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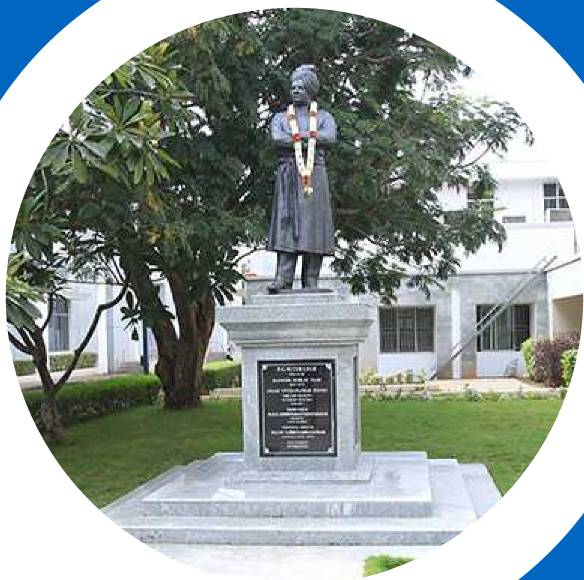

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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b-$\mathcal{H}_\beta$-open sets in HGTS

V. Chitra [1] **,R. Ramesh**[2]

Abstract - In this paper we introduce and study the notions of b-$\mathcal{H}_\beta$-open sets π^*-B-$\mathcal{H}_\beta$-sets, α^*-B-$\mathcal{H}_\beta$-sets and σ^*-B-$\mathcal{H}_\beta$-sets in hereditary generalized topological space. Also we obtained decompositions of (μ, λ)-continuity.

Keywords *hereditary generalized topology, α-$\mathcal{H}_\sigma$-open, σ-$\mathcal{H}_\sigma$-open and π-$\mathcal{H}_\sigma$-open sets, b-$\mathcal{H}_\sigma$-open sets.*
2010 Subject classification: *54A05*

1 Introduction and Preliminaries

In the year 2002, Csaszar [1] introduced very usefull notions of generalized topology and generalized continuity. Consider Z be a nonempty set and μ be a collection from the subsets of Z. Then μ is called a *generalized topology* (briefly GT) if $\emptyset \in \mu$ and an arbitrary union of elements from μ belongs to μ. A space Z is called a C_0-space [12], if $C_0 = Z$, where C_0 is the set of all representative elements of sets of μ. A subset L of a space (Z, μ) is called asμ-α-*open* [2] (resp. μ-σ-open [2], μ-π-open [2], μ-β-open [2], μ-b-open [11], μ-t-set [5], μ-t^*-set [5]), if $L \subset i_\mu c_\mu i_\mu(L)$ (resp. $L \subset c_\mu i_\mu(L)$, $L \subset i_\mu c_\mu(L)$, $L \subset c_\mu i_\mu c_\mu(L)$, $L \subset c_\mu i_\mu(L) \cup i_\mu c_\mu(L)$, $i_\mu c_\mu(L) = i_\mu(L)$, $i_\mu c_\mu i_\mu(L) = i_\mu(L)$). A subset L of Z is μ-locally closed set [4], $L = U \cap V$, where U is μ-open and V is μ-closed. A GTS (Z, μ) is called μ-extremally disconnected [**?**], if the μ-closure of every μ-open set is μ-open. A function $f : (Z, \mu) \to (W, \lambda)$ is said to be (μ, λ)-continuous [1], iff $M \in \lambda$ implies that $f^{-1}(M)$ is μ-open. A nonempty family $\mathcal{H}$ of subsets of Z is called as a *hereditary class* [3], if $L \in \mathcal{H}$ and $B \subset L$, then $B \in \mathcal{H}$. For each $L \subseteq Z$, $L^*(\mathcal{H}, \mu)$=$\{z \in Z : L \cap V \notin \mathcal{H}$ for All $V \in \mu$ such that $z \in V\}$[3]. For $L \subset Z$, define $c^*_\mu(L) = L \cup L^*(\mathcal{H}, \mu)$ and $\mu^* = \{L \subset Z : Z - L = c^*_\mu(Z - L)\}$. If $\mathcal{H}$ is a hereditary class on Z then $(Z, \mu, \mathcal{H})$ is called a hereditary generalized topological space ($H.G.T.S.$)

