VOLUME XII
ISBN No.: 978-93-94004-01-6
Physical Science

Estd. 1957

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An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

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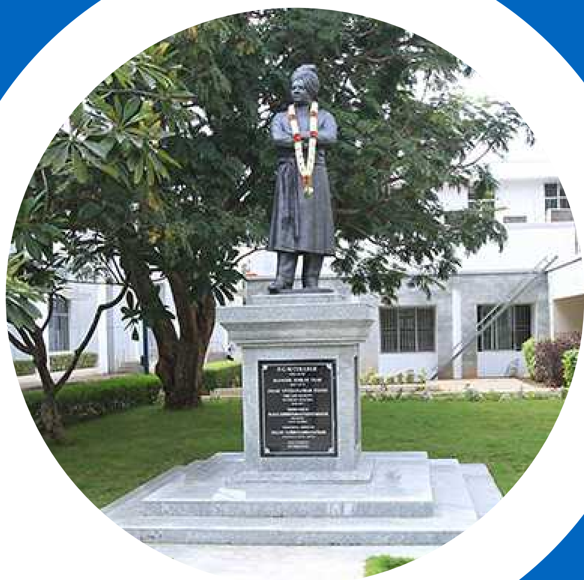

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EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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நன்றின்பால் உய்ப்பது அறிவு

Nallamuthu Gounder Mahalingam College, Pollachi- 1

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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COMPUTATIONAL APPROACH FOR TRANSIENT BEHAVIOUR OF M/ M (a, b) /1 BULK SERVICE QUEUEING SYSTEM WITH STARTING FAILURE

Shanthi[1] – Muthu Ganapathi Subramanian[2] – Gopal Sekar[3]

ABSTRACT: In this paper, the transient behaviour of single server bulk service queueing system with starting failure model has been considered. Arrival rate follows a Poisson process with parameter λ. Service will be given by batch and it follows an exponential distribution with parameter μ. Returning from failure state to service state follows an exponential distribution with parameter α. An infinitesimal generator matrix is formed for all transitions. Time dependent solutions and steady state solutions are acquired by using Eigen values and Eigen vectors. Numerical studies have been done for time dependent average number of customers in the queue, transient probabilities of server idle, busy and server is in failure for several values of t, λ, μ, α, a and b.

KEYWORDS: Bulk Service, Infinitesimal Matrix, Eigen values, Eigen vectors, Starting Failure, Exponential of a Matrix.

1. INTRODUCTION

The main objective of this research paper is to analyse the transient behaviour of bulk service queueing system by new computational approach. Queues with batch arrivals or batch services or both in batches are called bulk queues. The size of a batch may be fixed or a random variable. There are queueing situations in which arrival is single but service is in batch. Bulk service queues have potential applications in many areas e.g. In traffic signal systems, in computer networks where jobs are processed in batches, in restaurants, cinema halls, in transportation processes involving buses, airplanes, trains, ships, elevators, and so on.

In the study of queueing systems, determination of transient solution is very much essential to analyze the behavior of the system. Transient analysis is very useful for all queueing models to obtain optimal solutions which pave way to control the system. Even in the case of a simple M/M/1 queue, analytical approach to obtain transient behaviour is very difficult. Neuts (1967) explained about general class of bulk queues with Poisson input [7]. Neuts (1981) discussed about Matrix Geometric Solutions in Stochastic Models [8].

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Ammar, (2017) has studied transient solution of an M / M /1 vacation queue with a waiting server and impatient customers [1]. Krishna Kumar, Pavai Madheswari and Vijayakumar, (2002) have discussed feedback and starting failures in retrial queue [5]. Jinting Wang, Qing Zhao, (2007) have analysed retrial queue with general retrial times and starting failures [4]. Varalakshmi, Rajadurai, Saravanarajan and Chandrasekaran,(2016) have studied two phases of service, immediate Bernoulli feedbacks, single vacation and starting failures in retrial queue with general distribution [10].

Madhu Jain and Seema Agarwal, (2010) have explained retrial queue with starting failures and optional service [6]. Arivudainambi and Gowsalya, (2017) have Analysed retrial queue with Bernoulli vacation, two types of service and starting failure [2]. Rama Devi, Aankammarao and chandan (2019) have discussed Two-Phase, N policy, Server Failure and Second Optional Batch Service with Customers impatient behaviour [9]. Jinting wang, Peng-Fengzhou (2010) have studied batch arrival retrial queue with starting failures, feedback and admission control [3].

