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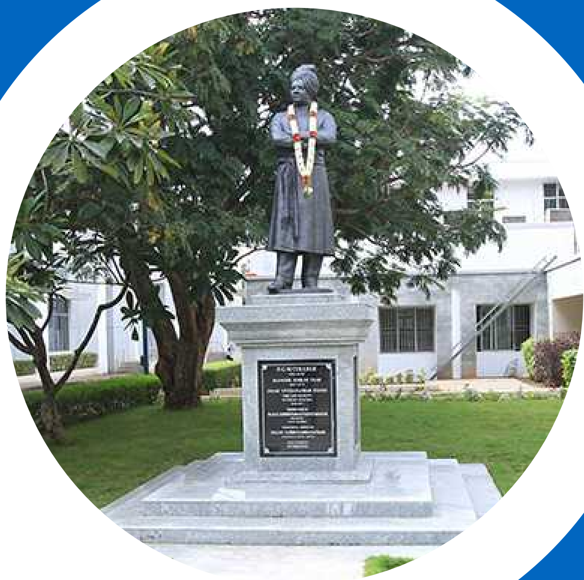

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EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

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The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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Generalized Anti Fuzzy Implicative Ideals of Near-Rings

M. Himaya Jaleela Begum[1], **P. Ayesha Parveen**[2],**J. Jayasudha** [3]

Abstract: In this paper, we introduced the concept of $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ -fuzzy implicative ideals of a near-ring and we discussed about its equivalent conditions. Some new characterizations are also given. In particular, We defined the level sets for ξ_p^ψ, ξ_p^ϕ and $[\xi]_p^\phi$

Keywords : Generalized Pythagorean closed sets, Generalized Pythagorean open sets, Generalized Pythagorean Fuzzy Closure, Generalized Pythagorean Fuzzy Interior. **2010 Subject classification:** 54A05, 54A10

1 INTRODUCTION

Fuzzy concept was first introduced by Zadeh[8]. A new type of fuzzy subgroup, that is, the $(\in, \in \vee q)$ fuzzy sub group, was introduced by Bhakat and Das[1] using the combined notions of belongingness and quasicoincidence of fuzzy points and fuzzy sets. The idea of beside to and non quasi-coincident relation was given by Saeid and Jun[5]. Kim[3] studied the notion of anti fuzzy ideals in near rings. Shabir and Rehman[6] introduced the concept of anti fuzzy left (right, lateral) ideals, anti fuzzy quasi ideals, bi ideals, anti fuzzy generalized bi-ideals in ternary semigroups. Tariq Anwar, Mammuhad Naeem, Saleem Abdullah[7] introduced generalized anti fuzzy ideals in near rings. In this paper, the concept of $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ -fuzzy implicative ideals of a near-ring is given with its equivalent conditions. We give the relationship between $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ -fuzzy implicative ideals and $(\Gamma_\psi, \Gamma_\psi)$ fuzzy implicative ideals of near rings. We bring the definition for three level sets ξ_p^ψ, ξ_p^ϕ and $[\xi]_p^\phi$.

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2 PRELIMINARIES

Throughout this paper, $\Re$ denotes Near ring.

Definition 2.1. A fuzzy set ξ in $\Re$ of the form

$$\xi(j) = \begin{cases} \upsilon \in [0,1) \text{ if } j = i, \\ 1 \qquad\quad \text{if } j \neq i, \end{cases}$$

is called an anti-fuzzy point with $'i'$ as support and υ as value and is denoted by i_υ.
A fuzzy set ξ in $\Re$ is said to be non unit if there exists $i \in \Re$ such that $\xi(i) < 1$.

Definition 2.2. An anti fuzzy point i_υ is said to beside to(correspondingly be non-quasi coincident with) a fuzzy set ξ, written as $i_\upsilon\ \Gamma\ \xi$(corresponding $i_\upsilon\ \Upsilon\ \xi$) if $i_\upsilon\ \Gamma\ \xi$ (correspondingly $\xi(i) + \upsilon < 1$). We say that Γ (correspondingly Υ) is a beside to (correspondingly non-quasi coincident with) relation between anti fuzzy points and fuzzy sets. If $i_\upsilon\ \Gamma\ \xi$ or $i_\upsilon\ \Upsilon\ \xi$,we say that $i_\upsilon\ \Gamma \vee \Upsilon\ \xi$ and $i_\upsilon\ \overline{\Gamma}\ \xi$(correspondingly $i_\upsilon\ \overline{\Upsilon}\ \xi$,$i_\upsilon\ \overline{\Gamma \vee \Upsilon}\ \xi$) means that $i_\upsilon\ \Gamma\ \xi$(correspondingly $i_\upsilon\ \Upsilon\ \xi$,$i_\upsilon\ \Gamma \vee \Upsilon\ \xi$) does not hold.