Definition 1.1. *[6] Consider L be a subset of $H.G.T.S.$ $(Z, \mu, \mathcal{H})$. Then $L^*_\beta(\mathcal{H}, \mu)$=$\{z \in Z : L \cap V \notin \mathcal{H}$ for all $V \in \mu$-β-open such that $z \in V\}$.*

Definition 1.2. *[3] A subset L of a $H.G.T.S.$ $(Z, \mu, \mathcal{H})$ is said to be*

1. *α-$\mathcal{H}$-open, if $L \subseteq i_\mu c^*_\mu i_\mu(L)$,*
2. *σ-$\mathcal{H}$-open, if $L \subseteq c^*_\mu i_\mu(L)$,*
3. *π-$\mathcal{H}$-open, if $L \subseteq i_\mu c^*_\mu(L)$,*
4. *β-$\mathcal{H}$-open, if $L \subseteq c_\mu i_\mu c^*_\mu(L)$,*
5. *strong β-$\mathcal{H}$-open, if $L \subseteq c^*_\mu i_\mu c^*_\mu(L)$,*
6. *μ^*-closed, if $c^*_\mu(L) \subset L$.*

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Definition 1.3. *A subset L of a H.G.T.S. $(Z, \mu, \mathcal{H})$ is said to be b-$\mathcal{H}$-open [7], if $L \subseteq i_\mu c^*_\mu(L) \cup c^*_\mu i_\mu(L)$.*

Proposition 1.4. *[6] Let L be a μ-σ-closed. Then $L^*_\sigma \subset L$.*

Let $(Z, \mu, \mathcal{H})$ be a hereditary generalized topological space. For $L \subset Z$, define $c^*_\beta(L) = L \cup L^*_\beta$ [6] and $c^*_\beta(L)$ is enlarging, monotone and idempotent.

Definition 1.5. *[8] A subset L of a H.G.T.S. $(Z, \mu, \mathcal{H})$ is said to be*

1. *α-$\mathcal{H}_\sigma$-open, if $L \subseteq i_\mu c^*_\sigma i_\mu(L)$,*
2. *σ-$\mathcal{H}_\sigma$-open, if $L \subseteq c^*_\sigma i_\mu(L)$,*
3. *π-$\mathcal{H}_\sigma$-open, if $L \subseteq i_\mu c^*_\sigma(L)$,*
4. *β-$\mathcal{H}_\sigma$-open, if $L \subseteq c_\mu i_\mu c^*_\sigma(L)$.*

Definition 1.6. *[10] A subset L of a H.G.T.S. $(Z, \mu, \mathcal{H})$ is said to be*

1. *α-$\mathcal{H}_\beta$-open, if $L \subseteq i_\mu c^*_\beta i_\mu(L)$,*
2. *σ-$\mathcal{H}_\beta$-open, if $L \subseteq c^*_\beta i_\mu(L)$,*
3. *π-$\mathcal{H}_\beta$-open, if $L \subseteq i_\mu c^*_\beta(L)$,*
4. *β-$\mathcal{H}_\beta$-open, if $L \subseteq c_\mu i_\mu c^*_\beta(L)$.*

Definition 1.7. *[10] A function $f : (Z, \mu, \mathcal{H}) \to (W, \lambda)$ is said to be $(\alpha$-$\mathcal{H}_\beta, \lambda)$-continuous (resp.$(\sigma$-$\mathcal{H}_\beta, \lambda)$-continuous, $(\pi$-$\mathcal{H}_\beta, \lambda)$-continuous), $f^{-1}(N)$ is α-$\mathcal{H}_\beta$-open (resp. σ-$\mathcal{H}_\beta$-open, π-$\mathcal{H}_\beta$-open) for each λ-open set N in (W, λ).*

