VOLUME XII
ISBN No.: 978-93-94004-01-6
Physical Science

Estd. 1957

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

Pollachi-642001

SUPPORTED BY

ABT Industries Ltd.,
Dairy Division

PROCEEDING

One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, Pollachi-642001.

Estd. 1957

Proceeding of the

One day International Conference on

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

ISBN No: 978-93-94004-01-6

978- 93- 94004- 01- 6

Nallamuthu Gounder Mahalingam College

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

www.ngmc.org

ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

EDITORIAL BOARD

International Conference on Emerging Trends in Science and Technology (ETIST 2021) Jointly Organized by Department of Biological Science, Physical Science and Computational Science Nallamuthu Gounder Mahalingam College, Affiliated to Bharathiar University, Tamilnadu, India.
Published by NGMC - November 2021 ISBN: 0000 - 0000

Horizontal trapezoidal intuitionistic fuzzy numbers in stress detection of cylindrical shells

J.Akila Padmasree[1] and R. Parvathi[2] and R.Santhi[3]

1, 2 Department of Mathematics, Vellalar College for Women, Erode - 638 012, Tamilnadu, India.

3 Department of Mathematics, Nallamuthu Gounder Mahalingam College,Pollachi - 642 001, Tamilnadu, India.

E-mail: akilaa6666@gmail.com, paarvathis@rediffmail.com E-mail: santhifuzzy@yahoo.co.in

Abstract

In this paper, an attempt is made to represent the uncertainity in fluid pressure using trapezoidal intuitionistic fuzzy numbers. Further, the horizontal membership functions are used as intuitionistic fuzzy pressure values by which the exact initiation of the stress, its distribution over the cylinder and the maximum point which it can attain so that the structure does not attain deformation can be predicted accurately. Horizontal relative trapezoidal intuitionistic fuzzy number (HRTrIFN) is applied in a few numerical problems of hydrodynamics in analysing the stress of thick and thin cylindrical shells and the result is compared with the traditional method using crisp values.

Keywords: Horizontal trapezoidal intuitionistic fuzzy number , thick and thin cylinders, fluid pressure
AMS classification(2010): 94D05, 97I80

1 Introduction

Intuitionistic fuzzy sets were introduced by Krassimir T. Atanassov [1]. The multi-dimensional relative distance measure interval arithmetic (RDM - IA) was developed in [7] which includes a multiplier $\alpha \in [0,1]$ that converts the membership and non-membership function from linear/curvilinear into planar form. The horizontal membership functions were introduced by Prof. A.Piegat in [8] which defines a fuzzy number not in the form of $\mu = f(x)$ but as $x = f(\mu)$. This form enables using fuzzy values in mathematical formulas of type $y = f(x_1, ..., x_n)$ together with crisp values. In [4], complex trapezoidal intuitionistic fuzzy numbers were introduced. A new definition of Horizontal relative trapezoidal intuitionistic fuzzy number (HRTrIFN) is defined in [5] with the geometric representation and its advantage over other intuitionistic fuzzy numbers is that it enables relatively easy aggregation of crisp and intuitionistic fuzzy values together in arithmetic operations. In [6], the stress and deflection of a composite bar is found using horizontal relative trapezoidal intuitionistic fuzzy number. The various analysis and applications of stress and strains in cylindrical shells are presented in [9].

In real life computations, uncertainity is natural because the processed values are from human measurements which cannot be really accurate. But in the mathematical modelling, these uncertainities are mostly avoided by the design and control of the experimental setup. A thin shell has the thickness very small when compared with its diameter. Thick cylinders are cylindrical shells whose wall thickness is large when compared with the diameter. Some examples of cylindrical shells are steam boilers, reservoirs, reactors, nuclear container tanks. The stress analysis of thick and thin cylinderical containers has uncertainity with both the system and the environment. In reality, the pressure cannot be defined exactly, it is based on the depth of the container, motion of the container and expressing them as a constant value may not be an accurate prediction. So, representing the internal and external pressure values as intuitionistic fuzzy number takes into consideration of all the uncertainities and provides a better result for the stress analysis in thin, thick and built-up cylinders. In this paper, the stress analysis of thin and thick cylindrical shells subjected to fluid pressure are studied. Based on the geometrical parameters of the shell and the forces present in it, the stress induced in the walls of the shells are found by using horizontal relative trapezoidal intuitionistic fuzzy number (HRTrIFN) as pressure values.

