOS in } X \text{ and } G \subseteq P\}$

 $b.cl(P) = \bigcap \{K : K \text{ is a PFCS in } X \text{ and } P \subseteq K \}$

c. $Fr(P) = cl(P) \cap cl(P^c)$

Remark 2.7. It is clear that,
a. int(P) is the biggest Pythagorean fuzzy open set contained in P,
b. cl(P) is the smallest Pythagorean fuzzy closed set containing P.

Remark 2.8. From the definition Pythagorean fuzzy union and intersection, it is obvious that pythagorean fuzzy interor, closure and boundary is a pythagorean fuzzy set.

3 Generalized Pythagorean fuzzy closed sets

Definition 3.1. Let (X, τ) be an intuitionistic fuzzy topological space. An intuitionistic fuzzy set A in (X, τ) is said to be generalized intuitionistic fuzzy closed (GIFC) if $IFcl(A) \subseteq G$ whenever $A \subseteq G$ and G is IFO

The complement of GIFC set is GIFO set.

Definition 3.2. Let $(X, \tau)_P$ be an Pythagorean Fuzzy topological space. An Pythagorean Fuzzy set A in $(X, \tau)_P$ is said to be generalized Pythagorean fuzzy closed (shortly GPFC) if $PFcl(A) \subseteq P$ whenever $A \subseteq P$ and P is PFO.

The complement of GPFC is GPFO

Definition 3.3. Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A be a PFS in X. Then Generalized Pythagorean fuzzy closure and Generalized Pythagorean Fuzzy interior of A are defined by (1) $GPFcl(A) = \bigcap \{G : G \text{ is } GPF \text{ closed set in } X \text{ and } A \subseteq G \}$ (2) $GPFint(A) = \bigcup \{G : G \text{ is } GPF \text{ open set in } X \text{ and } A \supseteq G \}$

Proposition 3.4. Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A and B be any two Pythagorean fuzzy sets in $(X, \tau)_P$. Then the Generalized Pythagorean Fuzzy closure operator satisfy the following properties.

1. $A \subseteq GPFcl(A)$

- 2. GPFcl(GPFcl(A)) = GPFcl(A)
- 3. $A \subseteq B \Rightarrow GPFcl(A) \subseteq GPFcl(B)$
- 4. $GPFcl(A \cup B) = GPFcl(A) \cup GPFcl(B)$
- 5. $GPFcl(1_X) = 1_X; GPFcl(0_X) = 0_X.$

Proof. (i), (ii), (iii) and (v) can be esily obtained by the definition of the GPFclosure. (iv) From $GPFcl(A) \subseteq GPFcl(A \cup B)$. We obtain $GPFcl(A) \cup GPFcl(B) \subseteq GPFcl(A \cup B)$. On the other hand, from the facts $A \subseteq GPFcl(A)$ and $B \subseteq GPFcl(B) \implies A \cup B \subseteq GPFcl(A) \cup GPFcl(B)$ and $GPFcl(A) \cup GPFcl(B) \in GPFcl(A) \in GPFcl(A) \cup GPFcl(B)$. Thus, proof of the axioms (iv) is obtained from these two inequalities. **Proposition 3.5.** Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A and B be any two Pythagorean fuzzy sets in $(X, \tau)_P$. Then the Generalized Pythagorean Fuzzy interior operator satisfy the following properties.

- 1. $GPFint(A) \subseteq A$
- 2. GPFint(GPFint(A)) = GPFint(A)
- 3. $A \subseteq B \Rightarrow GPFint(A) \subseteq GPFint(B)$
- 4. $GPFint(A \cap B) = GPFint(A) \cap GPFint(B)$
- 5. $GPFint(1_X) = 1_X; GPFint(0_X)) = 0_X$

 $\it Proof.$ (i), (ii), (iii) and (v) can be easily obtained from the definiton of the Generalized Pythagorean Fuzzy interior .

(iv) From $GPFint(A \cap B) \subseteq GPFint(A)$ and $GPFint(A \cap B) \subseteq GPFint(B)$.