Definition 1.8. *[9] A function $f : (Z, \mu, \mathcal{H}) \to (W, \lambda)$ is said to be $(\mu L, \lambda)$-continuous, if $f^{-1}(N)$ is μ-locally closed set for each λ-closed set N in (W, λ).*

2 b-$\mathcal{H}_\beta$-open sets

Definition 2.1. *A subset L of a H.G.T.S. $(Z, \mu, \mathcal{H})$ is said to be b-$\mathcal{H}_\beta$-open set, if $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L)$.*

Proposition 2.2. *In H.G.T.S. $(Z, \mu, \mathcal{H})$ all μ-open set is b-$\mathcal{H}_\beta$-open but not conversely.*

Proof. Let a subset L of H.G.T.S. $(Z, \mu, \mathcal{H})$ is μ-open. Then $L = i_\mu(L)$. Now $L \subseteq i_\mu(L) \subseteq i_\mu c^*_\beta(L) \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L)$. Hence L is b-$\mathcal{H}_\beta$-open.

Example 2.3. *Consider $Z = \{a, b, c, d\}$ $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}, Z\}$, $\mathcal{H} = \{\emptyset, \{a\}, \}$. Then $L = \{a, b, c\}$ is b-$\mathcal{H}_\beta$-open but not μ-open.*

Proposition 2.4. *Every b-$\mathcal{H}_\beta$-open is μ-b-open but not conversely.*

Proof. Let L be a b-$\mathcal{H}_\beta$-open. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\mu(L) \cup c^*_\mu i_\mu(L) \subseteq i_\mu c_\mu(L) \cup c_\mu i_\mu(L)$. Hence L is μ-b-open.

Proposition 2.5. *Every b-$\mathcal{H}_\beta$-open is b-$\mathcal{H}$-open but not conversely.*

Proof. Let L be a b-$\mathcal{H}_\beta$-open. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\mu(L) \cup c^*_\mu i_\mu(L)$. Hence L is b-$\mathcal{H}$-open.

Example 2.6. *Consider $Z = \{a,b,c,d,e\}$ $\mu = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\},$ $\{b,c\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}, \{a,b,c,d\}\}$, $\mathcal{H} = \{\emptyset, \{a\}\}$. Then $L = \{e\}$ is μ-b-open but not b-$\mathcal{H}_\beta$-open and $M = \{a,e\}$ is b-$\mathcal{H}$-open but not b-$\mathcal{H}_\beta$-open.*

Theorem 2.7. *If $L \subset Z$ is both b-$\mathcal{H}_\beta$-open and μ-σ-closed, then it is σ-$\mathcal{H}_\sigma$-open.*

Proof. Let L is both b-$\mathcal{H}_\beta$-open and μ-σ-closed. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L)$ and $c^*_\sigma(L) \subseteq L$ by Proposition 2.9 of [6]. Which implies $i_\mu c^*_\sigma(L) \subseteq i_\mu(L)$. Now $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L) \subseteq c^*_\sigma i_\mu(L) \cup i_\mu(L) = c^*_\sigma i_\mu(L)$. Hence σ-$\mathcal{H}_\sigma$-open.

Theorem 2.8. *If $L \subset Z$ is b-$\mathcal{H}_\beta$-open such that $i_\mu(L) = \emptyset$, then it is π-$\mathcal{H}_\sigma$-open.*

Proof. Let L be a b-$\mathcal{H}_\beta$-open and $i_\mu(L) = \emptyset$. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L) = i_\mu c^*_\sigma(L)$. Hence L is π-$\mathcal{H}_\sigma$-open.