The paper is organised as pre-requisites in section 2, necessity for the usage of intuitionistic fuzzy values is given in section 3, the subsection 3.1 discusses the stressess in thin cylinders with a numerical example and is compared with the crisp values, and the stress analysis of thick cylinders is studied in subsection 3.2. The section 3.3 gives the inference and advantages of using intuitionistic fuzzy values. Conclusion is given in section 4 with the advantages of the proposed method and the scope of further research.

2 Pre-requisites

[3] A Trapezoidal intuitionistic fuzzy number(TrIFN) A is an intuitionistic fuzzy set in R with the membership and non-membership fuctions as given below:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a} & x \in [a,b] \\ 1 & x \in [b,c] \\ \frac{d-x}{d-c} & x \in [c,d] \\ 0 & \text{otherwise} \end{cases} \qquad \nu_A(x) = \begin{cases} \frac{b-x}{b-a'} & x \in [a',b] \\ 0 & x \in [b,c] \\ \frac{x-c}{d'-c} & x \in [c,d'] \\ 1 & \text{otherwise} \end{cases}$$

where $b \leq c$ and $(b-a) \geq 0$ which gives $b \geq a$. $(d-c) \geq 0$ which gives $d \geq c$. Therefore, $a \leq b \leq c \leq d$. Also, $(b-a) \leq b-a'$. Therefore, $a' \leq a$ and similarly, $d \leq d'$. Hence, $a' \leq a \leq b \leq c \leq d \leq d'$. Thus $A_{TrIFN} = [a,b,c,d;a',b,c,d']$. [5] Let the parametric form of an intuitionistic fuzzy number $A = \langle [a,b,c,d;a',b,c,d'] \rangle$ be $X = \langle [\underline{x}, \overline{x}], [\underline{\underline{x}}, \overline{\overline{x}}] \rangle$. For every r-cut of membership values and r'-cut of non-membership values, let

$\underline{x(r)} = a + (b-a)r$ $\quad\overline{x(r)} = d - (d-c)r$ $\quad\underline{\underline{x(r')}} = a' + (b-a')r'$ $\quad\overline{\overline{x(r')}} = d' - (d'-c)r'$

where $\underline{x(r)} \leq \overline{x(r)}$, $\underline{\underline{x(r')}} \leq \overline{\overline{x(r')}}$ and $r, r' \in [0,1]$. Then a *horizontal relative trapezoidal intuitionistic fuzzy number* (HRTrIFN) H of A, is defined as

$$H = \{\langle x, x(\mu, \alpha_x), x(\nu, \beta_x) \rangle : x \in X, \mu, \nu, \alpha_x, \beta_x \in [0,1]\}$$

where $x(\mu, \alpha_x)$ denotes the horizontal relative membership function of H given by $x(\mu, \alpha_x) = [a + (b-a)\mu] + [(d-a) - (d-c+b-a)\mu]\alpha_x, \quad \mu, \alpha_x \in [0,1], x \in X$ and $x(\nu, \beta_x)$ denotes the horizontal relative non-membership function of H given by $x(\nu, \beta_x) = [a' + (b-a')\nu] + [(d'-a') - (d'-c+b-a')\nu]\beta_x, \quad \nu, \beta_x \in [0,1], x \in X$ and α_x and β_x are the relative distance measure of membership and non-membership functions of H respectively. For example, consider the trapezoidal intuitionistic fuzzy number $\langle [3,4,5,6;2,4,5,7] \rangle$. The horizontal relative membership function $x(\mu, \alpha_x) = [3+\mu] + [3-2\mu]\alpha_x$ and horizontal relative non-membership function $x(\nu, \beta_x) = [2+2\nu] + [5-4\nu]\beta_x$ are represented in Fig 1.

