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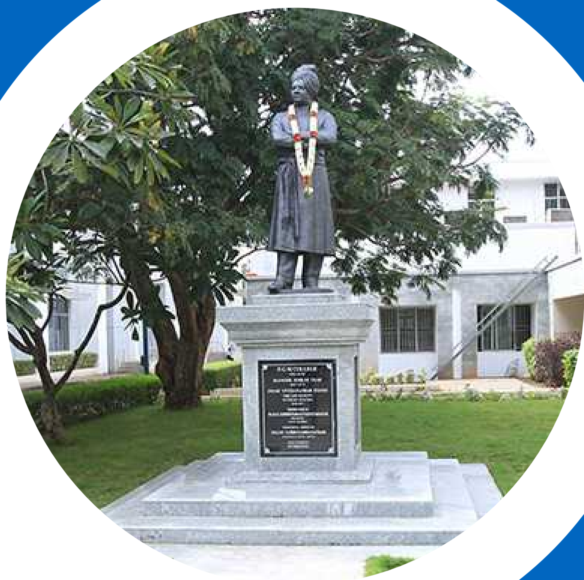

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One day International Conference
EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)
27th October 2021
Jointly Organized by
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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

PROPERTIES OF FUZZY BETA RARELY CONTINUOUS FUNCTIONS

[1]Dr.M.Saraswathi, [2]Dr. J. Jayasudha

Abstract: In this paper we introduce the concepts of fuzzy rarely β-continuous functions, fuzzy rarely slightly β-continuous functions and fuzzy rarely weakly β-continuous functions. Some interesting properties are investigated besides giving some examples.
Key Words: Fuzzy rare set, fuzzy rarely β-continuous, fuzzy rarely slightly β-continuous, fuzzy rarely weakly β-continuous functions.

1.Introduction

The study of fuzzy sets was introduced by Zadeh [8] in 1965. The idea was welcomed because it addresses the uncertainity, something classical cantor set theory could not address. Fuzzy set theory provides a natural way to deal with inaccuracy and a strict mathematical frame work for the study of uncertain phenomena and concepts. The concept of fuzzy topological space was introduced by C.L.Chang [4] in 1968. Continuity is one of the most important and fundamental properties that have been widely used in Mathematical Analysis. Jafari [6] introduced the notion of rare continuity as a generalisation of weak continuity. The concept of β-open sets was introduced in [1] and studied also by Allam and El Hakeim [2]. In [3] this concept has been generalized to fuzzy setting.

The purpose of this paper is to introduce the concepts of fuzzy rarely β-continuous functions, fuzzy rarely slightly β-continuity and fuzzy rarely weakly β-continuity. Relationship between these continuous functions are investigated besides giving some examples.

2. Preliminaries

Definition 2.1. A fuzzy topology on a set X is a collection δ of fuzzy set in X satisfying

i) $0 \in \delta$ and $1 \in \delta$

ii) μ and ρ belong to δ then so does $\mu \cap \rho \in \delta$

iii) if $\mu_i \in \delta$ for each $i \in I$ then $\bigcup_{i \in I} \mu i \in \delta$

δ is a fuzzy topology on X and the pair (X,δ) is called a fuzzy topological space. Every member of δ is called fuzzy open set. A fuzzy set is closed if and only if its complement is fuzzy open.

Definition 2.2. Let λ be any set in fuzzy topological space (X,δ). We define the closure of λ and the interior of λ as,

$cl\,\lambda = \bigcap\{\mu/\mu \geq \lambda, \mu \text{ is fuzzy closed}\}$

$Int\,\lambda = \bigcup\{ \sigma / \sigma \leq \lambda, \sigma \text{ is fuzzy open}\}$

Definition 2.3. A fuzzy set λ in fuzzy topological space (X,δ) is said to be fuzzy β-open if $\lambda \leq cl\,(int\,(cl(\lambda)))$. The complement of a fuzzy β-open sets is said to be fuzzy β-closed.

The fuzzy β-closure and fuzzy β-interior are defined as follows.

