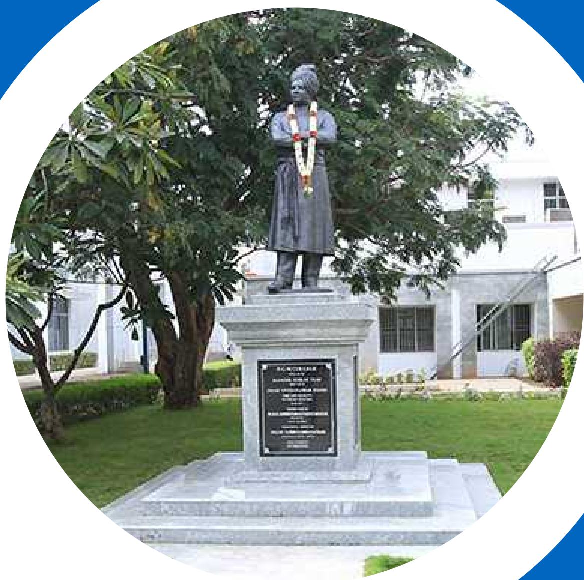
VOLUME XII
ISBN No.: 978-93-94004-01-6
Physical Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

Pollachi-642001



SUPPORTED BY

ABT Industries Ltd.,
Dairy Division

PROCEEDING

One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, Pollachi-642001.

Estd. 1957

Proceeding of the

One day International Conference on

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

Copyright © 2021 by Nallamuthu Gounder Mahalingam College

All Rights Reserved

ISBN No: 978-93-94004-01-6

978- 93- 94004- 01- 6

Nallamuthu Gounder Mahalingam College

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

www.ngmc.org

ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

EDITORIAL BOARD

Dr. V. Inthumathi

Associate Professor & Head, Dept. of Mathematics, NGM College

Dr. J. Jayasudha

Assistant Professor, Dept. of Mathematics, NGM College

Dr. R. Santhi

Assistant Professor, Dept. of Mathematics, NGM College

Dr. V. Chitra

Assistant Professor, Dept. of Mathematics, NGM College

Dr. S. Sivasankar

Assistant Professor, Dept. of Mathematics, NGM College

Dr. S. Kaleeswari

Assistant Professor, Dept. of Mathematics, NGM College

Dr. N.Selvanayaki

Assistant Professor, Dept. of Mathematics, NGM College

Dr. M. Maheswari

Assistant Professor, Dept. of Mathematics, NGM College

Mrs. A. Gnanasoundari

Assistant Professor, Dept. of Mathematics, NGM College

Dr. A.G. Kannan

Assistant Professor, Dept. of Physics, NGM College

THE GEODETIC NUMBER IN COMB PRODUCT OF GRAPHS

Dr. S. Sivasankar[1] **M. Gnanasekar**[2]

[1] *Assistant Professor, Department of Mathematics, NGM College, Pollachi*

sssankar@gmail.com

[2] *Research Scholar, Department of Mathematics, NGM College, Pollachi*

gnanasekar.kalam@gmail.com

Abstract

A set of vertices S of a graph G is a geodetic set, if every vertex of graph G lies in at least one interval between the vertices of S. The minimum size of a geodetic set in G is the geodetic number of G. The comb product of a two connected graphs G and H at vertex $o \in V(H)$, denoted by $G \triangleright_o H$, is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and identifying the i^{th} copy of H at the vertex o to the i^{th} vertex of G. In this paper, we determine an exact value of geodetic number in some classes of graphs.

Keywords: Distance, Geodesic, Geodetic Number , Geodetic closer, Comb product

Subject Classification: Graph Theory : 05C69

1.2 Introduction

By a simple and connected graph graph $G = (V, E)$, We further assume that G has no isolated vertices. For graph theoretic terminology we refer to Chartrand and Lesniak [8].