Result 2.3. *Let $\phi, \psi \in [0,1]$ be such that $\phi < \psi$.For a fuzzy point i_p and a fuzzy set ξ of $\Re$, we say that*
(1) $i_p\ \Gamma_\psi\ \xi$ if $\xi(i) \leq p\ <\ \psi$
(2) $i_p\ \Upsilon_\psi\ \xi$ if $\xi(i) + p\ <\ 2\phi$
(3) $i_p\ \Gamma_\psi \vee \Upsilon_\phi\ \xi$ if $i_p\ \Gamma_\psi\ \xi$ (or) $i_p\ \Upsilon_\phi\ \xi$

Definition 2.4. A non empty subset I of a near-ring $\Re$ is called an implicative ideals if it satisfies
1. $(I, +)$ is a normal subgroup of $(\Re, +)$,
2. $\Re I \subseteq I$,
3. $(i + k)j - ij \in I$ for any $k \in I$ and $i, j, K \in \Re$,
4. $((i(ji))k) \in I$ whenever $i \in I$ and $k \in I$ for all $i, j, k \in \Re$

3 $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ FUZZY IMPLICATIVE IDEALS OF NEAR RINGS

Definition 3.1. A fuzzy set ξ of $\Re$ is called a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$ if for all $i, j, k \in \Re$ and $p, n \in [0, \psi)$.
(I1a) $i_p\ \Gamma_\psi\ \xi$ and $j_n\ \Gamma_\psi\ \xi\ \Rightarrow (i + j)_{p \vee n}\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
(I1a*) $i_p\ \Gamma_\psi\ \xi\ \Rightarrow (-i)_p\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
(I1b) $i_p\ \Gamma_\psi\ \xi$ and $j_n\ \Gamma_\psi\ \xi\ \Rightarrow (ij)_{p \vee n}\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
(I1c) $i_p\ \Gamma_\psi\ \xi\ \Rightarrow (j + i - j)_p\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
(I1d) $j_p\ \Gamma_\psi\ \xi$ and $i \in \Re\ \Rightarrow (ij)_p\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
(I1e) $k_p\ \Gamma_\psi\ \xi\ \Rightarrow ((i + k)j - ij)_p\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
(I1f) $i_p\ \Gamma_\psi\ \xi$ and $k_n\ \Gamma_\psi\ \xi\ \Rightarrow ((i(ji))k)_{p \vee n}\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.

Example 3.2.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	a	b	c
c	a	0	c	b

Define the fuzzy set ξ of $\Re$ as $\xi(0) = \xi(b) = 0.4$,$\xi(a) = 0.3$,$\xi(c) = 0.5$ for $\phi = 0.1$,$\psi = 0.8$,$p = 0.5$,$n = 0.7$.

Theorem 3.3. *For a fuzzy set ξ in $\Re$,the following conditions are equivalent.*
a) $i_p\ \Gamma_\psi\ \xi$ *and* $k_n\ \Gamma_\psi\ \xi\ \Rightarrow ((i(ji))k)_{p\vee n}\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
b) $\xi((i(ji))k) \wedge \psi \leq \xi(i) \vee \xi(k) \vee \phi$.

Proof. $(a) \Rightarrow (b)$. Suppose there exists $i, j, k \in \Re$ and $p \in [0, \psi)$ such that $\xi((i(ji))k) \wedge \psi \leq \xi(i) \vee \xi(k) \vee \phi$.
$\Rightarrow \xi(i) \leq p < \psi$, $\xi(k) \leq n < \psi$ but $\xi((i(ji))k) > p$
and $\xi((i(ji))k) + p > 2p \geq 2\phi$ (i.e) $i_p\ \Gamma_\psi\ \xi$, $k_n\ \Gamma_\psi\ \xi$ but $((i(ji))k)_p\ \overline{\Gamma_\psi\ \vee\ \Upsilon_\phi}\ \xi$,
which is a contradiction. Therefore, $\xi((i(ji))k) \wedge \psi \leq \xi(i) \vee \xi(k) \vee \phi$.
$(b) \Rightarrow (a)$ Conversely, Suppose there exists $i, j, k \in \Re$ and $p, n \in [0, \psi)$ such that $i_p\ \Gamma_\psi\ \xi$, $k_n\ \Gamma_\psi\ \xi$ but $((i(ji))k)_{p\vee n}\ \overline{\Gamma_\psi\ \vee\ \Upsilon_\phi}\ \xi$. Then $\xi(i) \leq p$,$\xi(k) \leq n$ but $\xi((i(ji))k) > p \vee n$ and $\xi((i(ji))k) + p \vee n \geq 2\phi$ It follows that $\xi((i(ji))k) > \phi$
So,given, $\xi((i(ji))k) \wedge \psi > p \vee n \vee \phi \geq \xi(i) \vee \xi(k) \vee \phi$
(i.e)$\xi((i(ji))k) \wedge \psi > \xi(i) \vee \xi(k) \vee \phi$, which is a contradiction to our assumption.
Hence, $((i(ji))k)_{p\vee n}\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$. □