2. THE MATHEMATICAL MODEL AND ITS SOLUTIONS

A new computational method is used to estimate the Transient behaviour of Single server Bulk service queueing system with starting failure. We assume that the failure occur when the server starts the service. The server goes to the state of failure. After repair the server goes to the service state and starts the service immediately. According to the general bulk service rule pioneered by Neuts (1967), the server begins service only when a minimum of ‘a’ customers in the waiting room and a maximum service capacity is ‘b’.

The general considerations for Bulk Service with Starting Failure:

- After completion of the service if the number of customers in the queue is less than ‘a’ then the server remains idle and start the service only if the batch size reaches ‘a’.
- After completion of the service if the number of customers in the queue lies between ‘a’ and ‘b’ then all the customers in the queue will be taken for service and queue becomes empty. If the server starts service successfully then the customer gets service immediately and leaves the system. But the service not started successfully then the server goes to state of failure and after repair the server start the service for the batch of customers.
- After completion of the service if there are more than ‘b’ customers are waiting in the queue then the first ‘b’ customers are taken for service and the surviving customers will have to wait for service. If the server starts service successfully then the customer gets service immediately and leaves the system. But the service not started successfully then the server goes to state of failure and after repair the server start the service.

3. DESCRIPTION OF RANDOM PROCESS

Let N(t) be the random variable which represents the number of customers in the queue at time t and C(t) be the random variable which represents the server status at time t. The random process is described as

{ < N(t) ,C(t) > / {N(t) = 0,1,2,3,…,a-1 ; C(t) = 0} U { < N(t) ,C(t) >N(t) = 0,1,2,3,… ; C(t) = 1} U { < N(t) ,C(t) > N(t) = a,a+1,… ; C(t) = 2}

C (t) = 0 if the server is idle at time t

C (t) = 1 if the server is busy at time t

C (t) = 2 if the server is in failure state at time t

We define,

$P_{n0}(t)$: Probability that there are n customers in the queue when the server is idle at time t

$P_{n1}(t)$: Probability that there are n customers in the queue when the server is busy at time t

$P_{n2}(t)$: Probability that the server is in failure state when there are n customers in the queue at time t

The Chapman-Kolmogorov equations are

The server is idle

$$P'_{00}(t) = -\lambda P_{00}(t)+\mu_1 P_{01}(t) \tag{1.1}$$

$$P'_{n0}(t) = -\lambda P_{n0}(t)+\lambda P_{n-10}(t)+\mu P_{n1}(t) \quad \text{for} \quad n = 1,2,...,(a-1) \tag{1.2}$$

The server is busy

$$P'_{01}(t) = -(\lambda+\mu)P_{01}(t)+\mu\sum_{k=a}^{b} P_{k1}(t)+\alpha\sum_{k=a}^{b} P_{k2}(t) \tag{1.3}$$

$$P'_{n1}(t) = -(\lambda+\mu)P_{n1}(t)+\lambda P_{n-11}(t)+\mu P_{n+b1}(t)+\alpha P_{n+b2}(t) \text{ for } n = 1,2,3,... \tag{1.4}$$

The server is in failure

$$P'_{a2}(t) = -(\alpha+\lambda)P_{a2}(t)+\lambda P_{a-10}(t) \tag{1.5}$$

$$P'_{n2}(t) = -(\alpha+\lambda)P_{n2}(t)+\lambda P_{n-12}(t) \text{ for } n = a+1,a+2,... \tag{1.6}$$

The infinitesimal generator matrix **Q** for this model is given below

$$Q = \begin{pmatrix} A_{00} & A_{01} & A_{02} & A_{03} & A_{04} & ..... \\ A_{10} & A_{11} & A_{12} & A_{13} & A_{14} & ..... \\ A_{20} & A_{21} & A_{22} & A_{23} & A_{24} & ..... \\ A_{30} & A_{31} & A_{32} & A_{33} & A_{34} & ..... \\ . & . & . & . & . & ..... \\ . & . & . & . & . & ..... \end{pmatrix}$$