The notions of distance in graphs is a well-studied topic with several practical applications. For any two vertices u and v of a connected graph G, the distance $d_G(u, v)$ is the length of a shortest $u-v$ path in G. The eccentricity of a vertex u of a graph G is the maximum distance between u and any other vertex of G. The diameter of G, denoted by $diam(G)$, is the maximum eccentricity of vertices in G, and the radius is the minimum such eccentricity. The interval $I_G[u, v]$ between u and v is the set of all vertices on all shortest $u - v$ paths. Given a set $S \subseteq V(G)$, its geodetic closure $I_G[S]$ is the set of all vertices lying on some shortest path joining two vertices of S; that is,

$$I_G[S] = \{v \in V(G) : v \in I_G[x, y], x, y \in S\} = \bigcup_{x,y \in S} I_G[x, y].$$

Dr. S. Sivasankar[1] $Assistant Professor, Dept. of Mathematics, NGM College, Pollachi, E-mail: sssankar@gmail.com$
M.Gnanasekar[2], $Research Scholar, Dep. of Mathematics, NGM College, Pollachi, E-mail: gnanasekar.kalam@gmail.com$

A set $S \subseteq V(G)$ is called a geodetic set in G if $I_G[S] = V(G)$; that is, every vertex in G lies on some geodesic between two vertices from S. The geodetic number $g(G)$ of a graph G is the minimum cardinality of a geodetic set in G.

In chemistry [1], some families of chemical graphs can be measured as the comb product graphs. Let G and H be two connected graphs. Let o be a vertex of H. The comb product between G and H, denoted by $G \triangleright_o H$ H, is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and identify the i^{th} copy of H at the vertex o with the i^{th} vertex of G. By the definition of comb product, we can say that $V(G \triangleright_o H) = (a, v)|a \in V(G), v \in V(H)$ and $(a, v)(b, w) \in E(G \triangleright_o H)$ whenever $a = b$ and $vw \in E(H)$, or $ab \in E(G)$ and $v = w = o$. We consider two different vertices $a, b \in V(G)$ and a vertex $o \in V(H)$. We define $H(a) = (a, v)|v \in V(H)$, $G(o) = (v, o)|v \in V(G)$, and $P_G(a, b)$ is the shortest path from a to b in G.

We find four main results. The first result is related to $G \triangleright_o H$ when G is a connected graph and H is the classes of graphs such as Path, Cycle, Completed Graph, Trees and Wheel Graph. In the literature, the problem of the geodetic number of a graph was initiated by Harary, Loukakis, and Tsouros in 1986 and their result appeared as a published paper in 1993[10]. we collect the basic definitions in graphs and geodetic sets which are used in the subsequent chapters; for graph theoretic terminology we refer to the Chartrand and Lesniak [8]. few basic results on the geodetic number of cartesian and strong product of graphs. These results are in Bresar et al. [3] and in Caceres et al. [5].

Lemma 1.1. *[6] Every geodetic set of a graph contains its extreme vertices.*

Lemma 1.2. *[9] Let* $\deg : V(A) \to V(B)$ *be an isomorphism between graphs* A *and* B*. The set* S *is a geodetic set of* A *if and only if* $\deg(x)|x \in S$ *is a geodetic set of* B

Lemma 1.3. *[9] Let* $o \in V(H)$ *be the identifying vertex and* u, v *be two distinct vertices of* $G \triangleright_o H$*. For* $l \in 1, 2, ..., n,$ *if* $u \in V_l$ *and* $v \notin V_l$*, then every uv path in* $G \triangleright_o H$ *consists of* (g_l, o)*.*

Lemma 1.4. *[9] Let* $o \in V(H)$ *be the identifying vertex and* S *be a geodetic set of* $G \triangleright_o H$*. Then for* $l \in 1, 2, ..., n, (S \cap V_l) \cup (g_l, h_o)$ *is a geodetic set of* $G \triangleright_o H[V_l]$*.*

2.2 Geodetic Number for Comb product of graph

Let $C_P = G \triangleright_o H$. Let $V(G) = \{u_1, u_2, u_3, ..., u_m\}$ and $V(P_n) = \{v_1, v_2, v_3, ..., v_n\}$. From the definition of the Comb product of graphs $V(G \triangleright_o H) = (a, v)|a \in V(G), v \in V(H)$ and $(a, v)(b, w) \in E(G \triangleright_o H)$