Theorem 2.9. *If $L \subset Z$ is b-$\mathcal{H}_\beta$-open and $L \in \mathcal{H}$, then it is σ-$\mathcal{H}_\sigma$-open.*

Proof. Let L is b-$\mathcal{H}_\beta$-open and $L \in \mathcal{H}$. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L)$ and $c^*_\sigma(L) = L$ by Remark 2.10 of [6]. Now $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L) = i_\mu(L) \cup c^*_\sigma i_\mu(L) = c^*_\sigma i_\mu(L)$. Hence L is σ-$\mathcal{H}_\sigma$-open.

Theorem 2.10. *If $L \subset Z$ is b-$\mathcal{H}_\beta$-open and $\mathcal{H} = P(Z)$ then it is σ-$\mathcal{H}_\sigma$-open.*

Proof. Let L is b-$\mathcal{H}_\sigma$-open and $L \in \mathcal{H}$. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L)$ and $c^*_\sigma(L) = L$ by Remark 2.10 of [6]. Now $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L) = i_\mu(L) \cup c^*_\sigma i_\mu(L) = c^*_\sigma i_\mu(L)$. Hence L is σ-$\mathcal{H}_\sigma$-open.

Theorem 2.11. *If $L \subset Z$ is b-$\mathcal{H}_\beta$-open and $L \subset L^*_\beta$, then it is μ-β-open.*

Proof. Let L is b-$\mathcal{H}_\beta$-open and $L \subset L^*_\beta$. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L)$ and $c^*_\beta i_\mu(L) \subset c^*_\beta i_\mu c^*_\beta(L)$. Now $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu c^*_\beta(L) \subseteq c^*_\beta i_\mu c^*_\beta(L) \subseteq c^*_\mu i_\mu c^*_\mu(L) \subseteq c_\mu i_\mu c_\mu(L)$. Hence L is μ-β-open.

Theorem 2.12. *Let $(Z, \mu, \mathcal{H})$ be a strong H.G.T.S., where Z is C_0-space and μ-extremally disconnected space, $L \subset Z$. Then the following conditions are equivalent.*

1. *L is μ-open,*
2. *L is b-$\mathcal{H}_\beta$-open and μ-locally closed set.*

Proof. (1) $\Rightarrow$ (2) This is obvious from definitions.
(2) $\Rightarrow$ (1) Let L is b-$\mathcal{H}_\beta$-open and μ-locally closed set. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c_\mu(L) \cup c_\mu i_\mu(L)$ and $L = U \cap c_\mu(L)$. Now
$L \subset U \cap c_\mu(L)$.
$\subset U \cap [i_\mu c_\mu(L) \cup c_\mu i_\mu(L)]$
$\subset [U \cap i_\mu c_\mu(L)] \cup [U \cap c_\mu i_\mu(L)]$
$\subset [i_\mu(U) \cap i_\mu c_\mu(L)] \cup [i_\mu(U) \cap c_\mu i_\mu(L)]$
$\subset [i_\mu(U) \cap i_\mu c_\mu(L)] \cup [i_\mu(U) \cap i_\mu c_\mu(L)]$
$\subset [i_\mu(U \cap c_\mu(L))] \cup [i_\mu(U \cap c_\mu(L))]$
$= [i_\mu(L)] \cup [i_\mu(L)]$
$= i_\mu(L)$.
Hence L is μ-open.

Remark 2.13. *The notions of b-$\mathcal{H}_\beta$-open sets and μ-locally closed sets independent.*

Example 2.14. *Let $Z = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$ and $\mathcal{H} = \{\emptyset, \{a\}\}$. Then $L = \{a, d\}$ is μ-locally closed set but not b-$\mathcal{H}_\beta$-open.*

Example 2.15. *Let $Z = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a, b, c\}, \{c, d\}, Z\}$ and $\mathcal{H} = \{\emptyset, \{a\}, \{b\}\}$. Then $L = \{a, c, d\}$ is b-$\mathcal{H}_\beta$-open but not μ-locally closed set.*