The matrices $A_{00,}A_{01}, A_{10}, A_{11}, A_{20}, A_{02},...$ are described in the Infinitesimal generator matrix **Q** can be obtained from the following infinitesimal transition rates of process **X** as follows

$$q_{(0,j)(l,m)}=\begin{cases} -\lambda & if \quad (l,m)=(i,j) \quad for \;\; j=0 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=0 \\ -(\lambda+\mu) & if \quad (l,m)=(i,j) \quad for \;\; j=1 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=1 \\ \mu & if \quad (l,m)=(i,j-1) \quad for \;\; j=1 \\ 0 & otherwise \end{cases}$$

$$q_{(i,j)(l,m)}=\begin{cases} -\lambda & if \quad (l,m)=(i,j) \quad for \;\; j=0 \text{ and } i=1,2,...,a-2 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=0 \text{ and } i=1,2,...,a-2 \\ -(\lambda+\mu) & if \quad (l,m)=(i,j) \quad for \;\; j=1 \text{ and } i=1,2,...,a-2 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=1 \text{ and } i=1,2,...,a-2 \\ \mu & if \quad (l,m)=(i,j-1) \quad for \;\; j=1 \text{ and } i=1,2,...,a-2 \\ 0 & otherwise \end{cases}$$

$$q_{(i,j)(l,m)}=\begin{cases} -\lambda & if \quad (l,m)=(i,j) \quad for \;\; j=0 \text{ and } i=a-1 \\ \lambda & if \quad (l,m)=(i+1,j+2) \quad for \;\; j=0 \text{ and } i=a-1 \\ -(\lambda+\mu) & if \quad (l,m)=(i,j) \quad for \;\; j=1 \text{ and } i=a-1 \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=1 \text{ and } i=a-1 \\ \mu & if \quad (l,m)=(i,j-1) \quad for \;\; j=1 \text{ and } i=a-1 \\ 0 & otherwise \end{cases}$$

$$q_{(i,j)(l,m)}=\begin{cases} -(\lambda+\mu) & if \quad (l,m)=(i,j) \quad for \;\; j=1 \text{ and } i=atob \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=1 \text{ and } i=atob \\ \mu & if \quad (l,m)=(0,1) \quad for \;\; j=1 \text{ and } i=atob \\ -(\alpha+\lambda) & if \quad (l,m)=(i,j) \quad for \;\; j=2 \text{ and } i=atob \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=2 \text{ and } i=atob \\ \alpha & if \quad (l,m)=(0,1) \quad for \;\; j=2 \text{ and } i=atob \\ 0 & otherwise \end{cases}$$

$$q_{(i,j)(l,m)}=\begin{cases} -(\lambda+\mu) & if \quad (l,m)=(i,j) \quad for \;\; j=1 \text{ and } i=b+1,b+2,... \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=1 \text{ and } i=b+1,b+2,... \\ \mu & if \quad (l,m)=(i-b,j) \quad for \;\; j=1 \text{ and } i=b+1,b+2,... \\ -(\alpha+\lambda) & if \quad (l,m)=(i,j) \quad for \;\; j=2 \text{ and } i=b+1,b+2,... \\ \lambda & if \quad (l,m)=(i+1,j) \quad for \;\; j=2 \text{ and } i=b+1,b+2,... \\ \alpha & if \quad (l,m)=(i-b,j-1) \quad for \;\; j=2 \text{ and } i=b+1,b+2,... \\ 0 & otherwise \end{cases}$$

Remaining all other entries are zero.

Further, we can write the above equations (1.1), (1.2), (1.3), (1.4), (1.5) and (1.6) as

$X'(t) = AX(t)$ where $A = Q^T$

Where $[X(t)]^T = [\ P_{00}(t) \quad P_{01}(t) \quad P_{10}(t) \quad P_{10}(t) \quad , \ . \ . \ . \ . \ , \quad P_{a\text{-}10}(t) \quad P_{a\text{-}11}(t) \quad P_{a1}(t) \quad P_{a2}(t) \quad . \ . \ . \ .]$

Solving the above set of equation we get,

$$X(t)=e^{tA}\ X_0$$

When t = 0, $X_0 = X(0) = [1 \quad 0 \quad 0 \quad . \ . \ .]^T$

4. DESCRIPTION OF COMPUTATIONAL METHOD

The following effective computational procedure is used to find the Time dependent probabilities of number of customers in the queue at time t. The time dependent probabilities vector is denoted by

$$X(t) = [P_{00}(t), P_{01}(t), P_{10}(t), P_{11}(t), ..., P_{a-10}(t), P_{a-11}(t), P_{a1}(t), P_{a2}(t), P_{a+11}(t), P_{a+12}(t), ..., P_{M1}(t), P_{M2}(t)]^T$$

Step 1: Assume that the matrix Q is finite that is the number of customers in the queue at time t is M (sufficiently large).so that the loss probability is small. The only choice available for studying M is through algorithmic methods because of the intrinsic nature of the system. While a number of approaches are available for determining the cut-off point, M, the one that seems to perform well is to increase M until the largest individual change in the elements of **X (t)** for successive values is less than ε a predetermined infinitesimal value.