3 Stressess in cylindrical shells

Uncertainity in fluid pressure

The pressure at any point in a static fluid depends only on the depth at the point and the density of the fluid

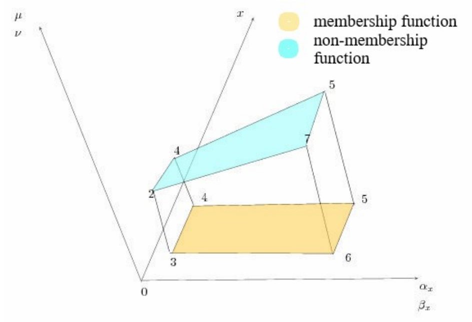

Figure 1: Horizontal membership and non-membership function

and the atmospheric pressure. Fluids located at deeper levels is subjected to more force than fluids near the surface. The atmospheric pressure changes with the height of the container and the internal and external temperature. When the fluid is in motion, the internal pressure changes with respect to the atmospheric changes and surrounding conditions. Example: moving cylindrical trucks, oil containers. The internal fluid pressure cannot be an accurate crisp prediction due to the atmospheric changes and also the external fluid pressure due to the environmental conditions. So considering these factors, the internal and external fluid pressure values are taken as a range of intuitionistic fuzzy values so the accurate prediction of stress along the cylindrical shells is possible. The degree of possibility that the parameter takes the value, degree of non-possibility that the parameter takes the value is predicted more accurately than the traditional method with single crisp value. It reflects the lack of information in a more clear way. When the thin cylindrical shells are subjected to internal pressure, two principal stresses called Hoop stress or circumferential stress, a stress in the tangential direction and longitudinal stress or axial stress, a stress parallel to the axis of cylindrical symmetry is developed in the material. Both the stresses are mutually perpendicular to each other and tensile in nature when there is internal pressure. Both these stresses will be compressive when the pressure is external.

3.1 Stresses in thin cylinders

Let the internal diameter of the cylindrical shell be 'd' its length be 'L' and the thickness of the wall be 't'. Consider an elementary portion 'ds' of which subtending an angle $'d\theta'$ radian with the centre at an angle $'\theta'$ from XX axis. Circumferential width of an elementary portion $ds = \frac{d}{2}.d\theta$. Surface area of this elementary portion $= \frac{d}{2}.d\theta$. Normal pressure on this surface area $= P.L.\frac{d}{2}.d\theta$. Vertical component of this pressure $= P.L.\frac{d}{2}.d\theta$. Therefore, total vertical upward force on the upper half of the cylinder is $V = \int_{\pi}^{0} \frac{p.d.L}{2} sin\theta d\theta$. $V = pdL$. Similarly, total vertical downward force on the lower part of the cylinder is $V = pdL$. These forces tend to burst the cylinder into halves circumferentially along XX axis. Area of cross-section resisting this bursting force is $2Lt$. Therefore stress developed in this area $= \frac{pd}{2t}$ and is called hoop stress or circumferential stress denoted by σ_c. Similarly longitudinal stress is $\sigma_l = \frac{pd}{4t}$. Any part of the cylinder is subjected to these two mutually perpendicular stressses.

Numerical example 1

A built-up cylindrical shell of 2 metre diameter and $20mm$ thickness is subjected to an internal pressure of $2.4MPa$. The efficiencies of circumferential and longitudinal joints are 70 percent and 80 percent respectively. Determine the hoop and axial stresses developed in the material of the cylinder.

Crisp method

Internal pressure $p = 2.4MPa = 2.4N/mm^2$, diameter of the cylinder $d = 2m = 2000mm$, thickness of the metal $t = 20mm$, efficiency of circumferential joint= 0.7, efficiency of longitudinal joint= 0.8. Circumferential stress or hoop stress $\sigma_c = \frac{pd}{2t\eta_l} = \frac{2.4\times2000}{2\times20\times0.8} = 150MPa$. Longitudinal or axial stress $\sigma_l = \frac{pd}{4t\eta_c} = \frac{2.4\times2000}{4\times20\times0.7} = 85.71MPa$.

