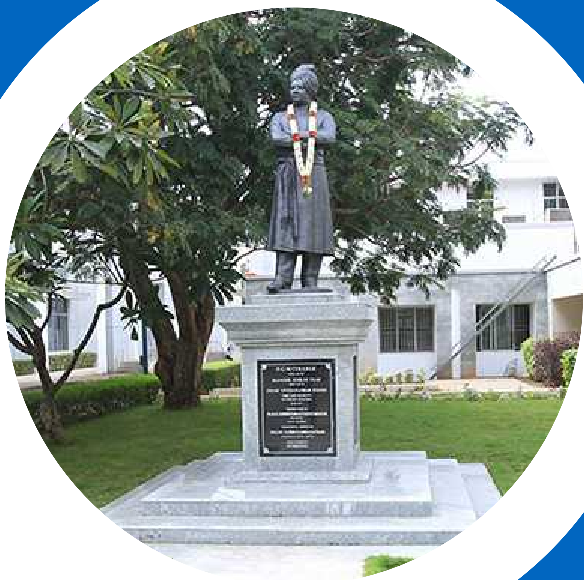
VOLUME XII
ISBN No.: 978-93-94004-01-6
Physical Science

Estd. 1957

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

Pollachi-642001



SUPPORTED BY

Riyasaa Labs

ABT Industries Ltd.,
Dairy Division

PROCEEDING

One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, Pollachi-642001.

Estd. 1957

Proceeding of the

One day International Conference on

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

ISBN No: 978-93-94004-01-6

978- 93- 94004- 01- 6

Nallamuthu Gounder Mahalingam College

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

www.ngmc.org

ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

EDITORIAL BOARD

International Conference on Emerging Trends in Science and Technology (ETIST 2021) Jointly Organized by Department of Biological Science, Physical Science and Computational Science Nallamuthu Gounder Mahalingam College, Affiliated to Bharathiar University, Tamilnadu, India.
Published by NGMC - November 2021

Intuitionistic fuzzy soft commutative ideals of BCK-algebras

Nana Liu[1], Chang Wang [2], V. Inthumathi[3].

Abstract - In this paper, we introduce the concept of intuitionistic fuzzy soft commutative ideal in BCK-algebra and discuss their important properties. In particular, the relations between intuitionistic fuzzy soft commutative ideal and intuitionistic fuzzy soft ideal are discussed. The "extended intersection", "restricted intersection", "union" and "AND" operations of intuitionistic fuzzy soft commutative ideal, and homomorphism of intuitionistic fuzzy soft commutative ideal are established. Besides, we will also discuss some further results of intuitionistic fuzzy soft ideal of BCK/BCI-algebras.

Keywords *vague set;* BCK-*algebra; commutative ideal; intuitionistic fuzzy soft ideal; intuitionistic fuzzy soft commutative ideal.*
2010 Subject classification: *06F35; 03G25*

1 Introduction

In 1966, Imai and Iséki [1, 2] introduced a new concept called a BCK/BCI-algebra, and since then many researchers have investigated various properties of this algebra. For the properties of BCK-algebras, we refer the reader to Iséki and Tanaka [3].

The fuzzy sets proposed by Zadeh [4] deal with problem by determining the degree to which an object belongs to a set. After the introduction of fuzzy sets by Zadeh, there have been a number of generalizations of this basic concept. Xi [5] applied the concept of fuzzy sets to BCK-algebras and gave some properties of it, and Molodtsov [6] introduced the concept of soft sets as a new mathematical tool for dealing with uncertainties that without the difficulties that plague the usual theoretical approach. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly, such as soft groups [7], soft semirings [8] and soft d-algebras [9]. The notion of intuitionistic fuzzy sets introduced by Atanassov [10] is one among them, while fuzzy sets give the degree of membership of an element in a given set, intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership. For more details about intuitionistic fuzzy sets, we refer the reader to [11].

For the general development of BCK-algebras, the ideal theory and its intuitionistic fuzzification play an important role. The notion of commutative ideal in BCK-algebras was first introduced by Meng [12] in 1991, and the intuitionistic fuzzification of commutative ideal in BCK-algebras was discussed by Jun et al. [13] in 2008, then Muhiuddin et al. [14] apply the fuzzy soft set theory to commutative ideal of BCK-algebras in 2021.

[1] Institute for Advanced Studies in History of Science, Northwest University ,Xi'an, Shaanxi 710127, China and School of Mathematics, Northwest University ,Xi'an, Shaanxi 710127, China
[2] Institute for Advanced Studies in History of Science, Northwest University ,Xi'an, Shaanxi 710127, China School of Mathematics, Northwest University ,Xi'an, Shaanxi 710127, China
[3] Associate Professor, Department of Mathematics,Nallamuthu Gounder Mahalingam College,Pollachi-642001, Coimbatore, Tamilnadu, India.
E.mail: inthumathi65@gmail.com

In this paper, we first investigate further properties of intuitionistic fuzzy soft ideal in BCK/BCI-algebras that were not studied in [15], then we introduce the notion of intuitionistic fuzzy soft commutative ideal in BCK-algebras, and investigate related properties. We provide relations between intuitionistic fuzzy soft commutative ideal and intuitionistic fuzzy soft ideal. The condition for intuitionistic fuzzy soft ideal to become intuitionistic fuzzy soft commutative ideal are discussed. In addition, we consider the "extended intersection", "restricted intersection", "union" and "AND" operations of intuitionistic fuzzy soft commutative ideal, and homomorphism of intuitionistic fuzzy soft commutative ideal.

We first review the definitions of the algebras we have studied, the basic definitions of intuitionistic fuzzy soft sets and some related operations in BCK-algebras.

2 Preliminaries

2.1 Basic results on BCK/BCI-algebras

In this section, we will recall some basic notions in BCK/BCI-algebra.

Definition 2.1. [2] An algebra $(X; *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following conditions:
(1) $((x * y) * (x * z)) * (z * y) = 0$,
(2) $(x * (x * y)) * y = 0$,
(3) $x * x = 0$,
(4) $x * y = 0, y * x = 0 \Rightarrow x = y$, for all $x, y, z \in X$.
If a BCI-algebra X satisfies the following identity:
(5) $0 * x = 0$, for all $x \in X$, then X is called a BCK-algebra.

In any BCK/BCI-algebra X one can define a partial order " $\leq$ " by putting $x \leq y$ if and only if $x * y = 0$.
In any BCK-algebra X the following holds:
(1) $x * 0 = x$;
(2) $x * y \leq x$;
(3) $(x * y) * z = (x * z) * y$;
(4) $(x * z) * (y * z) \leq x * y$;
(5) $x * (x * (x * y)) = x * y$;
(6) $x \leq y \Rightarrow x * z \leq y * z, z * y \leq z * x$.
A BCK-algebras X is said to be commutative if $x * (x * y) = y * (y * x)$ for all $x, y, z \in X$.
A nonempty subset A of a BCK/BCI-algebra X is called a BCK/BCI-subalgebra of X if $x * y \in A$ for all $x, y \in A$.
A nonempty subset A of a BCK/BCI-algebra X is called an ideal of X if it satisfies the following axioms:
(1) $0 \in A$;
(2) $x * y \in A, y \in A \Rightarrow x \in A$, for all $x \in X$.
A nonempty subset A of a BCK-algebra X is called a commutative ideal of X if it satisfies the following axioms:
(1) $0 \in A$;
(2) $(x * y) * z \in A, z \in A \Rightarrow x * (y * (y * x)) \in A$, for all $x, y, z \in X$.
Note that, in BCK-algebras, every commutative ideal is an ideal, but not the converse.

Definition 2.2. [5] A fuzzy set μ in BCK/BCI-algebras X is called a fuzzy ideal of X if it satisfies the following conditions:

(1) $\mu(0) \geq \mu(x)$;
(2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$;
for all $x, y \in X$.

Definition 2.3. [16] A fuzzy set μ in BCK-algebras X is called a fuzzy commutative ideal of X if it satisfies the following conditions:
(1) $\mu(0) \geq \mu(x)$;
(2) $\mu(x * (y * (y * x))) \geq \min\{\mu((x * y) * z), \mu(z)\}$;
for all $x, y, z \in X$.

Definition 2.4. [17] A mapping $f : X \rightarrow Y$ of BCK/BCI-algebras is called a homomorphism if $f(x * y) = f(x) * f(y)$ for all $x, y, z \in X$. Note that if $f : X \rightarrow Y$ is a homomorphism of BCK/BCI-algebras, then $f(0) = 0$.

Let $f : X \rightarrow Y$ is a homomorphism of BCK/BCI-algebras, for any intuitionistic fuzzy set $(\widetilde{F}, A)$ in Y, defined a new intuitionistic fuzzy set preimage $(\widetilde{F}, A)^f$ in X by $\mu_{\widetilde{F}}{}^f(x) = \mu_{\widetilde{F}}(f(x)), \gamma_{\widetilde{F}}{}^f(x) = \gamma_{\widetilde{F}}(f(x))$ for all $x \in X$.

2.2 Basic results on intuitionistic fuzzy soft sets

Molodtsov [6] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subset E$.

Definition 2.5. [6] A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \rightarrow P(U)$.

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\alpha \in A, F(\alpha)$ may be considered as the set of α -approximate elements of the soft set (F, A).

Definition 2.6. [10] Let U be an initial universe set and E be a set of parameters. Let $F(U)$ denote the set of all intuitionistic fuzzy sets in U. Then $(\widetilde{F}, A)$ is called an intuitionistic fuzzy soft set over U where $A \subseteq E$ and $\widetilde{F}$ is a mapping given by $\widetilde{F} : A \rightarrow F(U)$.

In general, for every $\alpha \in A, \widetilde{F}[\alpha]$ is an intuitionistic fuzzy set in U and it is called an intuitionistic fuzzy value set of parameter α. Clearly, $\widetilde{F}[\alpha]$ can be written as an intuitionistic fuzzy set such that $\widetilde{F}[\alpha] = \{< x, \mu_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{F}[\alpha]}(x) >| x$ where $\mu_{\widetilde{F}[\alpha]}(x)$ and $\gamma_{\widetilde{F}[\alpha]}(x)$ denotes the degree of membership and non-membership functions respectively. If for every $\alpha \in A, \mu_{\widetilde{F}[\alpha]}(x) = 1 - \gamma_{\widetilde{F}[\alpha]}(x)$ then $\widetilde{F}[\alpha]$ will be generated to be a standard fuzzy set and then $(\widetilde{F}, A)$ will be generated to be a traditional fuzzy soft set.

Definition 2.7. [18] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U, we say that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft subset of $(\widetilde{G}, B)$, denoted by $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$, if it satisfies:
(1) $A \subseteq B$;
(2) $\widetilde{F}[e]$ and $\widetilde{G}[e]$ are identical approximations, for all $e \in A$.

Definition 2.8. [18] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U, then "extended intersection" of $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H}, C)$ satisfying the following conditions:

$$\widetilde{H}[e] = \begin{cases} \widetilde{F}[e], & \text{if } e \in A \backslash B, \\ \widetilde{G}[e], & \text{if } e \in B \backslash A, \\ \widetilde{F}[e] \cap \widetilde{G}[e], & \text{if } e \in A \cap B. \end{cases}$$

where $C = A \cup B$ and for all $e \in C$. In this case, we write $(\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, B) = (\widetilde{H}, C)$.