Step 2: Find the Eigen values and Eigen vectors of this finite order matrix A.

Step3: Let $d_1, d_2, d_3, ..., d_{2(M+1)}$ be 2(M+1) Eigen values and $\overline{c_1}, \overline{c_2}, \overline{c_2}, ..., \overline{c_{2(M+1)}}$ be 2(M+1) Eigen vectors.

Step 4: Represent this Eigen vectors as column vectors of a matrix $C = \left(\overline{c_1}, \overline{c_2}, \overline{c_3}, ..., \overline{c_{2(M+1)}}\right)$

Step 5: Let $D = \begin{pmatrix} d_1 & 0 & 0 & . & . & 0 \\ 0 & d_2 & 0 & . & . & 0 \\ 0 & 0 & d_3 & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & d_{2(M+1)} \end{pmatrix}$

Step 6: Find the Exponential of the matrix tA using D and C. $e^{tA} = Ce^{tD}C^{-1}$

Step 7: Extract the first column of this Exponential matrix tA and store in X (t).

Step 8: This probability vector **X (t)** provides time dependent probabilities of number of customers in the queue at time t.

5. SYSTEM PERFORMANCE MEASURES

The following system measures are used to bring out the Transient behaviour of bulk service queueing model with starting failure under study. Numerical study has been dealt in very large scale to study the following measures for several values of t, λ, μ, α, a and b.

a. Probability that there are n customers in the queue when the server is idle at time t = $P_{n0}(t)$

b. Probability that there are n customers in the queue when the server is busy at time t = $P_{n1}(t)$

c. Probability that there are n customers in the queue when the server is in failure at time t = $P_{n2}(t)$

d. Probability that the server is idle at time t = $P_{\text{idle}}(t) = \sum_{n=0}^{a-1} P_{n0}(t)$

e. Probability that the server is busy at time t = $P_{\text{busy}}(t) = \sum_{n=0}^{\infty} P_{n1}(t)$

f. Probability that the server is failure at time t = $P_{\text{failure}}(t) = \sum_{n=a}^{\infty} P_{n2}(t)$

g. Average number of customers in the queue at time t = $L_q(t) = \sum_{n=0}^{a-1} nP_{n0}(t) + \sum_{n=0}^{\infty} nP_{n1}(t) + \sum_{n=a}^{\infty} nP_{n2}(t)$

6. NUMERICAL COMPUTATIONS

The Time dependent System performance measures and Transient probabilities of this model have been done and expressed in the form of tables, which are shown below for several values of t, λ, μ, α, a and b.

Table 1: Transient probability distribution of number of customers in the queue when the server is idle for various values of t, λ = 5, μ = 10, α = 5, a = 3 and b = 5.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$
0.3	0.2415	0.3430	0.2535
0.6	0.1465	0.2308	0.2704
0.9	0.1550	0.2160	0.2468
1.2	0.1566	0.2191	0.2449
1.5	0.1561	0.2193	0.2457
1.8	0.1561	0.2191	0.2457
2.1	0.1561	0.2191	0.2456
2.4	0.1561	0.2191	0.2456
2.7	0.1561	0.2191	0.2456
3.0	0.1561	0.2191	0.2456

Table 2: Transient probability distribution of number of customers in the queue when the server is idle for various values of t, λ = 5, μ = 10, α = 5, a = 4 and b = 12.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$	$P_{30}(t)$
0.4	0.1504	0.2781	0.2732	0.1811
0.8	0.1138	0.1590	0.2006	0.2236
1.2	0.1364	0.1754	0.1861	0.1953
1.6	0.1341	0.1796	0.1935	0.1966
2.0	0.1330	0.1778	0.1932	0.1982
2.4	0.1333	0.1778	0.1927	0.1979
2.8	0.1334	0.1779	0.1928	0.1978
3.2	0.1333	0.1779	0.1928	0.1979
3.6	0.1333	0.1779	0.1928	0.1978
4.0	0.1333	0.1779	0.1928	0.1978