Intuitionistic fuzzy values

Here we change the internal pressure value to intuitionistic fuzzy value as $p = \langle[2.1, 2.2, 2.4, 2.5; 2, 2.2, 2.4, 2.6]\rangle\, N/mm^2$, diameter of the cylinder $d = 2m = 2000mm$, thickness of the metal $t = 20mm$, efficiency of circumferential joint= 0.7, efficiency of longitudinal joint= 0.8.

Circumferential stress or hoop stress $\sigma_c = \frac{pd}{2t\eta_l} = \langle[150, 157, 171, 178; 142, 157, 171, 185]\rangle\, MPa$.

Longitudinal or axial stress $\sigma_l = \frac{pd}{4t\eta_c} = \langle[65.62, 68.75, 75, 78.12; 62.5, 68.75, 75, 81.25]\rangle\, MPa$.

3.2 Stresses in thick cylinders

Consider an annular thick cylinder of radius x and radial thickness dx subjected to internal pressure P_1 and external pressure P_2. The internal radii and external radii are r_1 and r_2 respectively. On any small element of this ring, p_x will be the radial stress and f_x will be hoop stress. Bursting force in vertical direction $= -2l(p_x dx + x dp_x)$ and resisting force $= 2 \times dx_x$. Equating the two gives $f_x + p_x + x\frac{dp_x}{dx} = 0$. Considering the assumption that their longitudinal strain is constant, $\frac{\sigma_l}{E} - v\frac{f_x}{E} + v\frac{p_x}{E} =$ constant. And the radial stress of the thick cylindrical shell is given by $p_x = \frac{b}{x^2} - a$. Since $f_x = p_x + 2a$, the hoop stress or circumferential stress of the thick cylindrical shell is given by $f_x = \frac{b}{x^2} + a$. These expressions for radial stress and hoop stress are called Lames expressions.

Numerical example 2

The internal and external diameters of a thick hollow cylinder are $80mm$ and $120mm$ respectively. It is subjected to an external pressure of $40N/mm^2$ and an internal pressure of $120N/mm^2$. Calculate the circumferential stress at the external and internal surfaces and determine the radial and circumferential stresses at the mean radius.

Crisp values

The Lame's expression for radial stress is $p_x = \frac{b}{x^2} - a$. At $x = 40$, $p_x = 120N/mm^2$ and at $x = 60$, $p_x = 40N/mm^2$. Substituting these values in (1), and solving for a, b gives $a = 24$ and $b = 230400$. Circumferential stress is given by $f_x = \frac{b}{x^2} + a$. At $x = 40$, $f_x = \frac{230400}{40^2} + 24 = 168N/mm^2$. At $x = 60$, $f_x = \frac{230400}{60^2} + 24 = 88N/mm^2$. At the mean radius, $\frac{40+60}{2} = 50mm$, the radial stress $= 68.16N/mm^2$ and circumferential stress $= 116.16N/mm^2$.

Intuitionistic fuzzy values

The external and internal pressure are taken as intuitionistic fuzzy values as below.

External pressure $= \langle [38, 39, 40, 41; 37, 39, 40, 42] \rangle$.

Internal pressure $= \langle [118, 119, 120, 121; 117, 119, 120, 122] \rangle$.

Rewriting the Lame's expression with intuitionistic fuzzy pressure values and applying horizontal functions gives the horizontal relative trapezoidal linear system of equations,

$b - 1600a{=}1888 + 16\mu + [48 - 32\mu]\alpha_x$

$b - 3600a{=}1368 + 36\mu + [108 - 22\mu]\alpha_x$

solving these equations give $a = 26 - \mu - (3 - 0.5\mu)\alpha_x$ and $b = 230400 + 5100\mu\alpha_x$.