Definition 2.9. [18] Let $(\widetilde{F},A)$ and $(\widetilde{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U such that $A\cap B\neq\emptyset$, then "restricted intersection" of $(\widetilde{F},A)$ and $(\widetilde{G},B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H},C)$ satisfying the condition: $\widetilde{H}[e]=\widetilde{F}[e]\cap\widetilde{G}[e]$.
where $C=A\cap B$ and for all $e\in C$. In this case, we write $(\widetilde{F},A)\widetilde{\cap}_r(\widetilde{G},B)=(\widetilde{H},C)$.

Definition 2.10. [18] Let $(\widetilde{F},A)$ and $(\widetilde{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U, then "union" of $(\widetilde{F},A)$ and $(\widetilde{G},B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H},C)$ satisfying the following conditions:

$$\widetilde{H}[e]=\begin{cases}\widetilde{F}[e], & \text{if } e\in A\backslash B,\\ \widetilde{G}[e], & \text{if } e\in B\backslash A,\\ \widetilde{F}[e]\cup\widetilde{G}[e], & \text{if } e\in A\cap B.\end{cases}$$

where $C=A\cup B$ and for all $e\in C$. In this case, we write $(\widetilde{F},A)\widetilde{\cup}(\widetilde{G},B)=(\widetilde{H},C)$.

Definition 2.11. [18] Let $(\widetilde{F},A)$ and $(\widetilde{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U, then $(\widetilde{F},A)AND(\widetilde{G},B)$ denoted by $(\widetilde{F},A)\widetilde{\wedge}(\widetilde{G},B)$ is defined by $(\widetilde{F},A)\widetilde{\wedge}(\widetilde{G},B)=(\widetilde{H},A\times B)$, where $\widetilde{H}[\alpha,\beta]=\widetilde{F}[\alpha]\cap\widetilde{G}[\beta]$ for all $(\alpha,\beta)\in A\times B$.

Definition 2.12. [18] Let $(\widetilde{F},A)$ and $(\widetilde{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U, then $(\widetilde{F},A)OR(\widetilde{G},B)$ denoted by $(\widetilde{F},A)\widetilde{\vee}(\widetilde{G},B)$ is defined by $(\widetilde{F},A)\widetilde{\vee}(\widetilde{G},B)=(\widetilde{H},A\times B)$, where $\widetilde{H}[\alpha,\beta]=\widetilde{F}[\alpha]\cup\widetilde{G}[\beta]$ for all $(\alpha,\beta)\in A\times B$.

Definition 2.13. [18] Let $(\widetilde{F},A)$ is an intuitionistic fuzzy soft set over a common universe U, we say that the complement of $(\widetilde{F},A)$ is denoted by $(\widetilde{F},A)^c$ and is defined as $\overline{\mu_{\widetilde{F}[\alpha]}}(x)=1-\mu_{\widetilde{F}[\alpha]}(x)$ and $\overline{\gamma_{\widetilde{F}[\alpha]}}(x)=1-\gamma_{\widetilde{F}[\alpha]}(x)$ for all $x\in X,\alpha\in A$.

Definition 2.14. [18] Let $(\widetilde{F},A)$ is an intuitionistic fuzzy soft set over a common universe U, then $\neg(\widetilde{F},A)=\left\{\mu_{\widetilde{F}[\alpha]}(x),\overline{\mu_{\widetilde{F}[\alpha]}}(x)\right\}$ and $\circ(\widetilde{F},A)=\left\{\overline{\gamma_{\widetilde{F}[\alpha]}}(x),\gamma_{\widetilde{F}[\alpha]}(x)\right\}$ for all $x\in X,\alpha\in A$.

3 Further properties of intuitionistic fuzzy soft ideals

In this section, X denotes BCK/BCI-algebras, we will give some properties of intuitionistic fuzzy soft ideals in BCK/BCI-algebras that were not studied in [15].

Definition 3.1. [17] An intuitionistic fuzzy set in BCK/BCI-algebra X is said an intuitionistic fuzzy BCK/BCI-subalgebra of X if satisfies:
(1) $\mu(x*y)\geq\min\{\mu(x),\mu(y)\}$;
(2) $\gamma(x*y)\leq\max\{\gamma(x),\gamma(y)\}$;
for all $x,y\in X$.

Definition 3.2. [15] Let $(\widetilde{F},A)$ be an intuitionistic fuzzy soft set over a BCK/BCI-algebra X where A is the subset of E. We say that $(\widetilde{F},A)$ is an intuitionistic fuzzy soft BCK/BCI-algebra over a BCK/BCI-algebra X if $\widetilde{F}[\alpha]$ is an intuitionistic fuzzy BCK/BCI-subalgebra in a BCK/BCI-algebra X for all $\alpha\in A$.

Definition 3.3. [15] Let $(\widetilde{F},A)$ be an intuitionistic fuzzy soft set, then $(\widetilde{F},A)$ is an intuitionistic fuzzy soft ideal over a BCK/BCI-algebra X if $\widetilde{F}[\alpha]=\left\{<x,\mu_{\widetilde{F}[\alpha]}(x),\gamma_{\widetilde{F}[\alpha]}(x)>\mid x\in X,\alpha\in A\right\}$ is an intuitionistic fuzzy ideal of X satisfies the following assertions:
(1) $\mu_{\widetilde{F}[\alpha]}(0)\geq\mu_{\widetilde{F}[\alpha]}(x)$;

(2) $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;
(3) $\mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$;
(4) $\gamma_{\widetilde{F}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}$;
for all $x, y, z \in X$ and $\alpha \in A$.

Based on the above definitions, we give the following theorems:

Theorem 3.1. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft* BCK/BCI-*algebras over a* BCK/BCI-*algebra X, then the "AND" $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft* BCK/BCI-*algebra over X.*

Proof. By means of Definition 2.11. we know that
$(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$, where $\widetilde{H}[\alpha,\beta] = \widetilde{F}[\alpha] \cap \widetilde{G}[\beta]$ for all $(\alpha,\beta) \in A \times B$.
For any $x, y \in X$, we have

$$\begin{aligned}
&\mu_{\widetilde{H}[\alpha,\beta]}(x * y) = \mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x * y)\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{G}[\beta]}(x * y)\right\}\\
&\geq \min\left\{\min\left\{\mu_{\widetilde{F}[\alpha]}(x), \mu_{\widetilde{F}[\alpha]}(y)\right\}, \min\left\{\mu_{\widetilde{G}[\beta]}(x), \mu_{\widetilde{G}[\beta]}(y)\right\}\right\}\\
&= \min\left\{\min\left\{\mu_{\widetilde{F}[\alpha]}(x), \mu_{\widetilde{G}[\beta]}(x)\right\}, \min\left\{\mu_{\widetilde{F}[\alpha]}(y), \mu_{\widetilde{G}[\beta]}(y)\right\}\right\}\\
&= \min\left\{\mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x), \mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(y)\right\}\\
&= \min\left\{\mu_{\widetilde{H}[\alpha,\beta]}(x), \mu_{\widetilde{H}[\alpha,\beta]}(y)\right\}
\end{aligned}$$

and

$$\begin{aligned}
&\gamma_{\widetilde{H}[\alpha,\beta]}(x * y) = \gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x * y)\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{G}[\beta]}(x * y)\right\}\\
&\leq \max\left\{\max\left\{\gamma_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{F}[\alpha]}(y)\right\}, \max\left\{\gamma_{\widetilde{G}[\beta]}(x), \gamma_{\widetilde{G}[\beta]}(y)\right\}\right\}\\
&= \max\left\{\max\left\{\gamma_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{G}[\beta]}(x)\right\}, \max\left\{\gamma_{\widetilde{F}[\alpha]}(y), \gamma_{\widetilde{G}[\beta]}(y)\right\}\right\}\\
&= \max\left\{\gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x), \gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(y)\right\}\\
&= \max\left\{\gamma_{\widetilde{H}[\alpha,\beta]}(x), \gamma_{\widetilde{H}[\alpha,\beta]}(y)\right\}.
\end{aligned}$$

Thus, $\widetilde{H}[\alpha,\beta] = \widetilde{F}[\alpha] \cap \widetilde{G}[\beta]$ is an intuitionistic fuzzy BCK/BCI-algebra of X for any $(\alpha,\beta) \in A \times B$.

Hence, $(\widetilde{H}, A \times B) = (\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft BCK/BCI-algebra of X for any $(\alpha,\beta) \in A \times B$. □

Lemma 3.1. *[15] Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft* BCK/BCI-*algebra over a* BCK/BCI-*algebra X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X if and only if it satisfies $x * y \leq z$, then*
(1) $\mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(y), \mu_{\widetilde{F}[\alpha]}(z)\right\}$;
(2) $\gamma_{\widetilde{F}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(y), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$;
for all $x, y, z \in X$ and $\alpha \in A$.

Theorem 3.2. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft ideals over a* BCK/BCI-*algebra X, then the "AND" $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft ideal over X.*

Proof. According to Theorem 3.1, let $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A\times B)$ is an intuitionistic fuzzy soft BCK/BCI-algebra over X, where $\widetilde{H}[\alpha,\beta](x) = \widetilde{F}[\alpha](x) \cap \widetilde{G}[\beta](x)$ for all $(\alpha,\beta) \in A \times B, x \in X$. For any $x \in X$, we have

$$\begin{aligned}
&\mu_{\widetilde{H}[\alpha,\beta]}(0) = \mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(0)\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}(0), \mu_{\widetilde{G}[\beta]}(0)\right\}
\end{aligned}$$

$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x), \mu_{\widetilde{G}[\beta]}(x)\right\}$
$= \mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x)$
$= \mu_{\widetilde{H}[\alpha,\beta]}(x)$
and
$\gamma_{\widetilde{H}[\alpha,\beta]}(0) = \gamma_{(\widetilde{F}[\alpha]\cap\tilde{G}[\beta])}(0)$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(0), \gamma_{\widetilde{G}[\beta]}(0)\right\}$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{G}[\beta]}(x)\right\}$
$= \gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x)$
$= \gamma_{\widetilde{H}[\alpha,\beta]}(x)$.
For any $x, y, z \in X$ be such that $x * y \leq z, (\alpha, \beta) \in A \times B$, we have
$\mu_{\widetilde{H}[\alpha,\beta]}(x) = \mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x)$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}(x), \mu_{\widetilde{G}[\beta]}(x)\right\}$
$\geq \min\left\{\min\left\{\mu_{\widetilde{F}[\alpha]}(y), \mu_{\widetilde{F}[\alpha]}(z)\right\}, \min\left\{\mu_{\widetilde{G}[\beta]}(y), \mu_{\widetilde{G}[\beta]}(z)\right\}\right\}$
$= \min\left\{\min\left\{\mu_{\widetilde{F}[\alpha]}(y), \mu_{\widetilde{G}[\beta]}(y)\right\}, \min\left\{\mu_{\widetilde{F}[\alpha]}(z), \mu_{\widetilde{G}[\beta]}(z)\right\}\right\}$
$= \min\left\{\mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(y), \mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(z)\right\}$
$= \min\left\{\mu_{\widetilde{H}[\alpha,\beta]}(y), \mu_{\widetilde{H}[\alpha,\beta]}(z)\right\}$
and
$\gamma_{\widetilde{H}[\alpha,\beta]}(x) = \gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x)$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{G}[\beta]}(x)\right\}$
$\leq \max\left\{\max\left\{\gamma_{\widetilde{F}[\alpha]}(y), \gamma_{\widetilde{F}[\alpha]}(z)\right\}, \max\left\{\gamma_{\widetilde{G}[\beta]}(y), \gamma_{\widetilde{G}[\beta]}(z)\right\}\right\}$
$= \max\left\{\max\left\{\gamma_{\widetilde{F}[\alpha]}(y), \gamma_{\widetilde{G}[\beta]}(y)\right\}, \max\left\{\gamma_{\widetilde{F}[\alpha]}(z), \gamma_{\widetilde{G}[\beta]}(z)\right\}\right\}$
$= \max\left\{\gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(y), \gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(z)\right\}$
$= \max\left\{\gamma_{\widetilde{H}[\alpha,\beta]}(y), \gamma_{\widetilde{H}[\alpha,\beta]}(z)\right\}$.