Table 3: Transient probability distribution of number of customers in the queue when the server is idle for various values of t, $\lambda = 7$, $\mu = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$
0.3	0.1532	0.2765	0.2780
0.6	0.1182	0.1815	0.2249
0.9	0.1214	0.1830	0.2151
1.2	0.1208	0.1830	0.2157
1.5	0.1209	0.1829	0.2156
1.8	0.1209	0.1829	0.2156
2.1	0.1209	0.1829	0.2156
2.4	0.1209	0.1829	0.2156
2.7	0.1209	0.1829	0.2156
3.0	0.1209	0.1829	0.2156

Table 4: Transient probability distribution of number of customers in the queue when the server is idle for various values of t, $\lambda = 7$, $\mu = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{00}(t)$	$P_{10}(t)$	$P_{20}(t)$	$P_{30}(t)$
0.3	0.1337	0.2631	0.2722	0.1896
0.6	0.0908	0.1356	0.1807	0.2117
0.9	0.1113	0.1509	0.1649	0.1766
1.2	0.1100	0.1560	0.1736	0.1791
1.5	0.1086	0.1541	0.1734	0.1812
1.8	0.1088	0.1539	0.1727	0.1808
2.1	0.1089	0.1541	0.1728	0.1807
2.4	0.1089	0.1541	0.1729	0.1807
2.7	0.1089	0.1540	0.1728	0.1807
3.0	0.1089	0.1540	0.1728	0.1807

Table 5: Transient probability distribution of number of customers in the queue when the server is busy for various values of t, $\lambda = 5$, $\mu = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$	$P_{71}(t)$
0.3	0.0284	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
0.6	0.0731	0.0249	0.0080	0.0024	0.0007	0.0002	0.0000	0.0000
0.9	0.0795	0.0315	0.0125	0.0049	0.0018	0.0007	0.0002	0.0001
1.2	0.0781	0.0317	0.0133	0.0057	0.0025	0.0011	0.0004	0.0002
1.5	0.0780	0.0315	0.0133	0.0058	0.0026	0.0012	0.0005	0.0002
1.8	0.0780	0.0315	0.0132	0.0058	0.0026	0.0012	0.0006	0.0003
2.1	0.0780	0.0315	0.0132	0.0058	0.0026	0.0012	0.0006	0.0003
2.4	0.0780	0.0315	0.0132	0.0058	0.0026	0.0012	0.0006	0.0003
2.7	0.0780	0.0315	0.0132	0.0058	0.0026	0.0012	0.0006	0.0003
3.0	0.0780	0.0315	0.0132	0.0058	0.0026	0.0012	0.0006	0.0003

Table 6: Transient probability distribution of number of customers in the queue when the server is busy for various values of t, $\lambda = 5$, $\mu = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$	$P_{71}(t)$
0.4	0.0211	0.0046	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000
0.8	0.0663	0.0204	0.0061	0.0018	0.0005	0.0001	0.0000	0.0000
1.2	0.0692	0.0233	0.0078	0.0026	0.0008	0.0003	0.0001	0.0000
1.6	0.0663	0.0222	0.0075	0.0025	0.0008	0.0003	0.0001	0.0000
2.0	0.0666	0.0222	0.0074	0.0025	0.0008	0.0003	0.0001	0.0000
2.4	0.0667	0.0223	0.0075	0.0025	0.0008	0.0003	0.0001	0.0000
2.8	0.0667	0.0223	0.0075	0.0025	0.0008	0.0003	0.0001	0.0000
3.2	0.0667	0.0223	0.0075	0.0025	0.0008	0.0003	0.0001	0.0000
3.6	0.0667	0.0223	0.0075	0.0025	0.0008	0.0003	0.0001	0.0000
4.0	0.0667	0.0223	0.0075	0.0025	0.0008	0.0003	0.0001	0.0000