3 π^*-B-$\mathcal{H}_\beta$-sets, α^*-B-$\mathcal{H}_\beta$-sets and σ^*-B-$\mathcal{H}_\beta$-sets

Definition 3.1. *A subset L of a H.G.T.S. $(Z, \mu, \mathcal{H})$ is called*

1. *π^*-$\mathcal{H}_\sigma$-set, if $i_\mu c^*_\beta(L) = i_\mu(L)$.*
2. *α^*-$\mathcal{H}_\sigma$-set, $i_\mu c^*_\beta i_\mu(L) = i_\mu(L)$.*
3. *σ^*-$\mathcal{H}_\sigma$-set, if $c^*_\beta i_\mu(L) = i_\mu(L)$.*

Remark 3.2. *The notions of π^*-$\mathcal{H}_\beta$-set (resp. α^*-$\mathcal{H}_\beta$-set, σ^*-$\mathcal{H}_\beta$-set) and π-$\mathcal{H}_\beta$-open (resp. α-$\mathcal{H}_\beta$-open, σ-$\mathcal{H}_\beta$-open) are independent.*

Example 3.3. *Let $Z = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$ and $\mathcal{H} = \{\emptyset, \{a\}\}$. Then $A = \{a, d\}$ is π^*-$\mathcal{H}_\beta$-set (resp. α^*-$\mathcal{H}_\beta$-set, σ^*-$\mathcal{H}_\beta$-set) but not π-$\mathcal{H}_\beta$-open (resp. α-$\mathcal{H}_\beta$-open, σ-$\mathcal{H}_\beta$-open).*

Example 3.4. *let $Z = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}, Z\}$ and $\mathcal{H} = \{\emptyset, \{a\}, \{c\}\}$. Then $A = \{a, b, c\}$ is π-$\mathcal{H}_\beta$-open (resp. α-$\mathcal{H}_\beta$-open, σ-$\mathcal{H}_\beta$-open) but not π^*-$\mathcal{H}_\beta$-set (resp. α^*-$\mathcal{H}_\beta$-set, σ^*-$\mathcal{H}_\beta$-set).*

Definition 3.5. *A subset L of H.G.T.S. $(Z, \mu, \mathcal{H})$ is called*

1. *π^*-B-$\mathcal{H}_\beta$-set, if $L = M \cap N$, where M is μ-open and N is π^*-$\mathcal{H}_\beta$-set.*
2. *α^*-B-$\mathcal{H}_\beta$-set, if $L = M \cap N$, where M is μ-open and N is α^*-$\mathcal{H}_\beta$-set.*
3. *σ^*-B-$\mathcal{H}_\beta$-set, if $L = M \cap N$, where M is μ-open and N is σ^*-$\mathcal{H}_\beta$-set*

Theorem 3.6. *If $L \subset Z$ is both b-$\mathcal{H}_\beta$-open and π^*-$\mathcal{H}_\sigma$-set, then it is σ-$\mathcal{H}_\sigma$-open.*

Proof. Let L is both b-$\mathcal{H}_\beta$-open and π^*-$\mathcal{H}_\sigma$-set. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L)$ and $i_\mu c^*_\sigma(L) = i_\mu(L)$. Now $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L) \subseteq c^*_\sigma i_\mu(L) \cup i_\mu(L) = c^*_\sigma i_\mu(L)$. Hence L is σ-$\mathcal{H}_\sigma$-open.

Theorem 3.7. *If $L \subset Z$ is both b-$\mathcal{H}_\beta$-open and σ^*-$\mathcal{H}_\sigma$-set, then it is π-$\mathcal{H}_\sigma$-open.*

Proof. Let L is both b-$\mathcal{H}_\beta$-open and σ^*-$\mathcal{H}_\sigma$-set. Then $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L)$ and $c^*_\sigma i_\mu(L) = i_\mu(L)$. Now $L \subseteq i_\mu c^*_\beta(L) \cup c^*_\beta i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup c^*_\sigma i_\mu(L) \subseteq i_\mu c^*_\sigma(L) \cup i_\mu(L) = i_\mu c^*_\sigma(L)$. Hence L is π-$\mathcal{H}_\sigma$-open.