It follows Lemma 3.1, that $(\widetilde{H}, A \times B) = (\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft ideal over X, for any $(\alpha, \beta) \in A \times B$. □

Theorem 3.3. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a* BCK/BCI-*algebra X such that $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$. If $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft ideal over X, then*
(1) $\mu_{\widetilde{G}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$;
(2) $\gamma_{\widetilde{G}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;
(3) $\mu_{\widetilde{G}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$;
(4) $\gamma_{\widetilde{G}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}$;
for all $x, y \in X$ and $\alpha \in A$.

Proof. Assume that $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft ideal over X, for any $x \in X$ and $\alpha \in A$, we have
$\mu_{\widetilde{G}[\alpha]}(0) \geq \mu_{\widetilde{G}[\alpha]}(x) \geq \mu_{\widetilde{F}[\alpha]}(x)$
and
$\gamma_{\widetilde{G}[\alpha]}(0) \leq \gamma_{\widetilde{G}[\alpha]}(x) \leq \gamma_{\widetilde{F}[\alpha]}(x)$.
which prove (1) and (2). Also for any $x, y \in X$ and $\alpha \in A$, we have
$\mu_{\widetilde{G}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{G}[\alpha]}(x * y), \mu_{\widetilde{G}[\alpha]}(y)\right\} \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$
and

$\gamma_{\widetilde{G}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{G}[\alpha]}(x * y), \gamma_{\widetilde{G}[\alpha]}(y)\right\} \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}.$
which prove (3) and (4), and the proof is complete. □

Remark 3.1. *For two intuitionistic fuzzy soft sets $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over a* BCK/BCI-*algebra X, where $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$, if $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft ideal over X, then $(\widetilde{F}, A)$ maybe not an intuitionistic fuzzy soft ideal over X. The following example will illustrate this viewpoint.*

Example 3.1. *Let $U = \{0, a, b, c, d\}$ with Cayley table given by:*

*	0	a	b	c	d
0	0	0	b	c	d
a	a	0	b	c	d
b	b	b	0	d	c
c	c	c	d	0	b
d	d	d	c	b	0

Then $(U; *, 0)$ is a BCI-*algebra.*
Consider a set of parameters $A = \{beautiful\}$, $B = \{fine, beautiful\}$, respectively.
Let $\{\widetilde{F}, A\}$ and $\{\widetilde{G}, B\}$ be two intuitionistic fuzzy soft sets over U which are defined by:

$\widetilde{F}$	0	a	b	c	d
beautiful	(0.5,0.4)	(0.5,0.4)	(0.4,0.6)	(0.4,0.6)	(0.2,0.7)

and

$\widetilde{G}$	0	a	b	c	d
fine	(0.8,0.2)	(0.8,0.2)	(0.6,0.4)	(0.7,0.3)	(0.6,0.4)
beautiful	(0.9,0)	(0.9,0)	(0.7,0.2)	(0.7,0.2)	(0.8,0.1)

respectively, and $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$. Then $\{\widetilde{G}, B\}$ is an intuitionistic fuzzy soft ideal over U, but $\{\widetilde{F}, A\}$ is not an intuitionistic fuzzy soft ideal over U. Since
$\mu_{\widetilde{F}[beautiful]}(d) = 0.2$
$< \min\left\{\mu_{\widetilde{F}[beautiful]}(d * b), \mu_{\widetilde{F}[beautiful]}(b)\right\}$
$= \min\left\{\mu_{\widetilde{F}[beautiful]}(c), \mu_{\widetilde{F}[beautiful]}(b)\right\}$
$= \min\{0.4, 0.4\} = 0.4$
and
$\gamma_{\widetilde{F}[beautiful]}(d) = 0.7$
$> \max\left\{\gamma_{\widetilde{F}[beautiful]}(d * b), \gamma_{\widetilde{F}[becutiful]}(b)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[beautiful]}(c), \gamma_{\widetilde{F}[beautiful]}(b)\right\}$
$= \max\{0.6, 0.6\} = 0.6.$

Theorem 3.4. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a* BCK/BCI-*algebra X. If I is a subset of A, then $\left(\widetilde{F}\big|_I, A\right)$ is an intuitionistic fuzzy soft ideal over a* BCK/BCI-*algebra X.*

Proof. Straightforward. □

The following example shows that there exists an intuitionistic fuzzy soft set $(\widetilde{F}, A)$ over a BCK/BCI-algebra X such that

(1) $(\widetilde{F}, A)$ is not an intuitionistic fuzzy soft ideal over X;

(2) there exists a subset I of A such that $\left(\widetilde{F}\big|_I, A\right)$ is an intuitionistic fuzzy soft ideal over X.

Example 3.2. *Let $U = \{0, a, b, c\}$ with Cayley table given by:*

$*$	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

Then $(U; *, 0)$ is a BCK*-algebra.*

Consider a set of parameters A = {amazing, smart, lovely}.

Let $\{\widetilde{F}, A\}$ be an intuitionistic fuzzy soft sets over U, then $\widetilde{F}$[amazing], $\widetilde{F}$[smart] and $\widetilde{F}$[lovely] are intuitionistic fuzzy sets. We define them as follows:

$\widetilde{F}$	0	a	b	c
amazing	(0.8,0.1)	(0.5,0.3)	(0.5,0.3)	(0.5,0.3)
smart	(0.7,0.2)	(0.4,0.4)	(0.6,0.3)	(0.4,0.4)
lovely	(0.3,0.7)	(0.3,0.5)	(0.5,0.3)	(0.6,0.2)

Then $(\widetilde{F}, A)$ is not an intuitionistic fuzzy soft ideal over U, since $\widetilde{F}$ [lovely] are not intuitionistic fuzzy ideal in U,

$\mu_{\widetilde{F}[lovely]}(0) = 0.3 < \mu_{\widetilde{F}[lovely]}(b) = 0.5$

and

$\gamma_{\widetilde{F}[lovely]}(0) = 0.7 > \gamma_{\widetilde{F}[lovely]}(c) = 0.2.$

But if we take I = {amazing, smart}, then $\left(\widetilde{F}\big|_I, A\right)$ is an intuitionistic fuzzy soft ideal over U.

Next, we consider other results that intuitionistic fuzzy soft ideal in BCK/BCI-algebra X.

Theorem 3.5. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a* BCK/BCI*-algebra X, then* $\neg(\widetilde{F}, A) = \{\mu_{\widetilde{F}[\alpha]}(x), \overline{\mu_{\widetilde{F}[\alpha]}}(x$ *is also an intuitionistic fuzzy soft ideal of X, for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a BCK/BCI-algebra X, we have

$\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$

$\Rightarrow 1 - \mu_{\widetilde{F}[\alpha]}(0) \leq 1 - \mu_{\widetilde{F}[\alpha]}(x)$

$\Rightarrow \overline{\mu_{\widetilde{F}[\alpha]}}(0) \leq \overline{\mu_{\widetilde{F}[\alpha]}}(x),$

for all $x \in X, \alpha \in A$.

Consider for any $x, y \in X, \alpha \in A$,

$\mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$

$\Rightarrow 1 - \overline{\mu_{\widetilde{F}[\alpha]}}(x) \geq \min\left\{1 - \overline{\mu_{\widetilde{F}[\alpha]}}(x * y), 1 - \overline{\mu_{\widetilde{F}[\alpha]}}(y)\right\}$

$\Rightarrow \overline{\mu_{\widetilde{F[\alpha]}}}(x) \leq 1 - \min\left\{1 - \overline{\mu_{\widetilde{F[\alpha]}}}(x * y), 1 - \overline{\mu_{\widetilde{F[\alpha]}}}(y)\right\}$
$\Rightarrow \overline{\mu_{\widetilde{F[\alpha]}}}(x) \leq \max\left\{\overline{\mu_{\widetilde{F[\alpha]}}}(x * y), \overline{\mu_{\widetilde{F[\alpha]}}}(y)\right\}$.
Hence, $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F[\alpha]}}(x), \overline{\mu_{\widetilde{F[\alpha]}}}(x)\right\}$ is an intuitionistic fuzzy soft ideal of X, for all $x \in X, \alpha \in A$. □

Theorem 3.6. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a* BCK/BCI-*algebra X, then $\circ(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F[\alpha]}}}(x), \gamma_{\widetilde{F[\alpha]}}(x\right.$ is also an intuitionistic fuzzy soft ideal of X, for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a BCK/BCI-algebra X, we have
$\gamma_{\widetilde{F[\alpha]}}(0) \leq \gamma_{\widetilde{F[\alpha]}}(x)$
$\Rightarrow 1 - \gamma_{\widetilde{F[\alpha]}}(0) \geq 1 - \gamma_{\widetilde{F[\alpha]}}(x)$
$\Rightarrow \overline{\gamma_{\widetilde{F[\alpha]}}}(0) \geq \overline{\gamma_{\widetilde{F[\alpha]}}}(x)$
for all $x \in X, \alpha \in A$.

Consider for any $x, y \in X, \alpha \in A$,
$\gamma_{\widetilde{F[\alpha]}}(x) \leq \max\left\{\gamma_{\widetilde{F[\alpha]}}(x * y), \gamma_{\widetilde{F[\alpha]}}(y)\right\}$
$\Rightarrow 1 - \overline{\gamma_{\widetilde{F[\alpha]}}}(x) \leq \max\left\{1 - \overline{\gamma_{\widetilde{F[\alpha]}}}(x * y), 1 - \overline{\gamma_{\widetilde{F[\alpha]}}}(y)\right\}$
$\Rightarrow \overline{\gamma_{\widetilde{F[\alpha]}}}(x) \geq 1 - \max\left\{1 - \overline{\gamma_{\widetilde{F[\alpha]}}}(x * y), 1 - \overline{\gamma_{\widetilde{F[\alpha]}}}(y)\right\}$
$\Rightarrow \overline{\gamma_{\widetilde{F[\alpha]}}}(x) \geq \min\left\{\overline{\gamma_{\widetilde{F[\alpha]}}}(x * y), \overline{\gamma_{\widetilde{F[\alpha]}}}(y)\right\}$.
Hence, $\circ(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F[\alpha]}}}(x), \gamma_{\widetilde{F[\alpha]}}(x)\right\}$ is an intuitionistic fuzzy soft ideal of X, for all $x \in X, \alpha \in A$. □

Theorem 3.7. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a* BCK/BCI-*algebra X if and only if $\mu_{\widetilde{F[\alpha]}}(x)$ and $\overline{\gamma_{\widetilde{F[\alpha]}}}(x)$ are fuzzy soft ideals of X for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a BCK/BCI-algebra X, clearly $\mu_{\widetilde{F[\alpha]}}(x)$ is a fuzzy soft ideal of X.