Dr. S. Sivasankar[1] $Assistant Professor, Dept. of Mathematics, NGM College, Pollachi, E-mail$: sssankar@gmail.com
M.Gnanasekar[2], $Research Scholar, Dep. of Mathematics, NGM College, Pollachi, E-mail$: gnanasekar.kalam@gmail.com

whenever $a = b$ and $vw \in E(H)$, or $ab \in E(G)$ and $v = w = o$. We consider two different vertices $a, b \in V(G)$ and a vertex $o \in V(H)$. We define $H(a) = (a, v)|v \in V(H)$, $G(o) = (v, o)|v \in V(G)$, and $P_G(a, b)$ is the shortest path from a to b in G

Lemma 2.5. *Let $C_P = G \triangleright_o H$ and let $S \in V(G)$, if any vertex of S is identifying in $V(H)$ then $I_G[S] = V(G \triangleright_o H)$*

Proof. Let $C_P = G \triangleright_o H$ and let $S \in V(G)$, let us assume that $\{u, v, w\} \in S$ by taking the comb product of $V(G \triangleright_o H) = (a, v)|a \in V(G), v \in V(H)$ and $(a, v)(b, w) \in E(G \triangleright_o H)$ whenever $a = b$ and $vw \in E(H)$, or $ab \in E(G)$ and $v = w = o$. We consider two different vertices $a, b \in V(G)$ and a vertex $o \in V(H)$. We define $H(a) = (a, v)|v \in V(H)$, $G(o) = (v, o)|v \in V(G)$, and $P_G(a, b)$ is the shortest path from a to b in G. So that $V(H)$ is an interior vertex of a shortest path between a vertex of S. □

We obtain four main results. The first result is related to $G \triangleright_o H$ when G is a connected graph and H is the family of graphs such as Path, Cycle, Completed Graph, Trees and Wheel Graph.

2.2 Geodetic Number for Comb product of graph G and H

Theorem 2.1. *$g(G \triangleright_o P_n) = m.g(G) - m$, where $m, n \geq 2$*

Proof. Let $C_P = G \triangleright_o P_n$ and let $V(G) = \{u_1, u_2, ..., u_m\}$ and $V(P_n) = \{v_1, v_2, ..., v_n\}$, let us assume that v_i and v_j be a pendent vertices. then we know that $S = \{v_i, v_j\}$ By the definition of comb product we can identifying $V(G)$ copies of H. Let us take v_i is an identifying vertex of all the copies. Then the resultant graph gives $V(G)$ copies of pendent vertices. Already we know that all the pendent vertices should be belongs to a geodetic set. by the Lemma 2.1 $V(G)$ copies of pendent vertices is geodetic closure of $V(H)$. Therefore $| S |= g(G).m - m$ □

In this way we can proof the following corollary.

Corollary 2.1. *$g(G \triangleright_o P_n) = m.g(G) - m$, where $m, n \geq 2$*

Theorem 2.2. *$g(G \triangleright_o C_n) = m.g(G) - m$, where $m, n \geq 3$*

Proof. Let $C_P = G \triangleright_o C_n$ and let $V(G) = \{u_1, u_2, ..., u_m\}$ and $V(C_n) = \{v_1, v_2, ..., v_n\}$

Case i: If n is even

Let us assume that H is an even cycle then the geodetic set of H is $S = \{v_1, v_{\frac{n}{2}}\}$. By the definition of comb product we can identifying $V(G)$ copies of H. Let us take v_1 is an identifying vertex of all the copies. Then the resultant graph gives $V(G)$ copies of $v_{\frac{n}{2}}$ vertex in $G \triangleright_o C_n$. Already we know that vertex $v_{\frac{n}{2}}$ is center of the graph Hso that the shortest path between the $V(G)$ copies of $v_{\frac{n}{2}}$ is geodetic closure of $V(H)$. Therefore $| S |\leq g(G).m - m$.