Proposition 3.8. *Let $(Z, \mu, \mathcal{H})$ be a strong H.G.T.S. and $L \subset Z$. Then the following holds:*

1. *If L is π^*-$\mathcal{H}_\beta$-set, then L is π^*-B-$\mathcal{H}_\beta$-set,*

2. If L is α^*-$\mathcal{H}_\beta$-set, then L is α^*-B-$\mathcal{H}_\beta$-set,

3. If L is σ^*-$\mathcal{H}_\beta$-set, then L is σ^*-B-$\mathcal{H}_\beta$-set.

Proof.

1. Let L be a π^*-$\mathcal{H}_\beta$-set. If we take $M = Z \in \mu$, then $L = M \cap L$ and hence L is a π^*-B-$\mathcal{H}_\beta$-set.
2. This is obvious.
3. Trivial.

Theorem 3.9. *Let $(Z, \mu, \mathcal{H})$ be a strong H.G.T.S., where Z is C_0-space and $L \subset Z$. Then the following conditions are equivalent.*

1. L is μ-open,

2. L is π-$\mathcal{H}_\beta$-open and π^*-B-$\mathcal{H}_\beta$-set,

3. α-$\mathcal{H}_\beta$-open and α^*-B-$\mathcal{H}_\beta$-set,

4. σ-$\mathcal{H}_\beta$-open and σ^*-B-$\mathcal{H}_\beta$-set.

Proof. (1) $\Rightarrow$ (2), (1) $\Rightarrow$ (3), (1) $\Rightarrow$ (4), are obvious.
(2) $\Rightarrow$ (1). Let L is both π-$\mathcal{H}_\beta$-open and π^*-B-$\mathcal{H}_\beta$-set. Then we have $L \subseteq i_\mu c^*_\beta(L) = i_\mu c^*_\beta(M \cap N)$, where $M \in \mu$ and N is π^*-$\mathcal{H}_\beta$-set. Hence $L \subseteq i_\mu c^*_\beta(M) \cap i_\mu c^*_\beta(N)$. Now $L \subseteq M \cap L \subseteq M \cap [i_\mu c^*_\beta(M) \cap i_\mu(N)] = M \cap i_\mu(N) = i_\mu(L)$. Hence L is μ-open.
(3) $\Rightarrow$ (1). Let L is both α-$\mathcal{H}_\beta$-open and α^*-B-$\mathcal{H}_\beta$-set. Then we have $L \subseteq i_\mu c^*_\beta i_\mu(L) = i_\mu c^*_\beta i_\mu(M \cap N)$, where $M \in \mu$ and N is t^*-$\mathcal{H}_\beta$-set. Hence $L \subseteq i_\mu c^*_\beta i_\mu(M) \cap i_\mu c^*_\beta i_\mu(N)$. Now $L \subseteq M \cap L \subseteq M \cap [i_\mu c^*_\beta i_\mu(M) \cap i_\mu(N)] = M \cap i_\mu(N) = i_\mu(L)$. Hence L is μ-open.
(3) $\Rightarrow$ (1). Let L is both σ-$\mathcal{H}_\beta$-open and σ^*-B-$\mathcal{H}_\beta$-set. Then we have $L \subseteq c^*_\beta i_\mu(L) = c^*_\beta i_\mu(M \cap N)$, where $M \in \mu$ and N is β^*-B-$\mathcal{H}_\beta$-set. Hence $L \subseteq c^*_\beta i_\mu(M) \cap c^*_\beta i_\mu(N)$. Now $L \subseteq M \cap L \subseteq M \cap [c^*_\beta i_\mu(M) \cap i_\mu(N)] = M \cap i_\mu(N) = i_\mu(L)$. Hence L is μ-open.