Let $x, y \in X, \alpha \in A$, then
$\overline{\gamma_{\widetilde{F[\alpha]}}}(0) = 1 - \gamma_{\widetilde{F[\alpha]}}(0) \geq 1 - \gamma_{\widetilde{F[\alpha]}}(x) = \overline{\gamma_{\widetilde{F[\alpha]}}}(x)$
and
$\overline{\gamma_{\widetilde{F[\alpha]}}}(x) = 1 - \gamma_{\widetilde{F[\alpha]}}(x)$
$\geq 1 - \max\left\{\gamma_{\widetilde{F[\alpha]}}(x * y), \gamma_{\widetilde{F[\alpha]}}(y)\right\}$
$= \min\left\{1 - \gamma_{\widetilde{F[\alpha]}}(x * y), 1 - \gamma_{\widetilde{F[\alpha]}}(y)\right\}$
$= \min\left\{\overline{\gamma_{\widetilde{F[\alpha]}}}(x * y), \overline{\gamma_{\widetilde{F[\alpha]}}}(y)\right\}$.
Hence, $\overline{\gamma_{\widetilde{F[\alpha]}}}(x)$ is a fuzzy soft ideal of X.

Conversely, assume that $\mu_{\widetilde{F[\alpha]}}(x)$ and $\overline{\gamma_{\widetilde{F[\alpha]}}}(x)$ are fuzzy soft ideals of X for all $x \in X, \alpha \in A$. For all $x \in X$, we have
$\mu_{\widetilde{F[\alpha]}}(0) \geq \mu_{\widetilde{F[\alpha]}}(x)$
and
$1 - \gamma_{\widetilde{F[\alpha]}}(0) = \overline{\gamma_{\widetilde{F[\alpha]}}}(0) \geq \overline{\gamma_{\widetilde{F[\alpha]}}}(x) = 1 - \gamma_{\widetilde{F[\alpha]}}(x)$.
Which show that $\gamma_{\widetilde{F[\alpha]}}(0) \leq \gamma_{\widetilde{F[\alpha]}}(x)$.

Now let $x, y \in X, \alpha \in A$, then
$\mu_{\widetilde{F[\alpha]}}(x) \geq \min\left\{\mu_{\widetilde{F[\alpha]}}(x * y), \mu_{\widetilde{F[\alpha]}}(y)\right\}$
and
$1 - \gamma_{\widetilde{F[\alpha]}}(x) = \overline{\gamma_{\widetilde{F[\alpha]}}}(x)$
$\geq \min\left\{\overline{\gamma_{\widetilde{F[\alpha]}}}(x * y), \overline{\gamma_{\widetilde{F[\alpha]}}}(y)\right\}$

$= \min\left\{1-\gamma_{\widetilde{F[\alpha]}}(x*y), 1-\gamma_{\widetilde{F[\alpha]}}(y)\right\}$
$= 1-\max\left\{\gamma_{\widetilde{F[\alpha]}}(x*y), \gamma_{\widetilde{F[\alpha]}}(y)\right\}$
and so
$\gamma_{\widetilde{F[\alpha]}}(x) \le \max\left\{\gamma_{\widetilde{F[\alpha]}}(x*y), \gamma_{\widetilde{F[\alpha]}}(y)\right\}$.
Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X. □

Theorem 3.8. *Let* $(\widetilde{F}, A)$ *be an intuitionistic fuzzy soft ideal over a* BCK/BCI-*algebra* X *if and only if* $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F[\alpha]}}(x), \overline{\mu_{\widetilde{F[\alpha]}}}(x)\right\}$ *and* $\circ(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F[\alpha]}}}(x), \gamma_{\widetilde{F[\alpha]}}(x)\right\}$ *are intuitionistic fuzzy soft ideals of* X*, for all* $x \in X, \alpha \in A$.

Proof. It is straightforward by Theorem 3.7. □

At the end of this section, we discuss the homomorphism between intuitionistic fuzzy soft ideals in BCK/BCI-algebra.

Theorem 3.9. *Let* $f : X \to Y$ *is an onto homomorphism of* BCK/BCI-*algebras. If an intuitionistic fuzzy soft set* $(\widetilde{F}, A)$ *of* Y *is an intuitionistic fuzzy soft ideal, then preimage* $(\widetilde{F}, A)^f$ *is also an intuitionistic fuzzy soft ideal of* X.

Proof. Since $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of Y, and $(\widetilde{F}, A)^f$ is the preimage of $(\widetilde{F}, A)$ under f of X, then $\mu_{\widetilde{F[\alpha]}}(f(x)) = \mu_{\widetilde{F[\alpha]}}{}^f(x), \gamma_{\widetilde{F[\alpha]}}(f(x)) = \gamma_{\widetilde{F[\alpha]}}{}^f(x)$ for all $x \in X, \alpha \in A$.

Since $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of Y, then for any $x \in X, \alpha \in A$, we have
$\mu_{\widetilde{F\alpha]}}{}^f(x) = \mu_{\widetilde{F[\alpha]}}(f(x)) \le \mu_{\widetilde{F[\alpha]}}(0) = \mu_{\widetilde{F[\alpha]}}(f(0)) = \mu_{\widetilde{F[\alpha]}}{}^f(0)$
and
$\gamma_{\widetilde{F[\alpha]}}{}^f(x) = \gamma_{\widetilde{F[\alpha]}}(f(x)) \ge \gamma_{\widetilde{F[\alpha]}}(0) = \gamma_{\widetilde{F[\alpha]}}(f(0)) = \gamma_{\widetilde{F[\alpha]}}{}^f(0)$.
Moreover,
$\min\left\{\mu_{\widetilde{F[\alpha]}}{}^f(x*y), \mu_{\widetilde{F[\alpha]}}{}^f(y)\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}(f(x*y)), \mu_{\widetilde{F[\alpha]}}(f(y))\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}(f(x)*f(y)), \mu_{\widetilde{F[\alpha]}}(f(y))\right\}$
$\le \mu_{\widetilde{F[\alpha]}}(f(x))$
$= \mu_{\widetilde{F[\alpha]}}{}^f(x)$
and
$\max\left\{\gamma_{\widetilde{F[\alpha]}}{}^f(x*y), \gamma_{\widetilde{F[\alpha]}}{}^f(y)\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}(f(x*y)), \gamma_{\widetilde{F[\alpha]}}(f(y))\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}(f(x)*f(y)), \gamma_{\widetilde{F[\alpha]}}(f(y))\right\}$
$\ge \gamma_{\widetilde{F[\alpha]}}(f(x))$
$= \gamma_{\widetilde{F[\alpha]}}{}^f(x)$.
Hence, $(\widetilde{F}, A)^f$ is also an intuitionistic fuzzy soft ideal of X, for any $x, y \in X, \alpha \in A$. □

If we strengthen the condition of f, then we can construct the converse of the above theorem as follows.

Theorem 3.10. *Let* $f : X \to Y$ *is an epimorphism of* BCK/BCI-*algebras. If an intuitionistic fuzzy soft set* $(\widetilde{F}, A)^f$ *is an intuitionistic fuzzy soft ideal of* X*, then* $(\widetilde{F}, A)$ *is also an intuitionistic fuzzy soft ideal of* Y.

Proof. Since $(\widetilde{F}, A)^f$ is an intuitionistic fuzzy soft ideal of X, and $(\widetilde{F}, A)^f$ is the preimage of $(\widetilde{F}, A)$ under f of X, then $\mu^f_{\widetilde{F_{[\alpha]}}}(x) = \mu_{\widetilde{F[\alpha]}}(f(x)), \gamma_{\widetilde{F[\alpha]}}{}^f(x) = \gamma_{\widetilde{F[\alpha]}}(f(x))$ for all $x \in X, \alpha \in A$.

Let $x, y \in Y, \alpha \in A$, there exist $a, b \in X$ such that $f(a) = x$ and $f(b) = y$. Now

$\mu_{\widetilde{F}[\alpha]}(x) = \mu_{\widetilde{F}[\alpha]}(f(a)) = \mu_{\widetilde{F}[\alpha]}{}^{f}(a) \leq \mu_{\widetilde{F}[\alpha]}{}^{f}(0) = \mu_{\widetilde{F}[\alpha]}(f(0)) = \mu_{\widetilde{F}[\alpha]}(0)$
and
$\gamma_{\widetilde{F}\alpha]}(x) = \gamma_{\widetilde{F}[\alpha]}(f(a)) = \gamma_{\widetilde{F}[\alpha]}{}^{f}(a) \geq \gamma_{\widetilde{F}[\alpha]}{}^{f}(0) = \gamma_{\widetilde{F}[\alpha]}(f(0)) = \gamma_{\widetilde{F}[\alpha]}(0).$
Moreover,
$\mu_{\widetilde{F}[\alpha]}(x)$
$= \mu_{\widetilde{F}[\alpha]}(f(a))$
$= \mu_{\widetilde{F}[\alpha]}{}^{f}(a)$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}{}^{f}(a * b), \mu_{\widetilde{F}[\alpha]}{}^{f}(b)\right\}$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}(f(a * b)), \mu_{\widetilde{F}[\alpha]}(f(b))\right\}$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}(f(a) * f(b)), \mu_{\widetilde{F}[\alpha]}(f(b))\right\}$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x)$
$= \gamma_{\widetilde{F}[\alpha]}(f(a))$
$= \gamma_{\widetilde{F}[\alpha]}{}^{f}(a)$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}{}^{f}(a * b), \gamma_{\widetilde{F}[\alpha]}{}^{f}(b)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(f(a * b)), \gamma_{\widetilde{F}[\alpha]}(f(b))\right\}$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(f(a) * f(b)), \gamma_{\widetilde{F}[\alpha]}(f(b))\right\}$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}.$
Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of Y. □

4 Intuitionistic fuzzy soft commutative ideals

In this section, X denotes a BCK-algebra unless otherwise is specified.

Definition 4.1. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft set. Then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X if $\widetilde{F}[\alpha] = \left\{< x, \mu_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{F}[\alpha]}(x) >\mid x \in X, \alpha \in A\right\}$ is an intuitionistic fuzzy commutative ideal of X satisfies the following assertions:
(1) $\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$;
(2) $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;
(3) $\mu_{\widetilde{F}[\alpha]}(x * (y * (y * x))) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$;
(4) $\gamma_{\widetilde{F}[\alpha]}(x * (y * (y * x))) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$;
for all $x, y, z \in X$ and $\alpha \in A$.