Table 7: Transient probability distribution of number of customers in the queue when the server is busy for various values of t, $\lambda = 7$, $\mu = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$	$P_{71}(t)$
0.3	0.0485	0.0149	0.0041	0.0010	0.0002	0.0000	0.0000	0.0000
0.6	0.0858	0.0410	0.0187	0.0080	0.0032	0.0012	0.0004	0.0001
0.9	0.0847	0.0436	0.0228	0.0119	0.0061	0.0030	0.0014	0.0007
1.2	0.0846	0.0434	0.0229	0.0123	0.0067	0.0037	0.0020	0.0011
1.5	0.0846	0.0434	0.0229	0.0123	0.0068	0.0038	0.0021	0.0012
1.8	0.0846	0.0434	0.0229	0.0123	0.0068	0.0038	0.0021	0.0012
2.1	0.0846	0.0434	0.0229	0.0123	0.0068	0.0038	0.0021	0.0012
2.4	0.0846	0.0434	0.0229	0.0123	0.0068	0.0038	0.0021	0.0012
2.7	0.0846	0.0434	0.0229	0.0123	0.0068	0.0038	0.0021	0.0012
3.0	0.0846	0.0434	0.0229	0.0123	0.0068	0.0038	0.0021	0.0012

Table 8: Transient probability distribution of number of customers in the queue when the server is busy for various values of t, $\lambda = 7$, $\mu = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{01}(t)$	$P_{11}(t)$	$P_{21}(t)$	$P_{31}(t)$	$P_{41}(t)$	$P_{51}(t)$	$P_{61}(t)$	$P_{71}(t)$
0.3	0.0218	0.0054	0.0012	0.0003	0.0001	0.0000	0.0000	0.0000
0.6	0.0737	0.0273	0.0097	0.0033	0.0011	0.0003	0.0001	0.0000
0.9	0.0796	0.0330	0.0135	0.0055	0.0022	0.0008	0.0003	0.0001
1.2	0.0760	0.0317	0.0132	0.0055	0.0023	0.0010	0.0004	0.0002
1.5	0.0760	0.0315	0.0131	0.0055	0.0023	0.0010	0.0004	0.0002
1.8	0.0762	0.0316	0.0132	0.0055	0.0023	0.0010	0.0004	0.0002
2.1	0.0762	0.0316	0.0132	0.0055	0.0023	0.0010	0.0004	0.0002
2.4	0.0762	0.0316	0.0132	0.0055	0.0023	0.0010	0.0004	0.0002
2.7	0.0762	0.0316	0.0132	0.0055	0.0023	0.0010	0.0004	0.0002
3.0	0.0762	0.0316	0.0132	0.0055	0.0023	0.0010	0.0004	0.0002

Table 9: Transient probability distribution of number of customers in the queue when the server is failure for various values of t, $\lambda = 5$, $\mu = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{32}(t)$	$P_{42}(t)$	$P_{52}(t)$	$P_{62}(t)$	$P_{72}(t)$	$P_{82}(t)$	$P_{92}(t)$
0.3	0.0901	0.0270	0.0070	0.0016	0.0003	0.0001	0.0000
0.6	0.1358	0.0636	0.0273	0.0107	0.0038	0.0012	0.0004
0.9	0.1261	0.0644	0.0324	0.0157	0.0072	0.0032	0.0013
1.2	0.1226	0.0617	0.0312	0.0157	0.0079	0.0039	0.0019
1.5	0.1228	0.0614	0.0307	0.0154	0.0077	0.0039	0.0019
1.8	0.1228	0.0614	0.0307	0.0154	0.0077	0.0038	0.0019
2.1	0.1228	0.0614	0.0307	0.0154	0.0077	0.0038	0.0019
2.4	0.1228	0.0614	0.0307	0.0154	0.0077	0.0038	0.0019
2.7	0.1228	0.0614	0.0307	0.0154	0.0077	0.0038	0.0019
3.0	0.1228	0.0614	0.0307	0.0154	0.0077	0.0038	0.0019

Table 10: Transient probability distribution of number of customers in the queue when the server is failure for various values of t, $\lambda = 5$, $\mu = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{42}(t)$	$P_{52}(t)$	$P_{62}(t)$	$P_{72}(t)$	$P_{82}(t)$	$P_{92}(t)$	$P_{102}(t)$
0.4	0.0635	0.0197	0.0054	0.0013	0.0003	0.0001	0.0000
0.8	0.1129	0.0541	0.0243	0.0102	0.0040	0.0014	0.0005
1.2	0.0998	0.0514	0.0263	0.0133	0.0065	0.0031	0.0014
1.6	0.0979	0.0489	0.0246	0.0124	0.0063	0.0032	0.0016
2.0	0.0990	0.0494	0.0247	0.0123	0.0061	0.0031	0.0015
2.4	0.0990	0.0495	0.0248	0.0124	0.0062	0.0031	0.0015
2.8	0.0989	0.0495	0.0247	0.0124	0.0062	0.0031	0.0015
3.2	0.0989	0.0495	0.0247	0.0124	0.0062	0.0031	0.0015
3.6	0.0989	0.0495	0.0247	0.0124	0.0062	0.0031	0.0015
4.0	0.0989	0.0495	0.0247	0.0124	0.0062	0.0031	0.0015