Remark 3.10. *The notions of π-$\mathcal{H}_\beta$-open and π^*-B-$\mathcal{H}_\beta$-set are independent.*

Remark 3.11. *The notions of α-$\mathcal{H}_\beta$-open and α^*-B-$\mathcal{H}_\beta$-set are independent.*

Remark 3.12. *The notions of σ-$\mathcal{H}_\beta$-open and σ^*-B-$\mathcal{H}_\beta$-set are independent.*

Example 3.13. *Let $Z = \{a, b, c, d, e\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$ and $\mathcal{H} = \{\emptyset, \{a\}\}$. Then $A = \{c, d\}$ is π^*-B-$\mathcal{H}_\beta$-set (resp. α^*-B-$\mathcal{H}_\beta$-set, σ^*-B-$\mathcal{H}_\beta$-set) but not π-$\mathcal{H}_\beta$-open (resp. α-$\mathcal{H}_\beta$-open, σ-$\mathcal{H}_\beta$-open).*

Example 3.14. *let $Z = \{a, b, c, d\}$, $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c, d\}, Z\}$ and $\mathcal{H} = \{\emptyset, \{a\}, \{c\}\}$. Then $A = \{a, b, c\}$ is π-$\mathcal{H}_\beta$-open (resp. α-$\mathcal{H}_\beta$-open, σ-$\mathcal{H}_\beta$-open) but not π^*-B-$\mathcal{H}_\beta$-set (resp. α^*-B-$\mathcal{H}_\beta$-set, σ^*-B-$\mathcal{H}_\beta$-set).*

4 Decomposition of (μ, λ)-continuity

Definition 4.1. *A function $f : (Z, \mu, \mathcal{H}) \to (W, \lambda)$ is said to be $(\pi^*$-B-$\mathcal{H}_\beta, \lambda)$-continuous (resp.$(\alpha^*$-$B$-$\mathcal{H}_\beta, \lambda)$-continuous, $(\sigma^*$-B-$\mathcal{H}_\beta, \lambda)$-continuous, if $f^{-1}(N)$ is π^*-B-$\mathcal{H}_\beta$-set (resp. α^*-B-$\mathcal{H}_\beta$-set, σ^*-B-$\mathcal{H}_\beta$-set, for each λ-open set N in (W, λ).*

Theorem 4.2. *Let $(Z, \mu, \mathcal{H})$ be a strong H.G.T.S. where Z is C_0-space and μ-extremally disconnected space, for a function $f : (Z, \mu, \mathcal{H}) \to (W, \lambda)$. Then the following conditions are equivalent.*

1. *f is (μ, λ)-continuous,*
2. *f is $(b$-$\mathcal{H}_\beta, \lambda)$-continuous and $(\mu L, \lambda)$-continuous.*

Proof. Proof is trivial from Theorem 2.12.

Theorem 4.3. *Let $(Z, \mu, \mathcal{H})$ be a strong H.G.T.S., for a function $f : (Z, \mu, \mathcal{H}) \to (W, \lambda)$. Then the following conditions are equivalent.*

1. *f is (μ, λ)-continuous,*
2. *f is $(\pi$-$\mathcal{H}_\beta, \lambda)$-continuous and $(\pi^*$-B-$\mathcal{H}_\beta, \lambda)$-continuous,*
3. *f is $(\alpha$-$\mathcal{H}_\beta, \lambda)$-continuous and $(\alpha^*$-B-$\mathcal{H}_\beta, \lambda)$-continuous,*
4. *f is $(\sigma$-$\mathcal{H}_\beta, \lambda)$-continuous and $(\sigma^*$-B-$\mathcal{H}_\beta, \lambda)$-continuous.*

Proof. Proof is trivial from Theorem 3.9.

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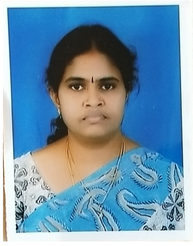

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