Let us illustrate this definition using the following example.

Example 4.1. *Let $U = \{0, a, b, c\}$ with Cayley table given by:*
Then $(U; *, 0)$ is a BCK-*algebra.*
Consider a set of parameters A = {content, sad, clam}.
Let $\{\widetilde{F}, A\}$ is an intuitionistic fuzzy soft sets over U, then $\widetilde{F}$[content], $\widetilde{F}$[sad] and $\widetilde{F}$[clam] are intuitionistic fuzzy sets. We define them as follows:
Then $\{\widetilde{F}, A\}$ is an intuitionistic fuzzy soft commutative ideal over U based on parameter "content", "sad" and "clam".

$*$	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

$\widetilde{F}$	0	a	b	c
content	(0.8,0.1)	(0.6,0.3)	(0.6,0.3)	(0.7,0.2)
sad	(0.7,0.2)	(0.4,0.3)	(0.4,0.3)	(0.3,0.5)
clam	(0.8,0.2)	(0.5,0.4)	(0.5,0.4)	(0.5,0.4)

Theorem 4.1. *For any* BCK-*algebra X, every intuitionistic fuzzy soft commutative ideal is order preserving.*

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal over X based on the parameter $\alpha \in A$, Let $x, y \in X$ be such that $x \le y$, then for all $z \in X$, putting $y = 0$ and $z = y$ in

$\mu_{\widetilde{F}[\alpha]}(x * (y * (y * x))) \ge \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$

and

$\gamma_{\widetilde{F}[\alpha]}(x * (y * (y * x))) \le \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$.

Then we have

$\mu_{\widetilde{F}[\alpha]}(x * (0 * (0 * x))) \ge \min\left\{\mu_{\widetilde{F}[\alpha]}((x * 0) * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$

$\Rightarrow \mu_{\widetilde{F}[\alpha]}(x * 0) \ge \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$

$\Rightarrow \mu_{\widetilde{F}[\alpha]}(x) \ge \min\left\{\mu_{\widetilde{F}[\alpha]}(0), \mu_{\widetilde{F}[\alpha]}(y)\right\}$

$\Rightarrow \mu_{\widetilde{F}[\alpha]}(x) \ge \mu_{\widetilde{F}[\alpha]}(y)$

and

$\gamma_{\widetilde{F}[\alpha]}(x * (0 * (0 * x))) \le \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * 0) * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}$

$\Rightarrow \gamma_{\widetilde{F}[\alpha]}(x * 0) \le \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}$

$\Rightarrow \gamma_{\widetilde{F}[\alpha]}(x) \le \max\left\{\gamma_{\widetilde{F}[\alpha]}(0), \gamma_{\widetilde{F}[\alpha]}(y)\right\}$

$\Rightarrow \gamma_{\widetilde{F}[\alpha]}(x) \le \gamma_{\widetilde{F}[\alpha]}(y)$,

for all $x, y, z \in X, \alpha \in A$.

Hence, intuitionistic fuzzy soft commutative ideal $(\widetilde{F}, A)$ is order preserving. □

Theorem 4.2. *For any* BCK-*algebra X, every intuitionistic fuzzy soft commutative ideal is an intuitionistic fuzzy soft ideal.*

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal over X based on the parameter $\alpha \in A$, then

$\mu_{\widetilde{F}[\alpha]}(x) = \mu_{\widetilde{F}[\alpha]}(x * (0 * (0 * x)))$

$\ge \min\left\{\mu_{\widetilde{F}[\alpha]}((x * 0) * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$

$= \min\left\{\mu_{\widetilde{F}[\alpha]}(x * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$

and

$\gamma_{\widetilde{F}[\alpha]}(x) = \gamma_{\widetilde{F}[\alpha]}(x * (0 * (0 * x)))$

$\le \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * 0) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$

$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\},$
for all $x, z \in X$.

Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal over X based on the parameter $\alpha \in A$. □

The converse of Theorem 4.2 is not true in general as shown in the example given next.

Example 4.2. *Let $U = \{0, a, b, c, d\}$ with Cayley table given by:*

*	0	a	b	c	d
0	0	0	0	0	0
a	a	0	a	0	0
b	b	b	0	0	0
c	c	c	c	0	0
d	d	d	d	c	0

Then $(U; *, 0)$ is a BCK-*algebra.*

Consider a set of parameters $A = \{eye, hair, ear\}$.

Let $\{\widetilde{F}, A\}$ be an intuitionistic fuzzy soft set over U. Then $\widetilde{F}[eye]$, $\widetilde{F}[hair]$ and $\widetilde{F}[ear]$ are intuitionistic fuzzy sets. We define them as follows:

$\widetilde{F}$	0	a	b	c	d
eye	(0.9,0.1)	(0.2,0.7)	(0.5,0.4)	(0.2,0.7)	(0.2,0.7)
hair	(0.8,0.2)	(0.7,0.3)	(0.7,0.3)	(0.3,0.5)	(0.3,0.5)
ear	(0.7,0.3)	(0.6,0.4)	(0.2,0.6)	(0.2,0.6)	(0.2,0.6)

Then $\{\widetilde{F}, A\}$ is an intuitionistic fuzzy soft ideal over U based on the parameter "eye", "hair" and "ear", but $\{\widetilde{F}, A\}$ is not an intuitionistic fuzzy soft commutative ideal over U based on the parameter "eye". Since

$\mu_{\widetilde{F}[eye]}(a * (c * (c * a))) = \mu_{\widetilde{F}[eye]}(a * (c * c))$
$= \mu_{\widetilde{F}[eye]}(a * 0) = \mu_{\widetilde{F}[eye]}(a) = 0.2$
$< \min\left\{\mu_{\widetilde{F}[eye]}((a * c) * b), \mu_{\widetilde{F}[eye]}(b)\right\}$
$= \min\left\{\mu_{\widetilde{F}[eye]}(0 * b), \mu_{\widetilde{F}[eye]}(b)\right\}$
$= \min\left\{\mu_{\widetilde{F}[eye]}(0), \mu_{\widetilde{F}[eye]}(b)\right\}$
$= \min\{0.9, 0.5\} = 0.5$

and

$\gamma_{\widetilde{F}[eye]}(a * (c * (c * a))) = \gamma_{\widetilde{F}[eye]}(a * (c * c))$
$= \gamma_{\widetilde{F}[eye]}(a * 0) = \gamma_{\widetilde{F}[eye]}(a) = 0.7$
$> \max\left\{\gamma_{\widetilde{F}[eye]}((a * c) * b), \gamma_{\widetilde{F}[eye]}(b)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[eye]}(0 * b), \gamma_{\widetilde{F}[eye]}(b)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[eye]}(0), \gamma_{\widetilde{F}[eye]}(b)\right\}$
$= \max\{0.1, 0.4\} = 0.4.$

In the following theorem, we can see that the converse of Theorem 4.2 holds in a commutative BCK-algebra.

Theorem 4.3. *In a commutative* BCK-*algebra X, every intuitionistic fuzzy soft ideal is an intuitionistic fuzzy soft commutative ideal.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a commutative BCK-algebra X.
Let $x, y, z \in X$, then

$((x * (y * (y * x))) * ((x * y) * z)) * z$
$= ((x * (y * (y * x))) * z) * ((x * y) * z)$
$\leq (x * (y * (y * x))) * (x * y)$
$= (x * (x * y)) * (y * (y * x)) = 0.$

That is, $(x * (y * (y * x))) * ((x * y) * z) \leq z$. Let $\alpha \in A$ be a parameter and $x, y, z \in X$, it follows from Lemma 3.1 that

$\mu_{\widetilde{F[\alpha]}}(x * (y * (y * x))) \geq \min\left\{\mu_{\widetilde{F[\alpha]}}((x * y) * z), \mu_{\widetilde{F[\alpha]}}(z)\right\}$

and

$\gamma_{\widetilde{F[\alpha]}}(x * (y * (y * x))) \leq \max\left\{\gamma_{\widetilde{F[\alpha]}}((x * y) * z), \gamma_{\widetilde{F[\alpha]}}(z)\right\}.$

Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal of X. □

Next, we provide a condition for an intuitionistic fuzzy soft ideal to be an intuitionistic fuzzy soft commutative ideal in BCK-algebra X.

Theorem 4.4. *Every intuitionistic fuzzy soft commutative ideal* $\{\widetilde{F}, A\}$ *over a* BCK-*algebra X satisfies the following assertion:*

(1) $\mu_{\widetilde{F[\alpha]}}(x * (y * (y * x))) \geq \mu_{\widetilde{F[\alpha]}}(x * y)$;
(2) $\gamma_{\widetilde{F[\alpha]}}(x * (y * (y * x))) \leq \gamma_{\widetilde{F[\alpha]}}(x * y)$.

Proof. Let if we take $z = 0$ in

$\mu_{\widetilde{F[\alpha]}}(x * (y * (y * x))) \geq \min\left\{\mu_{\widetilde{F[\alpha]}}((x * y) * z), \mu_{\widetilde{F[\alpha]}}(z)\right\}$

and

$\gamma_{\widetilde{F[\alpha]}}(x * (y * (y * x))) \leq \max\left\{\gamma_{\widetilde{F[\alpha]}}((x * y) * z), \gamma_{\widetilde{F[\alpha]}}(z)\right\}.$

Then

$\mu_{\widetilde{F[\alpha]}}(x * (y * (y * x)))$
$\geq \min\left\{\mu_{\widetilde{F[\alpha]}}((x * y) * 0), \mu_{\widetilde{F[\alpha]}}(0)\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}(x * y), \mu_{\widetilde{F[\alpha]}}(0)\right\}$
$= \mu_{\widetilde{F[\alpha]}}(x * y)$

and

$\gamma_{\widetilde{F[\alpha]}}(x * (y * (y * x)))$
$\leq \max\left\{\gamma_{\widetilde{F[\alpha]}}((x * y) * 0), \gamma_{\widetilde{F[\alpha]}}(0)\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}(x * y), \gamma_{\widetilde{F[\alpha]}}(0)\right\}$
$= \gamma_{\widetilde{F[\alpha]}}(x * y).$

for all $x, y \in X, \alpha \in A$, and the proof is complete. □

Theorem 4.5. *If an intuitionistic fuzzy soft ideal* $\{\widetilde{F}, A\}$ *over a* BCK-*algebra X satisfies the Theorem 4.4 conditions, then* $\{\widetilde{F}, A\}$ *is an intuitionistic fuzzy soft commutative ideal over X.*

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal over X based on the parameter $\alpha \in A$, then

$\mu_{\widetilde{F[\alpha]}}(x * (y * (y * x)))$
$\geq \mu_{\widetilde{F[\alpha]}}(x * y)$

$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*(y*(y*x)))$
$\leq \gamma_{\widetilde{F}[\alpha]}(x*y)$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$.
Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal over X. □

Theorem 4.6. *Suppose that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal over a* BCK-*algebra X, then the following are equivalent:*
(1) $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal;
(2) $\mu_{\widetilde{F}[\alpha]}(x*(y*(y*x))) \geq \mu_{\widetilde{F}[\alpha]}(x*y)$ and $\gamma_{\widetilde{F}[\alpha]}(x*(y*(y*x))) \leq \gamma_{\widetilde{F}[\alpha]}(x*y)$ for all $x, y \in X, \alpha \in A$;
(3) $\mu_{\widetilde{F}[\alpha]}(x*(y*(y*x))) = \mu_{\widetilde{F}[\alpha]}(x*y)$ and $\gamma_{\widetilde{F}[\alpha]}(x*(y*(y*x))) = \gamma_{\widetilde{F}[\alpha]}(x*y)$ for all $x, y \in X, \alpha \in A$.