Theorem 3.4. *For a fuzzy set ξ in $\Re$,the following conditions are equivalent.*
a) $i_p\ \Gamma_\psi\ \xi$ *and* $j_n\ \Gamma_\psi\ \xi\ \Rightarrow (i + j)_{p\vee n}\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
b) $\xi(i + j) \wedge \psi \leq \xi(i) \vee \xi(j) \vee \phi$ *forall* $i, j \in \Re$ *and* $p, n \in [0, \psi)$.

Theorem 3.5. *For a fuzzy set ξ in $\Re$,the following conditions are equivalent.*
a) $i_p\ \Gamma_\psi\ \xi \Rightarrow (-i)_p\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
b) $\xi(-i) \wedge \psi \leq \xi(i) \vee \phi$ *forall* $i, j \in \Re$ *and* $p, n \in [0, \psi)$.

Theorem 3.6. *For a fuzzy set ξ in $\Re$,the following conditions are equivalent.*
a) $i_p\ \Gamma_\psi\ \xi$ *and* $j_n\ \Gamma_\psi\ \xi\ \Rightarrow (ij)_{p\vee n}\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
b) $\xi(ij) \wedge \psi \leq \xi(i) \vee \xi(j) \vee \phi$*forall* $i, j \in \Re$ *and* $p, n \in [0, \psi)$.

Theorem 3.7. *For a fuzzy set ξ in $\Re$,the following conditions are equivalent.*
a) $i_p\ \Gamma_\psi\ \xi \Rightarrow (j + i - j)_p\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
b) $\xi(j + i - j) \wedge \psi \leq \xi(i) \vee \phi$.

Theorem 3.8. *For a fuzzy set ξ in $\Re$,the following conditions are equivalent.*
a) $k_p\ \Gamma_\psi\ \xi \Rightarrow ((i + k)j - ij)_p\ \Gamma_\psi \vee\ \Upsilon_\phi\ \xi$.
b) $\xi((i + k)j - ij) \wedge \psi \leq \xi(k) \vee \phi$ *forall* $i, j \in \Re$ *and* $p, n \in [0, \psi)$.

Theorem 3.9. *Any* $(\Gamma_\psi, \Gamma_\psi \vee\ \Upsilon_\phi)$ *fuzzy implicative ideal of* $\Re$ *such that* $p, n \in [\phi, \psi)$ *for all* $i, j, k \in \Re$ *is a* $(\Gamma_\psi, \Gamma_\psi)$ *fuzzy implicative ideal of* $\Re$.

Proof. Given, ξ be a $(\Gamma_\psi, \Gamma_\psi \vee\ \Upsilon_\phi)$ fuzzy implicative ideal of $\Re$ with $p \in [\phi, \psi)$ for all $i, j, k \in \Re$.
Let $i_p\ \Gamma_\psi\ \xi$,$k_n\ \Gamma_\psi$. We have, $\xi((i(ji))k) \wedge \psi \leq \xi(i) \vee \xi(k) \vee \phi = p \vee n \vee \phi$
$\xi((i(ji))k) \leq p \vee n < \psi = p \vee n$ since $p, n \in [\phi, \psi)$
Therefore, $((i(ji))k)_{p\vee n}\ \Gamma_\psi\ \xi$. Hence, ξ is an $(\Gamma_\psi, \Gamma_\psi)$ fuzzy implicative ideal of $\Re$. Conversely, Let $\xi(i) = p$,$\xi(k) = n$ where $p, n \in [\phi, \psi)$
then $\xi(i) \leq p < \psi$,$\xi(k) \leq n < \psi \Rightarrow i_p\ \Gamma_\psi\ \xi$,$k_n\ \Gamma_\psi\ \xi$.
Since ξ is an $(\Gamma_\psi, \Gamma_\psi)$ fuzzy implicative ideal of $\Re$, $((i(ji))k)_{p\vee n}\ \Gamma_\psi\ \xi$.
Now,$\xi((i(ji))k) \wedge \psi \leq p \vee n \wedge \psi = p \vee n = p \vee n \vee \phi = \xi(i) \vee \xi(k) \vee \phi$
Therefore,$\xi((i(ji))k) \leq \xi(i) \vee \xi(k) \vee \phi$.
Hence,ξ is an $(\Gamma_\psi, \Gamma_\psi \vee\ \Upsilon_\phi)$ fuzzy implicative ideal of $\Re$. □

Theorem 3.10. *The union of any family of* $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ *fuzzy implicative ideal of* $\Re$ *is a* $(\Gamma_\psi, \Gamma_\psi)$ *fuzzy implicative ideal of* $\Re$.