Table 11: Transient probability distribution of number of customers in the queue when the server is failure for various values of t, $\lambda = 7$, $\mu = 10$, $\alpha = 5$, a = 3 and b = 5.

t	$P_{32}(t)$	$P_{42}(t)$	$P_{52}(t)$	$P_{62}(t)$	$P_{72}(t)$	$P_{82}(t)$	$P_{92}(t)$
0.3	0.1369	0.0573	0.0206	0.0064	0.0018	0.0004	0.0001
0.6	0.1386	0.0836	0.0478	0.0254	0.0124	0.0056	0.0023
0.9	0.1260	0.0746	0.0445	0.0266	0.0156	0.0089	0.0049
1.2	0.1258	0.0734	0.0429	0.0252	0.0148	0.0088	0.0052
1.5	0.1258	0.0734	0.0428	0.0250	0.0146	0.0085	0.0050
1.8	0.1258	0.0734	0.0428	0.0250	0.0146	0.0085	0.0050
2.1	0.1258	0.0734	0.0428	0.0250	0.0146	0.0085	0.0050
2.4	0.1258	0.0734	0.0428	0.0250	0.0146	0.0085	0.0050
2.7	0.1258	0.0734	0.0428	0.0250	0.0146	0.0085	0.0050
3.0	0.1258	0.0734	0.0428	0.0250	0.0146	0.0085	0.0050

Table 12: Transient probability distribution of number of customers in the queue when the server is failure for various values of t, $\lambda = 7$, $\mu = 10$, $\alpha = 5$, a = 4 and b = 12.

t	$P_{42}(t)$	$P_{52}(t)$	$P_{62}(t)$	$P_{72}(t)$	$P_{82}(t)$	$P_{92}(t)$	$P_{102}(t)$
0.3	0.0757	0.0262	0.0079	0.0021	0.0005	0.0001	0.0000
0.6	0.1272	0.0714	0.0370	0.0176	0.0077	0.0031	0.0011
0.9	0.1065	0.0649	0.0393	0.0232	0.0132	0.0071	0.0036
1.2	0.1038	0.0605	0.0356	0.0212	0.0127	0.0075	0.0044
1.5	0.1056	0.0614	0.0357	0.0208	0.0121	0.0071	0.0042
1.8	0.1055	0.0616	0.0359	0.0209	0.0122	0.0071	0.0041
2.1	0.1054	0.0615	0.0359	0.0209	0.0122	0.0071	0.0042
2.4	0.1054	0.0615	0.0359	0.0209	0.0122	0.0071	0.0042
2.7	0.1054	0.0615	0.0359	0.0209	0.0122	0.0071	0.0042
3.0	0.1054	0.0615	0.0359	0.0209	0.0122	0.0071	0.0042

Table 13: System performance measures for various values of t, $\lambda = 5$, $\mu = 10$, $\alpha = 5$, a = 3 and b = 5.

t	Pidle(t)	Pbusy(t)	Pfailure(t)	Lq(t)
0.3	0.8380	0.0360	0.1260	1.2844
0.6	0.6477	0.1093	0.2430	1.7275
0.9	0.6177	0.1313	0.2510	1.7808
1.2	0.6207	0.1330	0.2463	1.7879
1.5	0.6210	0.1333	0.2457	1.7915
1.8	0.6208	0.1335	0.2457	1.7924
2.1	0.6208	0.1335	0.2456	1.7925
2.4	0.6208	0.1335	0.2456	1.7925
2.7	0.6208	0.1335	0.2456	1.7925
3.0	0.6208	0.1335	0.2456	1.7925

Table 14: System performance measures for various values of t, $\lambda = 5$, $\mu = 10$, $\alpha = 5$, a = 4 and b = 12.