Proof. To prove (1) ⇒ (2), assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal of X, from Theorem 4.4, we know that the condition (2) is holds.

To prove (2) ⇒ (3), from the property of BCK-algebras $x*y \leq x$, we can observe that $y*(y*x) \leq y$, then $x*y \leq x*(y*(y*x))$, for all $x, y \in X$, applying Theorem 4.1, then we have
$\mu_{\widetilde{F}[\alpha]}(x*(y*(y*x))) \leq \mu_{\widetilde{F}[\alpha]}(x*y)$
and
$\gamma_{\widetilde{F}[\alpha]}(x*(y*(y*x))) \geq \gamma_{\widetilde{F}[\alpha]}(x*y)$,
for all $\alpha \in A$. It follows from condition (2) that
$\mu_{\widetilde{F}[\alpha]}(x*(y*(y*x))) = \mu_{\widetilde{F}[\alpha]}(x*y)$
and
$\gamma_{\widetilde{F}[\alpha]}(x*(y*(y*x))) = \gamma_{\widetilde{F}[\alpha]}(x*y)$,
for all $x, y \in X, \alpha \in A$. Hence the condition (3) is holds.

To prove (3) ⇒ (1), Since $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal over a BCK-algebra X, then
$\mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$,
for all $x, y, z \in X, \alpha \in A$. Combining (3), we have
$\mu_{\widetilde{F}[\alpha]}(x*(y*(y*x))) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*(y*(y*x))) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$,
for all $x, y, z \in X, \alpha \in A$.

Obviously, $(\widetilde{F}, A)$ is also satisfies $\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$ and $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$ for all $x \in X, \alpha \in A$.

Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal over X. Hence, the condition (1) is holds. The proof is complete. □

Next, we discuss other properties of intuitionistic fuzzy soft commutative ideal in BCK-algebra X.

Theorem 4.7. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a* BCK-*algebra X such that $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$. If $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over X, then*
(1) $\mu_{\widetilde{G}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$;
(2) $\gamma_{\widetilde{G}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;
(3) $\mu_{\widetilde{G}[\alpha]}(x*(y*(y*x))) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$;

(4) $\gamma_{\widetilde{G}[\alpha]}(x*(y*(y*x))) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$;
for all $x, y, z \in X$ *and* $\alpha \in A$.

Proof. Assume that $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over X.
For any $x \in X$ and $\alpha \in A$, we have:
$\mu_{\widetilde{G}[\alpha]}(0) \geq \mu_{\widetilde{G}[\alpha]}(x) \geq \mu_{\widetilde{F}[\alpha]}(x)$
and
$\gamma_{\widetilde{G}[\alpha]}(0) \leq \gamma_{\widetilde{G}[\alpha]}(x) \leq \gamma_{\widetilde{F}[\alpha]}(x)$,
which prove (1) and (2). Also for any $x, y \in X$ and $\alpha \in A$, we have:
$\mu_{\widetilde{G}[\alpha]}(x*(y*(y*x)))$
$\geq \min\left\{\mu_{\widetilde{G}[\alpha]}((x*y)*z), \mu_{\widetilde{G}[\alpha]}(z)\right\}$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
and
$\gamma_{\widetilde{G}[\alpha]}(x*(y*(y*x)))$
$\leq \max\left\{\gamma_{\widetilde{G}[\alpha]}((x*y)*z), \gamma_{\widetilde{G}[\alpha]}(z)\right\}$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$.
which prove (3) and (4), and the proof is complete. □

Remark 4.1. *For two intuitionistic fuzzy soft sets* $(\widetilde{F}, A)$ *and* $(\widetilde{G}, B)$ *over a* BCK-*algebra X, where* $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$*, if* $(\widetilde{G}, B)$ *is an intuitionistic fuzzy soft commutative ideal over X, then* $(\widetilde{F}, A)$ *maybe not an intuitionistic fuzzy soft commutative ideal over X. The following example will illustrate this viewpoint.*

Example 4.3. *Let U be a* BCK-*algebra which is given in Example* 4.1.
Consider sets of parameters A = {*pen*}, *B* = {*pen, eraser*}*, respectively. Let* $(\widetilde{F}, A)$ *and* $(\widetilde{G}, B)$ *be two intuitionistic fuzzy soft sets over U which are defined by:*

$\widetilde{F}$	0	a	b	c
pen	(0.6,0.4)	(0.2,0.7)	(0.3,0.6)	(0.1,0.9)

and

$\widetilde{G}$	0	a	b	c
pen	(0.7,0.2)	(0.4,0.5)	(0.4,0.5)	(0.5,0.3)
eraser	(0.8,0.2)	(0.6,0.3)	(0.6,0.3)	(0.6,0.3)

respectively, and $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$*. Then* $(\widetilde{G}, B)$ *is an intuitionistic fuzzy soft commutative ideal over U, but* $(\widetilde{F}, A)$ *is not an intuitionistic fuzzy soft commutative ideal over U, Since*
$\mu_{\widetilde{F}[pen]}(a*(c*(c*a)))$
$= \mu_{\widetilde{F}[pen]}(a*(c*c))$
$= \mu_{\widetilde{F}[pen]}(a*0)$
$= \mu_{\widetilde{F}[pen]}(a) = 0.2$
$< \min\left\{\mu_{\widetilde{F}[pen]}((a*c)*b), \mu_{\widetilde{F}[pen]}(b)\right\}$
$= \min\left\{\mu_{\widetilde{F}[pen]}(a*b), \mu_{\widetilde{F}[pen]}(b)\right\}$

$= \min\left\{\mu_{\widetilde{F}[pen]}(0), \mu_{\widetilde{F}[pen]}(b)\right\}$
$= \min\{0.6, 0.3\} = 0.3$
and
$\gamma_{\widetilde{F}[pen]}(a * (c * (c * a)))$
$= \gamma_{\widetilde{F}[pen]}(a * (c * c))$
$= \gamma_{\widetilde{F}[pen]}(a * 0)$
$= \gamma_{\widetilde{F}[pen]}(a) = 0.7$
$> \max\left\{\gamma_{\widetilde{F}[pen]}((a * c) * b), \gamma_{\widetilde{F}[pen]}(b)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[pen]}(a * b), \gamma_{\widetilde{F}[pen]}(b)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[pen]}(0), \gamma_{\widetilde{F}[pen]}(b)\right\}$
$= \max\{0.4, 0.6\} = 0.6.$

Theorem 4.8. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft commutative ideal over a* BCK*-algebra X, If I is a subset of A, then $\left(\widetilde{F}\big|_I, A\right)$ is an intuitionistic fuzzy soft commutative ideal over X.*

Proof. Straightforward. □

The following example shows that there exists an intuitionistic fuzzy soft set $(\widetilde{F}, A)$ over a BCK-algebra X such that
(1) $(\widetilde{F}, A)$ is not an intuitionistic fuzzy soft commutative ideal over X.
(2) there exists a subset I of A such that $\left(\widetilde{F}\big|_I, A\right)$ is an intuitionistic fuzzy soft commutative ideal over X.

Example 4.4. *Let U be a* BCK*-algebra which is given in Example* 4.1.
Consider a set of parameters A = {red, green, yellow}. Then $\widetilde{F}$[red], $\widetilde{F}$[green] and $\widetilde{F}$[yellow] are intuitionistic fuzzy sets over U. We define them as follows:

$\widetilde{F}$	0	a	b	c
red	(0.8,0.1)	(0.7,0.3)	(0.5,0.4)	(0.5,0.4)
green	(0.7,0.2)	(0.4,0.5)	(0.4,0.5)	(0.4,0.5)
yellow	(0.9,0.1)	(0.6,0.3)	(0.6,0.3)	(0.4,0.5)

Then $(\widetilde{F}, A)$ is not an intuitionistic fuzzy soft commutative ideal over U, since $\widetilde{F}$[red] is not intuitionistic fuzzy commutative ideal in U,
$\mu_{\widetilde{F}[red]}(b * (c * (c * b)))$
$= \mu_{\widetilde{F}[red]}(b) = 0.5$
$< \min\left\{\mu_{\widetilde{F}[red]}((b * c) * a), \mu_{\widetilde{F}[red]}(a)\right\}$
$= \min\left\{\mu_{\widetilde{F}[red]}(a), \mu_{\widetilde{F}[red]}(a)\right\}$
$= \min\{0.7, 0.7\} = 0.7$
and
$\gamma_{\widetilde{F}[red]}(b * (c * (c * b)))$
$= \gamma_{\widetilde{F}[red]}(b) = 0.4$
$> \max\left\{\gamma_{\widetilde{F}[red]}((b * c) * a), \gamma_{\widetilde{F}[red]}(a)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[red]}(a), \gamma_{\widetilde{F}[red]}(a)\right\}$
$= \max\{0.3, 0.3\} = 0.3.$
But if we take I = {green, yellow}, then $\left(\widetilde{F}\big|_I, A\right)$ is an intuitionistic fuzzy soft commutative ideal over U.

Next, we consider other results of intuitionistic fuzzy soft commutative ideal in BCK-algebra X.

Theorem 4.9. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft commutative ideal over a* BCK*-algebra X, then* $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F}[\alpha]}(x), \overline{\mu_{\widetilde{F}[\alpha]}}(x)\right\}$ *is also an intuitionistic fuzzy soft commutative ideal of X, for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X, we have

$\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$
$\Rightarrow 1 - \mu_{\widetilde{F}[\alpha]}(0) \leq 1 - \mu_{\widetilde{F}[\alpha]}(x)$
$\Rightarrow \overline{\mu_{\widetilde{F}[\alpha]}}(0) \leq \overline{\mu_{\widetilde{F}[\alpha]}}(x)$,

for all $x \in X, \alpha \in A$.

Consider for any $x, y, z \in X, \alpha \in A$,

$\mu_{\widetilde{F}[\alpha]}(x * (y * (y * x))) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
$\Rightarrow 1 - \overline{\mu_{\widetilde{F}[\alpha]}}(x * (y * (y * x))) \geq \min\left\{1 - \overline{\mu_{\widetilde{F}[\alpha]}}((x * y) * z), 1 - \overline{\mu_{\widetilde{F}[\alpha]}}(z)\right\}$
$\Rightarrow \overline{\mu_{\widetilde{F}[\alpha]}}(x * (y * (y * x))) \leq 1 - \min\left\{1 - \overline{\mu_{\widetilde{F}[\alpha]}}((x * y) * z), 1 - \overline{\mu_{\widetilde{F}[\alpha]}}(z)\right\}$
$\Rightarrow \overline{\mu_{\widetilde{F}[\alpha]}}(x * (y * (y * x))) \leq \max\left\{\overline{\mu_{\widetilde{F}[\alpha]}}((x * y) * z), \overline{\mu_{\widetilde{F}[\alpha]}}(z)\right\}$.