$\beta\text{-}cl\,(\lambda) = \bigcap\,\{\eta:\lambda \leq \rho, \rho \text{ is } \beta\text{-closed}\}$

$\beta\text{-}int\,(\lambda) = \bigcup\,\{\eta:\lambda \geq \rho, \rho \text{ is } \beta\text{-open}\}$

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Definition 2.4. A fuzzy set in X is called fuzzy singleton if and only if it takes the value 0 for all y ∈ X except one, say, x ∈X. If its value at x is ∈(0<∈≤1), We denote this fuzzy singleton by x_ε,where the point x is called its support.

Definition 2.5. A fuzzy set λ in a fuzzy topological space (X,δ) is said to be

i) fuzzy β-open if λ≤ cl (int (cl(λ)))
ii) fuzzy pre-open if λ≤int (cl(λ))
iii) fuzzy semi-open if λ≤cl (int(λ))
iv) fuzzy regular open if λ=int (cl(λ))

3. Main results

Definition 3.1. A fuzzy set R is called **fuzzy rare set** if int(R)=φ

Definition 3.2. A fuzzy set R is called **fuzzy nowhere dense** set if int(cl(R))= φ

Definition 3.3. Let (X,G) and (Y,H) be two fuzzy topological spaces. A function f: (X,G)→(Y,H) is called

i) **fuzzy β-continuous** if for each fuzzy point x∈X and each open set V containing f(x) there exists a β-open set U∈SPO(x) containing x such that f(U)≤V.
ii) **fuzzy slightly β-continuous** if for each point x∈X and each clopen set V containing f(x) there exists a β-open set U of x containing x such that f(U)≤V.
iii) **fuzzy weakly β-continuous** if for each fuzzy point x∈X and each open set V containing f(x) there exists U∈SPO(x) containing x such that f(U) ≤cl(V).

Definition 3.4. Let (X,G) and (Y,H) be two fuzzy topological spaces. A function f: (X,G)→(Y,H) is called

i) **fuzzy rarely β-continuous** if for each fuzzy point x∈X and each fuzzy open set V in (Y,H) containing f(x) there exists a fuzzy rare set W with V∩Int(cl(W))=φ a fuzzy β-open set U in (X,T) such that f(U)≤V∩W.
ii) **fuzzy rarely slightly β-continuous** if each fuzzy point x∈X and each clopen set V containing f(x) there exists a fuzzy rare β-open set W with V∩cl(int(W))=φ containing x such that f(U)≤V.
iii) **fuzzy rarely weakly β-continuous** if for each fuzzy point x∈X and each open set V containing f(x) there exists a fuzzy rare set W with V∩cl(int(W))=φ such that f(U)≤V.

Example 3.1. Let X={a,b,c}. Define the fuzzy sets A,B and C as follows:

A={x,($\frac{a}{1},\frac{b}{0},\frac{c}{0}$)}, B={x, ($\frac{a}{0},\frac{b}{1},\frac{c}{0}$) } and C={x, ($\frac{a}{0},\frac{b}{0},\frac{c}{1}$) }

Then T={ φ, I_x ,A} and S={ φ, I_x,B} are fuzzy topologies on X.

Define f: (X,T)→(X,S) as a identity function. Then f is a rarely β-continuous function.

Proposition 3.1. Let (X,G) and (Y,H) be any two fuzzy topological spaces. For a function f: (X,G)→(Y,H) the following statements are equivalent:

i) The function f is fuzzy rarely β-continuous at x_i in (X,G).
ii) For each fuzzy point x_i∈X, f is fuzzy rarely slightly β-continuous.
iii) For each open set V containing f(x) and for each fuzzy rare set W, f is fuzzy rarely weakly β-continuous.

Proof: (i)⇒(ii) Let K be the fuzzy open set in (Y,H) containing $f(x_i) \in X \leq$ cl (int(K)), then there exists a fuzzy rare set L with cl(Int(K)) ∪ cl(R)=φ and a fuzzy β-clopen set U in (X,G) containing x_i such that f(U) ≤cl(int(K)) ∪ L. We have cl(f(U)∩K)=cl(f(U))∩cl(int(G))=φ

(ii)⇒(iii) Let K be a fuzzy open set in (Y,H) containing $f(x_i)$. Then there exists a fuzzy β-open set L containing x_i such that cl(f(K)≤int(K). We have let U be a fuzzyβ-clopen set in (X,G) containing $f(x_i)$ and for each fuzzy rare set W such that cl(K)∩W=φ. Then cl(int(K)∩W=φ and f(U) ≤ K then f is weakly β-continuous.