We claim that $| S | | \geq g(G).m - m$, Suppose We assume that $\{S - v_{\frac{n}{2}}\}$ is geodetic set in $G \triangleright_o C_n$, and $v_{\frac{n}{2}} \in i^{th}$ copy $V(G)$ then that i^{th} copy intermediate vertices are does not belongs to the geodetic closure $I_{G \triangleright_o C_n}[S]$ by the definition of the geodetic set its a contradiction. So $| S | | \geq g(G).m - m$ Therefore $g(G \triangleright_o C_n) = m.g(G) - m$

Case ii: If n is odd

Let us assume that H is an odd cycle then the geodetic set of H is $S = \{v_1, v_{\frac{n}{2}, v_{\frac{n}{2}+1}}\}$. By the definition of comb product we can identifying $V(G)$ copies of H. Let us take v_1 is an identifying vertex of all the copies. Then the resultant graph gives $V(G)$ copies of $\{v_{\frac{n}{2}}, v_{\frac{n}{2}+1}\}$ vertex in $G \triangleright_o C_n$. Already we know that vertex shortest path between $(v_{\frac{n}{2}, v_{\frac{n}{2}+1}})$ geodetic closer H, so that the shortest path between the $V(G)$ copies of $(v_{\frac{n}{2}, v_{\frac{n}{2}+1}})$ is geodetic closure of $V(H)$. Therefore $|S| \leq g(G).m - m$

We claim that $| S | | \geq g(G).m - m$, Suppose We assume that $\{S - v_{\frac{n}{2}}\}$ is geodetic set in $G \triangleright_o C_n$, and $v_{\frac{n}{2}} \in i^{th}$ copy $V(G)$ then that i^{th} copy intermediate vertices are does not belongs to the geodetic closure $I_{G \triangleright_o C_n}[S]$ by the definition of the geodetic set its a contradiction. So $| S | | \geq g(G).m - m$ Therefore $g(G \triangleright_o C_n) = m.g(G) - m$

□

Theorem 2.3. *$g(G \triangleright_o K_n) = m.g(G) - m$, where $m, n \geq 3$*

Proof. Let $C_P = G \triangleright_o K_n$ and let $V(G) = \{u_1, u_2, ..., u_m\}$ and $V(K_n) = \{v_1, v_2, ..., v_n\}$, we know that $S \in V(G), S = \{v_1, v_2, ..., v_n\}$ By the definition of comb product we can identifying $V(G)$ copies of H. Let us take v_1 is an identifying vertex of all the copies. Then the resultant graph gives $V(G)$ copies of $\{S/v_1\}$. Already we know that distance between any vertex is 1 so that the Lemma 2.1 $V(G)$ copies of $\{S/v_1\}$ vertices is geodetic closure of $V(H)$. Therefore $| S |\leq g(G).m - m$

Dr. S. Sivasankar[1] $AssistantProfessor, Dept.of Mathematics, NGMCollege, Pollachi, E-mail: sssankar@gmail.com$
M.Gnanasekar[2], $ResearchScholar, Dep.of Mathematics, NGMCollege, Pollachi, E-mail: gnanasekar.kalam@gmail.com$

We claim that $| S | | \geq g(G).m - m$, Suppose We assume that $\{S - v_{\frac{n}{2}}\}$ is geodetic set in $G \triangleright_o C_n$, and $v_{\frac{n}{2}} \in i^{th}$ copy $V(G)$ then that i^{th} copy intermediate vertices are does not belongs to the geodetic closure $I_{G \triangleright_o C_n}[S]$ by the definition of the geodetic set its a contradiction. So $| S | | \geq g(G).m - m$ Therefore $g(G \triangleright_o C_n) = m.g(G) - m$

□

Theorem 2.4. *$g(G \triangleright_o W_{1,n}) = m.g(G) - m$, where $m, n \geq 4$*

Proof. Let $C_P = G \triangleright_o W_{1,n}$ and let $V(G) = \{u_1, u_2, ..., u_m\}$ and $V(W_{1,n}) = \{v_1, v_2, ..., v_n\}$, we know that $S \in V(G), S = \{v_1, v_3, v_5..., v_{n-1}\}$ By the definition of comb product we can identifying $V(G)$ copies of H. Let us take v_1 is an identifying vertex of all the copies. Then the resultant graph gives $V(G)$ copies of $\{S/v_1\}$. Already we know that distance between any vertex is 1 so that the Lemma 2.1 $V(G)$ copies of $\{S/v_1\}$ vertices is geodetic closure of $V(H)$. Therefore $| S | \leq g(G).m - m$