Hence, $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F}[\alpha]}(x), \overline{\mu_{\widetilde{F}[\alpha]}}(x)\right\}$ is an intuitionistic fuzzy soft commutative ideal of X, for all $x \in X, \alpha \in A$. □

Theorem 4.10. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft commutative ideal over a* BCK*-algebra X, then* $\circ(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F}[\alpha]}}(x), \gamma_{\widetilde{F}[\alpha]}(x)\right\}$ *is also an intuitionistic fuzzy soft commutative ideal of X, for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X, we have

$\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$
$\Rightarrow 1 - \gamma_{\widetilde{F}[\alpha]}(0) \geq 1 - \gamma_{\widetilde{F}[\alpha]}(x)$
$\Rightarrow \overline{\gamma_{\widetilde{F}[\alpha]}}(0) \geq \overline{\gamma_{\widetilde{F}[\alpha]}}(x)$

for all $x \in X, \alpha \in A$.

Consider for any $x, y, z \in X, \alpha \in A$,

$\gamma_{\widetilde{F}[\alpha]}(x * (y * (y * x))) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$
$\Rightarrow 1 - \overline{\gamma_{\widetilde{F}[\alpha]}}(x * (y * (y * x))) \leq \max\left\{1 - \overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), 1 - \overline{\gamma_{\widetilde{F}[\alpha]}}(z)\right\}$
$\Rightarrow \overline{\gamma_{\widetilde{F}[\alpha]}}(x * (y * (y * x))) \geq 1 - \max\left\{1 - \overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), 1 - \overline{\gamma_{\widetilde{F}[\alpha]}}(z)\right\}$
$\Rightarrow \overline{\gamma_{\widetilde{F}[\alpha]}}(x * (y * (y * x))) \geq \min\left\{\overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), \overline{\gamma_{\widetilde{F}[\alpha]}}(z)\right\}$

Hence, $\circ(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F}[\alpha]}}(x), \gamma_{\widetilde{F}[\alpha]}(x)\right\}$ is an intuitionistic fuzzy soft commutative ideal of X, for all $x \in X, \alpha \in A$. □

Theorem 4.11. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft commutative ideal over a* BCK*-algebra X if and only if $\mu_{\widetilde{F}[\alpha]}(x)$ and $\overline{\gamma_{\widetilde{F}[\alpha]}}(x)$ are fuzzy soft commutative ideals of X for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X, clearly $\mu_{\widetilde{F}[\alpha]}(x)$ is a fuzzy soft commutative ideal of X.

Let $x, y, z \in X, \alpha \in A$, then

$\overline{\gamma_{\widetilde{F}[\alpha]}}(0) = 1 - \gamma_{\widetilde{F}[\alpha]}(0) \geq 1 - \gamma_{\widetilde{F}[\alpha]}(x) = \overline{\gamma_{\widetilde{F}[\alpha]}}(x)$

and

$\overline{\gamma_{\widetilde{F}[\alpha]}}(x * (y * (y * x))) = 1 - \gamma_{\widetilde{F}[\alpha]}(x * (y * (y * x)))$
$\geq 1 - \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$
$= \min\left\{1 - \gamma_{\widetilde{F}[\alpha]}((x * y) * z), 1 - \gamma_{\widetilde{F}[\alpha]}(z)\right\}$

$= \min\left\{\overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), \overline{\gamma_{\widetilde{F}[\alpha]}}(z)\right\}.$

Hence, $\overline{\gamma_{\widetilde{F}[\alpha]}}(x)$ is a fuzzy soft commutative ideal of X.

Conversely, assume that $\mu_{\widetilde{F}[\alpha]}(x)$ and $\overline{\gamma_{\widetilde{F}[\alpha]}}(x)$ are fuzzy soft commutative ideals of X for all $x \in X, \alpha \in A$. For all $x \in X$, we have

$\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$

and

$1 - \gamma_{\widetilde{F}[\alpha]}(0) = \overline{\gamma_{\widetilde{F}[\alpha]}}(0) \geq \overline{\gamma_{\widetilde{F}[\alpha]}}(x) = 1 - \gamma_{\widetilde{F}[\alpha]}(x).$

Which show that $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$.

Now let $x, y, z \in X, \alpha \in A$, then

$\mu_{\widetilde{F}[\alpha]}(x * (y * (y * x))) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$

and

$1 - \gamma_{\widetilde{F}[\alpha]}(x * (y * (y * x))) = \overline{\gamma_{\widetilde{F}[\alpha]}}(x * (y * (y * x)))$

$\geq \min\left\{\overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), \overline{\gamma_{\widetilde{F}[\alpha]}}(z)\right\}$

$= \min\left\{1 - \gamma_{\widetilde{F}[\alpha]}((x * y) * z), 1 - \gamma_{\widetilde{F}[\alpha]}(z)\right\}$

$= 1 - \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$

and so

$\gamma_{\widetilde{F}[\alpha]}(x * (y * (y * x))) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}.$

Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal of X. □

Theorem 4.12. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft commutative ideal over a* BCK-*algebra X if and only if $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F}[\alpha]}(x), \overline{\mu_{\widetilde{F}[\alpha]}}(x)\right\}$ and $\circ(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F}[\alpha]}}(x), \gamma_{\widetilde{F}[\alpha]}(x)\right\}$ are intuitionistic fuzzy soft commutative ideals of X, for all $x \in X, \alpha \in A$.*

Proof. It is straightforward by Theorem 4.11. □

Theorem 4.13. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft commutative ideals over a* BCK-*algebra X, then the "extended intersection" $(\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over* X.

Proof. Let $(\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, B) = (\widetilde{H}, C)$ be the "extended intersection" of intuitionistic fuzzy soft commutative ideal $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over X, where $C = A \cup B$. For any $e \in C$,

if $e \in A \backslash B$, then $\widetilde{H}[e] = \widetilde{F}[e]$ is an intuitionistic fuzzy commutative ideal in X because $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X;

if $e \in B \backslash A$, then $\widetilde{H}[e] = \widetilde{G}[e]$ is an intuitionistic fuzzy commutative ideal in X because $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X;

if $A \cap B \neq \emptyset$, then $\widetilde{H}[e] = \widetilde{F}[e] \cap \widetilde{G}[e]$ is an intuitionistic fuzzy commutative ideal for all $e \in A \cap B$, since the intersection of two intuitionistic fuzzy commutative ideals is an intuitionistic fuzzy commutative ideal.

Therefore $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X. □

The following two corollaries are straightforward results of Theorem 4.13.

Corollary 4.1. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, A)$ be two intuitionistic fuzzy soft commutative ideals over a* BCK-*algebra X, then the "extended intersection" $(\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, A)$ is an intuitionistic fuzzy soft commutative ideal over X.*

Corollary 4.2. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft commutative ideals over a* BCK-*algebra X, then the "restricted intersection" $(\widetilde{F}, A)\widetilde{\cap}_r(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over X.*

Theorem 4.14. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft commutative ideals over a* BCK-*algebra X, if A and B are disjoint, then the "union" $(\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over X.*

Proof. Let $(\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B) = (\widetilde{H}, C)$ be the "union" of intuitionistic fuzzy soft commutative ideal $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over X. Since A and B are disjoint, then for all $e \in C$, either $e \in A\backslash B$ or $e \in B\backslash A$, by means of Definition 2.10,

if $e \in A\backslash B$, then $\widetilde{H}[e] = \widetilde{F}[e]$ is an intuitionistic fuzzy commutative ideal in X because $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X;

if $e \in B\backslash A$, then $\widetilde{H}[e] = \widetilde{G}[e]$ is an intuitionistic fuzzy commutative ideal in X because $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X.

Hence $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over a BCK-algebra X. □

Theorem 4.15. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft commutative ideals over a* BCK*-algebra X, then the "AND " $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft commutative ideal over X.*

Proof. Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft commutative ideals over a BCK-algebra X based on parameter $\alpha \in A$ and $\beta \in B$, respectively, then $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ are two intuitionistic fuzzy soft ideals over X based on the parameter $\alpha \in A$ and $\beta \in B$, respectively.

By Theorem 3.2, we know that $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$ is an intuitionistic fuzzy soft ideal over X based on the parameter $(\alpha, \beta) \in A \times B$.
For any $x, y \in X$, we have

$$\begin{aligned}
&\mu_{\widetilde{H}[\alpha,\beta]}(x * (y * (y * x)))\\
&= \mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x * (y * (y * x)))\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}(x * (y * (y * x))), \mu_{\widetilde{G}[\beta]}(x * (y * (y * x)))\right\}\\
&\geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{G}[\beta]}(x * y)\right\}\\
&= \mu_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x * y)\\
&= \mu_{\widetilde{H}[\alpha,\beta]}(x * y)
\end{aligned}$$

and

$$\begin{aligned}
&\gamma_{\widetilde{H}[\alpha,\beta]}(x * (y * (y * x)))\\
&= \gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x * (y * (y * x)))\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * (y * (y * x))), \gamma_{\widetilde{G}[\beta]}(x * (y * (y * x)))\right\}\\
&\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{G}[\beta]}(x * y)\right\}\\
&= \gamma_{(\widetilde{F}[\alpha]\cap\widetilde{G}[\beta])}(x * y)\\
&= \gamma_{\widetilde{H}[\alpha,\beta]}(x * y).
\end{aligned}$$

It follows from Theorem 4.5 that $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$ is an intuitionistic fuzzy soft commutative ideal over X based on the parameter (α, β). □

At the end of the paper, we discuss the homomorphism between intuitionistic fuzzy soft commutative ideals.

Theorem 4.16. *Let $f : X \to Y$ is an onto homomorphism of* BCK*-algebras. If an intuitionistic fuzzy soft set $(\widetilde{F}, A)$ of Y is an intuitionistic fuzzy soft commutative ideal, then preimage $(\widetilde{F}, A)^f$ is also an intuitionistic fuzzy soft commutative ideal of X.*

Proof. Since $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal of Y, and $(\widetilde{F}, A)^f$ is the preimage of $(\widetilde{F}, A)$ under f of X, then $\mu_{\widetilde{F}[\alpha]}(f(x)) = \mu_{\widetilde{F}[\alpha]}{}^f(x), \gamma_{\widetilde{F}[\alpha]}(f(x)) = \gamma_{\widetilde{F}[\alpha]}{}^f(x)$ for all $x \in X, \alpha \in A$.