Proof. Let $\{\xi_f\}_{f\in F}$ be any family of $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideal of $\Re$ and $\xi = \bigcup_{f\in F}$. Let $i, j, k \in \Re$.
Now,$\xi((i(ji))k) \wedge \psi = (\bigcup_{f\in F}((i(ji))k) \wedge \psi = \bigcup_{f\in F}(\xi_f((i(ji))k) \wedge \psi) \le \bigcup_{f\in F}(\xi_f(i) \vee \xi_f(k) \vee \phi) = (\bigcup_{f\in F}\xi_f)(i) \vee (\bigcup_{f\in F}\xi_f)(k) \vee \phi) = \xi(i) \vee \xi(k) \vee \phi$
Therefore,$\xi((i(ji))k) \wedge \psi \le \xi(i) \vee \xi(k) \vee \phi$ □

Definition 3.11. For any fuzzy set ξ in $\Re$ and $p \in [0, 1)$ we define
$\xi_p^\psi = \{i \in \Re / i_p\ \Gamma_\psi\ \xi\}$,$\xi_p^\phi = \{i \in \Re / i_p\ \Upsilon_\phi\ \xi\}$ and $[\xi]_p^\phi = \{i \in \Re / i_p\ \Gamma_\psi \vee \Upsilon_\phi\ \xi\}$
It is clear that $[\xi]_p^\phi = \xi_p^\psi \cup \xi_p^\phi$ where ξ_p^ψ, ξ_p^ϕ and $[\xi]_p^\phi$ are called Γ_ψ-level set,Υ_ϕ-level set and $\Gamma_\psi \vee \Upsilon_\phi$-level set of ξ respectively.

Theorem 3.12. *Let* ξ *be a fuzzy set in* $\Re$. *Then* ξ *is a* $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ *fuzzy implicative ideals of* $\Re$ *iff* $[\xi]_p^\phi \neq \phi$ *is an implicative ideals of* $\Re$ *for all* $p \in [0, \psi)$

Proof. Given,ξ is a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$. Let $p \in [0, \psi)$ be such that $[\xi]_p^\phi \neq \phi$. Let $i, k \in [\xi]_p^\phi$ then $i_p\ \Gamma_\psi \vee \Upsilon_\phi\ \xi$,$k_p\ \Gamma_\psi \vee \Upsilon_\phi\ \xi$.
We can consider four cases.
(i) $\xi(i) \le p$ and $\xi(k) \le p$ (ii) $\xi(i) \le p$ and $\xi(k) + p < 2\phi$
(iii) $\xi(i) + p < 2\phi$ and $\xi(k) \le p$ (iv) $\xi(i) + p < 2\phi$ and $\xi(k) + p < 2\phi$
Case (i): $\xi(i) \le p$ and $\xi(k) \le p$. For $p \in [0, \phi)$ then $2\phi - p > \phi > p$
Now,$\xi((i(ji))k) \wedge \psi \le \xi(i) \vee \xi(k) \vee \phi = p \vee p\phi = \phi < 2\phi - p$
(or) $\xi((i(ji))k) \le p \vee (2\phi - p) \vee \phi = 2\phi - p$ (or) $\xi((i(ji))k) \le (2\phi - p) \vee (2\phi - p) \vee \phi = 2\phi - p$
Therefore,$\xi((i(ji))k) < 2\phi - p$ (i.e)$\xi(i(ji))k + p < 2\phi$
Hence, $((i(ji))k)_p\ \Upsilon_\phi\ \xi$. Therefore,$((i(ji))k)_p\ \Gamma_\psi \vee \Upsilon_\phi\ \xi$
For $p \in [\phi, \psi)$ then $2\phi - p < \phi \le p$
Now, $\xi((i(ji))k) \le \xi(i) \vee \xi(k) \vee \phi = p \vee p\phi = p$ (or) $\xi((i(ji))k) \le p \vee (2\phi - p) \vee \phi = p$
(or) $\xi((i(ji))k) \le (2\phi - p) \vee (2\phi - p) \vee \phi = \phi \le p$. Therefore,$\xi((i(ji))k) \le \phi$
Hence, $((i(ji))k)_p\ \Gamma_\psi\ \xi$. Therefore,$((i(ji))k)_p\ \Gamma_\psi \vee \Upsilon_\phi\ \xi$
simiarly, we can prove the other cases. We can easily prove the converse.
Therefore, ξ is an $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$ □

4 CONCLUSION

In this research article, We talked about certain characterizations of$(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of near-rings and also we discussed about its level sets.

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