We obtain $GPFint(A \cap B) \subseteq GPFint(A) \cap GPFint(B)$. On the other hand, from the facts $GPFint(A) \subseteq A$ and $GPFint(B) \subseteq B \implies GPFint(A) \cap GPFint(B) \subseteq A \cap B$ and $GPFint(A) \cap GPFint(B) \in \tau_P$. We have $GPFint(A) \cap GPFint(B) \subseteq GPFint(A \cap B)$. Thus, proof of the axioms (iv) is obtained from these two inequalities.

Proposition 3.6. Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. Let A and B be any two Pythagorean fuzzy sets in $(X, \tau)_P$. Then the following properties hold.

- 1. 1 GPFcl(A) = GPFint(1 A)
- 2. 1 GPFint(A) = GPFcl(1 A)

Proposition 3.7. If A and B are GPF-closed sets, then $A \cup B$ is a GPF-closed set.

Remark 3.8. The intersection of two GPF-closed sets need not be GPF-closed set.

Proposition 3.9. Let $(X, \tau)_P$ be a Pythagorean Fuzzy topological space. If B is GPF-closed and $B \subseteq A \subseteq PFcl(B)$ then A is GPF-closed.

Proposition 3.10. In an Pythagorean fuzzy topological space $(X, \tau)_P$, $\tau_P = T_P$ (The family of all Pythagorean fuzzy closed Sets) iff every Pythagorean fuzzy closed set of $(X, \tau)_P$ is a GPF closed set.

Proof. Suppose that every Pythagorean fuzzy set A of $(X, \tau)_P$ is GPF closed. Let $A \in \tau_P$. Since $A \subseteq A$ and A is GPF-closed, $PFcl(A) \subseteq A$. But $A \subseteq PFcl(A)$. Hence, PFcl(A) = A. Thus, $A \in \tau_P$. Therefore, $\tau_P \subseteq T_P$. If $B \in T$, then $1_X - B \in \tau_P \subseteq T_P$ and hence $B \in \tau_P$. That is $T_P \subseteq \tau_P$. Therefore $\tau_P = T_P$

Conversely, Suppose that A be a Pythagorean Fuzzy set in $(X, \tau)_P$. Let B be a Pythagorean fuzzy open set in $(X, \tau)_P$ such that $A \subseteq B$. By hypothesis, B is Pythagorean fuzzy closed set. By the definition of Pythagorean fuzzy closure $PFcl(A) \subseteq B$. Therefore A is GPF-closed. \Box

Proposition 3.11. If $GPFint(A) \subseteq B \subseteq A$ and if A is GPF-open then B is also GPF-open.

Proposition 3.12. Let $(X, \tau)_P$ be a Pythagorean fuzzy topological space. A Pythagorean fuzzy set A is GPF-open iff $B \subseteq GPFint(A)$, whenever B is Pythagorean fuzzy closed and $B \subseteq A$.

Proof. Let A be a GPF-open set and B be a Pythagorean fuzzy closed set, such that $B \subseteq A$. Now, $B \subseteq A \implies 1_X - A \subseteq 1_X - B$ and $1_X - A$ is a GPF-closed set $\implies PFcl(1_X - A) \subseteq 1_X - B$. That is, $B = 1_X - (1_X - B) \subseteq 1_X - PFcl(1_X - A)$. But $1_X - PFcl(1_X - A) = PFint(A)$. Thus, $B \subseteq PFint(A)$. Conversely, suppose that A be a Pythagorean fuzzy set, such that $B \subseteq PFint(A)$ whenever B is Pythagorean fuzzy closed and $B \subseteq A$. Let $1_X - A \subseteq B$ whenever B is Pythagorean fuzzyopen. Now, $1_X - A \subseteq B \implies 1_X - B \subseteq A$. Hence by assumption, $1_X - B \subseteq PFint(A)$. That is, $1_X - PFint(A) \subseteq B$. But $1_X - PFint(A) = PFcl(1_X - A)$. Hence, $PFcl(1_X - A) \subset B$. That is, $1_X - A$ is GPF-closed. Therefore, A is GPF-open. \Box

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Biography



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