t	Pidle(t)	Pbusy(t)	Pfailure(t)	Lq(t)
0.4	0.8828	0.0268	0.0904	1.7726
0.8	0.6971	0.0952	0.2077	2.2636
1.2	0.6931	0.1041	0.2028	2.1979
1.6	0.7038	0.0998	0.1964	2.1902
2.0	0.7022	0.1000	0.1978	2.1976
2.4	0.7018	0.1003	0.1980	2.1972
2.8	0.7020	0.1002	0.1978	2.1968
3.2	0.7020	0.1002	0.1978	2.1969
3.6	0.7019	0.1002	0.1979	2.1969
4.0	0.7020	0.1002	0.1978	2.1969

Table 15: System performance measures for various values of t, $\lambda = 7$, $\mu = 10$, $\alpha = 5$, a = 3 and b = 5.

t	Pidle(t)	Pbusy(t)	Pfailure(t)	Lq(t)
0.3	0.7076	0.0689	0.2235	1.6580
0.6	0.5246	0.1585	0.3169	2.0632
0.9	0.5194	0.1747	0.3058	2.1288
1.2	0.5195	0.1778	0.3027	2.1463
1.5	0.5194	0.1786	0.3021	2.1500
1.8	0.5194	0.1787	0.3019	2.1507
2.1	0.5194	0.1788	0.3019	2.1509
2.4	0.5194	0.1788	0.3018	2.1509
2.7	0.5194	0.1788	0.3018	2.1509
3.0	0.5194	0.1788	0.3018	2.1509

Table 16: System performance measures for various values of t, $\lambda = 7$, $\mu = 10$, $\alpha = 5$, a = 4 and b = 12.

t	Pidle(t)	Pbusy(t)	Pfailure(t)	Lq(t)
0.3	0.8587	0.0287	0.1126	1.8868
0.6	0.6189	0.1154	0.2657	2.5134
0.9	0.6037	0.1351	0.2611	2.4954
1.2	0.6186	0.1303	0.2511	2.4907
1.5	0.6173	0.1300	0.2527	2.5009
1.8	0.6162	0.1306	0.2532	2.5020
2.1	0.6164	0.1306	0.2530	2.5016
2.4	0.6165	0.1305	0.2530	2.5017
2.7	0.6165	0.1305	0.2530	2.5017
3.0	0.6165	0.1305	0.2530	2.5017

7. RESULTS & DISCUSSION

Table 1 to Table 4 show Transient probabilities of number of customers in the queue when the server is idle for several values of t, λ, μ, α, a and b.

We infer the following

- As the value of t increases the Transient Probabilities $P_{n0}(t) \to P_{n0}$

Table 5 to Table 8 show Transient probabilities of number of customers in the queue when the server is busy for several values of t, λ, μ, α, a and b.

We infer the following

- As the value of t increases the Transient Probabilities $P_{n1}(t) \to P_{n1}$
- The sequence $\{P_{n1}(t)\} \to 0$ as n $\to \infty$ for all values of t

Table 9 to Table 12 show Transient probabilities of number of customers in the queue when the server is failure for several values of t, λ, μ, α, a and b.

We infer the following

- As the value of t increases the Transient Probabilities $P_{n2}(t) \to P_{n2}$
- The sequence $\{P_{n2}(t)\} \to 0$ as n $\to \infty$ for all values of t

Table 13 to Table 16 show Time dependent System performance measures for several values of t, λ, μ, α, a and b.

We infer the following

- As the value of t increases and for several values of t, λ, μ, α, a and b, $P_{\text{idle}}(t) \to P_{\text{idle}}, P_{\text{busy}}(t) \to P_{\text{busy}}, P_{\text{failure}}(t) \to P_{\text{failure}}$, and $L_q(t) \to L_q$
- If α tends to ∞ this model coincides with bulk service queueing system.

8. CONCLUSION

A new computational approach was used to evaluate the Transient behaviour of Bulk service queueing system with starting failure model using infinite generator matrix and Eigen vectors and Eigen values. Numerical studies have been analysed in elaborate manner. In this model we have provided transient probability distribution of number of customers in the queue at time t and also time dependent system measures.

ACKNOWLEDGEMENT

I would like to Thank My Research Guide Dr. A. Muthu Ganapathi Subramanian, for his Support and Encouragement. Also, I would like to Thank Dr. Gopal Sekar for his continuous support and help.

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