We claim that $| S | | \geq g(G).m - m$, Suppose We assume that $\{S - v_{\frac{n}{2}}\}$ is geodetic set in $G \triangleright_o C_n$, and $v_{\frac{n}{2}} \in i^{th}$ copy $V(G)$ then that i^{th} copy intermediate vertices are does not belongs to the geodetic closure $I_{G \triangleright_o C_n}[S]$ by the definition of the geodetic set its a contradiction. So $| S | | \geq g(G).m - m$ Therefore $g(G \triangleright_o C_n) = m.g(G) - m$

□

References

[1] M. Azari and A. Iranmanesh, Chemical graphs constructed from rooted product and their Zagreb indices, MATCH Commun. Math. Comput. Chem. 70 (2013), 901–919.

[2] B. Bresar, S. Klavzar and A. Douglas F. Rall, Dominating direct product of graphs, Discrete Math. 307 (2007) 1636-1642.

[3] B. Bresar, S. Klavzar and A. Tepeh Horvat, On the geodetic number and related metric sets in Cartesian product graphs, Discrete Math. 308 (2008) 5555-5561.

[4] B. Bresar, S. Klavzar, T. K. Sumenjak and A. Tepeh Horvat, The geodetic number of lexicographic product of graphs, Discrete Math. 311 (2011) 1693-1698.

Dr. S. Sivasankar[1] *AssistantProfessor, Dept.of Mathematics, NGMCollege, Pollachi, E-mail* : *sssankar@gmail.com*
M.Gnanasekar[2], *ResearchScholar, Dep.of Mathematics, NGMCollege, Pollachi, E-mail* : *gnanasekar.kalam@gmail.com*

[5] J. Caceres, C. Hernando, M. Mora, I. M. Pelayo, M. L. Puertas, On the geodetic and the hull numbers in strong product graphs, Comput. Math. Appl. 60 (2010) 3020-3031.

[6] G. Chartrand, F. Harary and P. Zhang, On the geodetic number of a graph, Diss. Math. Graph theory 20 (2000) 129-138.

[7] G. Chartrand, F. Harary and P. Zhang, Geodetic sets in graph, Networks 39 (2002) 1-6.

[8] G. Chartrand and L. Lesniak (2005), Graphs and Digraphs, Fourth Edition, CRC Press, Boca Raton.

[9] Dimas Agus Fahrudin, Suhadi Wido Saputro (2020), The geodetic-dominating number of comb product graphs, Electronic Journal of Graph Theory and Applications 8 (2) (2020), 373–381.

[10] F. Harary, E. Loukakis and C. Tsouros, The geodetic number of a graph, Math. Comput. Modelling 17 (1993) 89-95.

[11] T. Jiang, I. Pelayo and D. Pritikin, Geodesic convexity and Cartesian products in graphs, manuscript, 2004.

[12] W. Imrich and S. Klavzar. Products graphs: Structure and Recognition. Wiley-Interscience, New York, 2000.

Dr. S. Sivasankar, presently working as an Assistant Professor in Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi, Tamilnadu, India. He has a Ph.D., in Mathematics from AnnamalaiUniversity, Annamalainagar. He has around a decade of Teaching experience and two decades of research experience, and the area of research is Graph Theory. He guided more than 14 M.Phil. Students and presently 03 of them doing Ph.D. degree.

Mr. M. Gnanasekar, presently pursuing Ph.D in Mathematics under the guidance of Dr. S. Sivasankar, Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi, Tamilnadu, India. Also, He is working as Assistant Professor in Department of Mathematics, Rathinam College of Arts and Science. Echanari, Coimbatore.

Dr. S. Sivasankar[1] *AssistantProfessor, Dept.ofMathematics, NGMCollege, Pollachi, E − mail : sssankar@gmail.com*
M.Gnanasekar[2], *ResearchScholar, Dep.ofMathematics, NGMCollege, Pollachi, E − mail : gnanasekar.kalam@gmail.com*