Since $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal of Y, then for any $x \in X, \alpha \in A$, we have

$$\mu_{\widetilde{F}\alpha]}{}^f(x) = \mu_{\widetilde{F}[\alpha]}(f(x)) \leq \mu_{\widetilde{F}[\alpha]}(0) = \mu_{\widetilde{F}[\alpha]}(f(0)) = \mu_{\widetilde{F}[\alpha]}{}^f(0)$$

and

$$\gamma_{\widetilde{F}[\alpha]}{}^f(x) = \gamma_{\widetilde{F}[\alpha]}(f(x)) \geq \gamma_{\widetilde{F}[\alpha]}(0) = \gamma_{\widetilde{F}[\alpha]}(f(0)) = \gamma_{\widetilde{F}[\alpha]}{}^f(0).$$

Moreover,

$\min\left\{\mu_{\widetilde{F[\alpha]}}{}^{f}((x*y)*z), \mu_{\widetilde{F[\alpha]}}{}^{f}(z)\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}(f((x*y)*z)), \mu_{\widetilde{F[\alpha]}}(f(z))\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}(f(x*y)*f(z)), \mu_{\widetilde{F[\alpha]}}(f(z))\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}((f(x)*f(y))*f(z)), \mu_{\widetilde{F[\alpha]}}(f(z))\right\}$
$\leq \mu_{\widetilde{F[\alpha]}}(f(x)*(f(y)*(f(y)*f(x))))$
$= \mu_{\widetilde{F[\alpha]}}(f(x)*(f(y)*(f(y*x))))$
$= \mu_{\widetilde{F[\alpha]}}(f(x)*(f(y*(y*x))))$
$= \mu_{\widetilde{F[\alpha]}}(f(x*(y*(y*x))))$
$= \mu_{\widetilde{F[\alpha]}}{}^{f}(x*(y*(y*x)))$
and
$\max\left\{\gamma_{\widetilde{F[\alpha]}}{}^{f}((x*y)*z), \gamma_{\widetilde{F[\alpha]}}{}^{f}(z)\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}(f((x*y)*z)), \gamma_{\widetilde{F[\alpha]}}(f(z))\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}(f(x*y)*f(z)), \gamma_{\widetilde{F[\alpha]}}(f(z))\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}((f(x)*f(y))*f(z)), \gamma_{\widetilde{F[\alpha]}}(f(z))\right\}$
$\geq \gamma_{\widetilde{F[\alpha]}}(f(x)*(f(y)*(f(y)*f(x))))$
$= \gamma_{\widetilde{F[\alpha]}}(f(x)*(f(y)*(f(y*x))))$
$= \gamma_{\widetilde{F[\alpha]}}(f(x)*(f(y*(y*x))))$
$= \gamma_{\widetilde{F[\alpha]}}(f(x*(y*(y*x))))$
$= \gamma_{\widetilde{F[\alpha]}}{}^{f}(x*(y*(y*x)))$.
Hence, $(\widetilde{F}, A)^f$ is also an intuitionistic fuzzy soft commutative ideal of X, for any $x, y, z \in X, \alpha \in A$. □

If we strengthen the condition of f, then we can construct the converse of the above theorem as follows.

Theorem 4.17. *Let $f : X \to Y$ is an epimorphism of* BCK*-algebras. If an intuitionistic fuzzy soft set $(\widetilde{F}, A)^f$ is an intuitionistic fuzzy soft commutative ideal of X, then $(\widetilde{F}, A)$ is also an intuitionistic fuzzy soft commutative ideal of Y.*

Proof. Since $(\widetilde{F}, A)^f$ is an intuitionistic fuzzy soft commutative ideal of X, and $(\widetilde{F}, A)^f$ is the preimage of $(\widetilde{F}, A)$ under f of X, then $\mu^{f}_{\widetilde{F}_{[\alpha]}}(x) = \mu_{\widetilde{F[\alpha]}}(f(x)), \gamma_{\widetilde{F[\alpha]}}{}^{f}(x) = \gamma_{\widetilde{F[\alpha]}}(f(x))$ for all $x \in X, \alpha \in A$.

Let $x, y, z \in Y, \alpha \in A$, there exist $a, b, c \in X$ such that $f(a) = x, f(b) = y$ and $f(c) = z$.
Now,
$\mu_{\widetilde{F[\alpha]}}(x) = \mu_{\widetilde{F[\alpha]}}(f(a)) = \mu_{\widetilde{F[\alpha]}}{}^{f}(a) \leq \mu_{\widetilde{F[\alpha]}}{}^{f}(0) = \mu_{\widetilde{F[\alpha]}}(f(0)) = \mu_{\widetilde{F[\alpha]}}(0)$
and
$\gamma_{\widetilde{F\alpha]}}(x) = \gamma_{\widetilde{F[\alpha]}}(f(a)) = \gamma_{\widetilde{F[\alpha]}}{}^{f}(a) \geq \gamma_{\widetilde{F[\alpha]}}{}^{f}(0) = \gamma_{\widetilde{F[\alpha]}}(f(0)) = \gamma_{\widetilde{F[\alpha]}}(0)$.
Moreover,
$\mu_{\widetilde{F[\alpha]}}(x*(y*(y*x)))$
$= \mu_{\widetilde{F[\alpha]}}(f(a)*(f(b)*(f(b)*f(a))))$
$= \mu_{\widetilde{F[\alpha]}}(f(a)*(f(b)*(f(b*a))))$
$= \mu_{\widetilde{F[\alpha]}}(f(a)*(f(b*(b*a))))$
$= \mu_{\widetilde{F[\alpha]}}(f(a*(b*(b*a))))$
$= \mu_{\widetilde{F[\alpha]}}{}^{f}(a*(b*(b*a)))$
$\geq \min\left\{\mu_{\widetilde{F[\alpha]}}{}^{f}((a*b)*c), \mu_{\widetilde{F[\alpha]}}{}^{f}(c)\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}(f((a*b)*c)), \mu_{\widetilde{F[\alpha]}}(f(c))\right\}$

$= \min\left\{\mu_{\widetilde{F[\alpha]}}(f(a*b)*f(c)), \mu_{\widetilde{F[\alpha]}}(f(c))\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}((f(a)*f(b))*f(c)), \mu_{\widetilde{F[\alpha]}}(f(c))\right\}$
$= \min\left\{\mu_{\widetilde{F[\alpha]}}((x*y)*z), \mu_{\widetilde{F[\alpha]}}(z)\right\}$
and
$\gamma_{\widetilde{F[\alpha]}}(x*(y*(y*x)))$
$= \gamma_{\widetilde{F[\alpha]}}(f(a)*(f(b)*(f(b)*f(a))))$
$= \gamma_{\widetilde{F[\alpha]}}(f(a)*(f(b)*(f(b*a))))$
$= \gamma_{\widetilde{F[\alpha]}}(f(a)*(f(b*(b*a))))$
$= \gamma_{\widetilde{F[\alpha]}}(f(a*(b*(b*a))))$
$= \gamma_{\widetilde{F[\alpha]}}{}^{f}(a*(b*(b*a)))$
$\leq \max\left\{\gamma_{\widetilde{F[\alpha]}}{}^{f}((a*b)*c), \gamma_{\widetilde{F[\alpha]}}{}^{f}(c)\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}(f((a*b)*c)), \gamma_{\widetilde{F[\alpha]}}(f(c))\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}(f(a*b)*f(c)), \gamma_{\widetilde{F[\alpha]}}(f(c))\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}((f(a)*f(b))*f(c)), \gamma_{\widetilde{F[\alpha]}}(f(c))\right\}$
$= \max\left\{\gamma_{\widetilde{F[\alpha]}}((x*y)*z), \gamma_{\widetilde{F[\alpha]}}(z)\right\}.$
Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft commutative ideal of Y. □

5 Conclusion

In this paper, we introduced the notion of intuitionistic fuzzy soft commutative ideal, and investigated related properties. Meanwhile, we discussed relations between intuitionistic fuzzy soft commutative ideal and intuitionistic fuzzy soft ideal over a BCK-algebras, we also discuss some results of intuitionistic fuzzy soft commutative ideal in BCK-algebras. Based on these results, we will apply intuitionistic fuzzy soft sets to another type of ideals in BCK-algebras, and discuss the relevant results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgements

The works described in this paper are supported by the National Natural Science Foundation of China under Grant nos. 11501444, 11726019.

References

[1] Y.Imai, K.Iséki, On axiom system of propositional calculus, Proceedings of the Japan Academy 42(5)(1966)19-22.

[2] K.Iséki, An algebra related with a propositional calculus, Proceedings of the Japan Academy 42(1)(1966)26-29.

[3] K.Iséki, S.Tanaka, An introduction to the theory of BCK-algebra, Math. Japon 23(1978)1-26.

[4] L.A.Zadeh, Fuzzy sets, Information and control 8(3)(1965)338-353.

[5] O.G.Xi, Fuzzy BCK-algebras, Math. Japon 36(1991)935-942.

[6] D.Molodtsov, Soft set theory-First results, Computers and Mathematics with Applications 37(1999)19-31.

[7] H.Aktas, N.Agman, Soft sets and soft groups, Information Sciences 177(13)(2007)2726-2735.

[8] F.Feng, B.Y.Jun, X.Z.Zhao, Soft Semirings, Computers and Mathematics with Applications 56(10)(2008)2621-2628.

[9] Y.B.Jun, K.J.Le, C.H.Park, Soft Set Theory Applied to Ideals in d-algebras, Computers and Mathematics with Applications 57(3)(2009)367-378.

[10] K.T.Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and Systems 20(1)(1986)87-96.

[11] K.T.Atanassov, New operations defined over the intutionistic fuzzy sets, Fuzzy Sets and Systems 61(1994)137-142.

[12] J.Meng, Commutative ideals in BCK-algebras, Pure and Applied Mathematics 7(2)(1991)49-53.

[13] Y.B.Jun, D.S.Lee, C.H.Park, Intuitionistic fuzzy commutative ideals of BCK-algebras, The Pure and Applied Mathematics 15(1)(2008)73-84.

[14] G.Muhiuddin, D.Al-Kadi, H.Harizavi, A.S.Albjedi, Fuzzy soft set theory applied to commutative ideals of BCK-algebras, International Journal of Advanced and Applied Sciences 8(6)(2021)48-56.

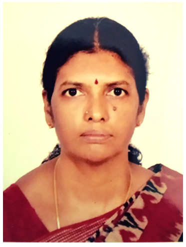
[15] M.Balamurugan, G.Balasubramanian, C.Ragavan, Intuitionistic Fuzzy Soft Ideals in BCK/BCI-algebras, Materials today: proceedings 16(2019)496-503.

[16] Y.B.Jun, E.H.Roh, Fuzzy commutative ideals of BCK-algebras, Fuzzy Sets and Systems 64(3)(1994)401-405.

[17] Y.B.Jun, K.H.Kim, Intuitionistic fuzzy ideals of BCK-algebras, International Journal of Mathematics and Mathematical Sciences 24(12)(2000)839-849.

[18] P.K.Maji, More on intuitionistic fuzzy soft sets, Lect. International Conference on Rough Sets 5908(2009)231-240.

BIOGRAPHY



Dr. V. Inthumathi received Doctoral degree in the field of Ideal topological spaces from Bharathiar University in 2012. She is working as an Associate Professor in Department of Mathematics in Nallamuthu Gounder Mahalingam college, Pollachi, India. She has 24 years of teaching experience. she has published about 45 research articles in reputed journals. She guided 23 M.Phil scholars and guiding 4 Ph.D. scholars. Currently she is doing research in the area of soft topological spaces and Nano ideal topological spaces.