Circumferential stress is given by $f_x = \frac{b}{x^2} + a$. At $x = 40$, $f_x = \frac{230400+5100\mu\alpha_x}{40^2} + 26 - \mu - (3 - 0.5\mu)\alpha_x$. At $x = 60$, $f_x = \frac{230400+5100\mu\alpha_x}{60^2} + 26 - \mu - (3 - 0.5\mu)\alpha_x$. At the mean radius, $\frac{40+60}{2} = 50mm$, the radial stress $= p_x = \frac{230400+5100\mu\alpha_x}{50^2} - 26 - \mu - (3 - 0.5\mu)\alpha_x N/mm^2$ and circumferential stress $= 230400 + 5100\mu\alpha_x 50^2 + 26 - \mu - (3 - 0.5\mu)\alpha_x N/mm^2$. Giving values for α and μ, values of stress at different points in the cylindrical shell can be found.

3.3 Inference

Since the internal and external pressure is subjected to various changes, the distribution of stress over the volume of the cylindrical shells cannot be a certain single crisp value. So, the intuitionistic fuzzy valued output which is distributed over an interval clearly reflects the uncertainity of the origin of stress, its distribution and variation and the maximum value it can attain inside the specific cylindrical shell respective to its shape, size and the environmental conditions which is not the case in the traditional method of calculating stress with a single crisp output value. The maximum stress point which it can attain so that the structure does not attain deformation can be predicted accurately. With the varying values of α and μ, the stress at any point in the shell can be calculated with the same single expression resulting in a multi-dimensional solution with easy computation and time efficiency.

4 Conclusion

In this paper, the horizontal relative trapezoidal numbers are used in the stress analysis of cylindrical shells. The main perspective of the paper is to apply these concepts in relatively simple numerical problems that interprets the idea of uncertainity easily and more accurately. Further scope of these concepts when applied to engineering problems reduces computational complexity and provides more insight on the lack of information in real life models.

References

[1] Atanassov, K. T. *Intuitionistic Fuzzy Sets: Theory and Applications*, Physica-Verlag, Heidelberg, 1999.

[2] Burillo.P, Bustince.H and Mohedano.V, Some definition of intuitionistic fuzzy number, Fuzzy Based Expert Systems, Fuzzy Bulgarian Enthusiasts, Sofia, Bulgaria, 1994, 28 - 30.

[3] Parvathi, R., Malathi, C. (2012). Arithmetic operations on symmetric trapezoidal intuitionistic fuzzy numbers, International Journal of Soft Computing and Engineering, 2(2), 268273.

[4] Parvathi. R., Akila padmasree. J , Complex trapezoidal intuitionistic fuzzy numbers, *Notes on intuitionistic fuzzy sets*, **24** (2018) No.4 (50-62), DOI:10.7546/nifs.2018.24.4.50-62

[5] Akila Padmasree J, Parvathi R. (2022) A novel approach of complex intuitionistic fuzzy linear systems in an electrical circuit, *In: Kahraman C., Cebi S., Cevik Onar., Oztaysi B., Tolga A.C., Sari I.U.(eds) Intelligent and Fuzzy Techniques for Emerging Conditions and Digital Transformation. INFUS 2021. Lecture Notes in Networks and Systems Series, vol. 308. Springer, Cham. https://doi.org/10.1007/978-3-030-85577-2*$_1$3

[6] Akila Padmasree.J, Parvathi.R, An innovative approach of intuitionistic fuzzy numbers in calculation of stress and deflection of a composite bar, *Turkish journal of computer and mathematics education,* ***Volume 12*** *No. 14 (2021) (2916-2927)*

[7] Piegat A.,Landowski M., (2017c), On fuzzy RDM-arithmetic, *Hard and soft computing for artificial intelligence,multimedia and security,Advances in intelligent systems and computing*, **534**(3-16), Springer International Publishing.

[8] Piegat A.,Landowski M., (2015), Horizontal membership functions and examples of its applications, *International Journal of Fuzzy Systems*, **17**(1) (22-30).

[9] Allan F.Bower., *Applied Mechanics of solids*, CRC press, Taylor and Francis group, http://www.taylorandfrancis.com