(iii)⇒(i) Assume that K be a fuzzy open set in (Y,H) containing $f(x_i)$. Then there exists a fuzzy rare set W with Int(K)∩W=φ such that $x_i \in cl(f^{-1}(int(K))\cap W)$. Let $V=cl(f^{-1}(int(K))\cap W$. Hence K is a fuzzy β-open set in (Y,H). Therefore f(K) ≤ cl(int(cl(K)))∩W. Hence we have cl(f(K)∩W)=φ ⇒ f(K)≤V∩W and f is fuzzy rarely β-continuous.

Proposition 3.2. Let (X,G) and (Y,H) be any two fuzzy topological spaces. Then the function f:(X,G)→(Y,H) is fuzzy rarely β-continuous if and only if $f^{-1}(K) \leq int_\beta(f^{-1}(K\cap L))$ where K is fuzzy open set and L is a fuzzy rare set.

Proof: Suppose that K be a fuzzy rarely β open set in (Y,H) containing $f(x_i)$. Then K∩int(cl(L))= φ and V be a fuzzy β open set in (Y,H) containing x_i such that f(V)≤K∩L. Hence $x_i \in V \leq f^{-1}(K\cap L) \Rightarrow f^{-1}(K) \leq int_\beta\, f^{-1}(K\cap L)$.

Definition 3.5. Let (X,G) and (Y,H) be fuzzy topological spaces and f:X → Y be a map. Then the map f is said to be **fuzzy stronglyβ*-continuous** if for each fuzzy semi open set λ in Y, $f^{-1}(\lambda)$ is fuzzy β-open set in X.

Example 3.2. Let X={a,b},Y={c,d}. Let μ and λ be fuzzy sets in X and Y defined by μ(a)=0.1, μ(b)=0.2, μ(c)=0.4 and μ(d)=0.5. Let δ_1={0, μ,1} and δ_2={0, λ,1} be the fuzzy topologies on sets X and Y respectively. The map f:X → Y defined as $f(a_i)=b_i$, i=1,2 is fuzzy strongly β-continuous.

Proposition 3.3. Let (X,G) and (Y,H) be fuzzy topological spaces and f:X → Y be a map. Then f is fuzzy strongly β^*-continuous iff for each fuzzy set λ in X, s(int(f(λ)) ≤f(β-int(λ))

Proof: Let f:(X,G) → (Y,H) be a bijective map. Suppose f is fuzzy strongly β^*-continuous. If λ is a fuzzy set in X then f(λ) is a fuzzy set in Y. Since f is fuzzy strongly β^*-continuous, we have f^{-1}(s-cl(f(λ)) ≤β-cl(f^{-1}(f(λ))). Since f is bijective β-cl(f^{-1}(f(λ)))=β-cl(λ). Since f is onto, we have s-cl(f(λ)) ≤f(β-cl(λ).

Conversely, let μ be a fuzzy semi-open set in Y. Then s-cl(μ)=μ. Also f^{-1}(μ) is a fuzzy set in X. Further since f is one-one, α-cl(f^{-1}(μ)) ≤f^{-1}(μ). Thus f^{-1}(μ) is a fuzzy β-open set in X and the map is fuzzy strongly β^*-continuous.

References

1. Abd. El.Monseb, S.N.El-Deeb and R.A.Mahmould, β-open sets and β-continuous mapping. Bull.Fac.Sci.Assiut Univ.(1982).
2. A.A.Allam and Abd.El.Hakkim,On β-compact spaces, Bull.Calcutta Math. Soc.81(1989) 179-182.
3. G.Balasubramanian. On fuzzy β-compact spaces and fuzzy β-extremely disconnected spaces. Kybernetika 33(1997), 271-277.
4. C.L.Chang, Fuzzy topological spaces, J.Math.Anal.Appl.24(1968),182.190.
5. ErdalEkici, On the forms of continuity for fuzzy functions.
6. S. Jafari, Rarely α-continuity, Bulietin of the Malaysian mathematical science society, 2(28)2,(2005),157-161.
7. Levine.N.,Decomposition of continuity in topological spaces, Amer. Math.Monthly(60),(1961) 44-46.

I.A.Zadeh, Fuzzy sets, Information and control, Vol.8, no.3,pp338-353.1965