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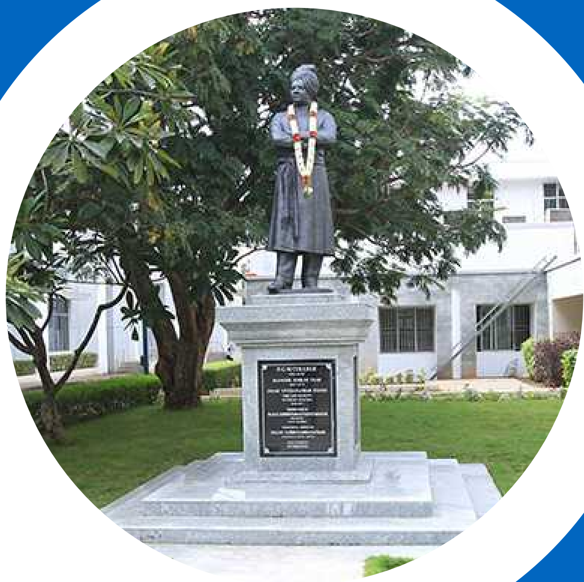

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EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on "Emerging Trends in Science and Technology (ETIST-2021)" is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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Intuitionistic fuzzy soft positive implicative ideals of BCK-algebras

Nana Liu[1], Chang Wang [2], V. Inthumathi[3].

Abstract - The aim of this paper is to apply the concept of intuitionistic fuzzy soft set to positive implicative ideal in BCK-algebras, and introduce the notion of intuitionistic fuzzy soft positive implicative ideal in BCK-algebras with several related properties are investigated. Furthermore, the operations, namely "extended intersection", "restricted intersection", "union" and "AND" on intuitionistic fuzzy soft positive implicative ideal are discussed. Finally, the homomorphism of intuitionistic fuzzy soft positive implicative ideal of BCK-algebras are given.

Keywords BCK-*algebra; positive implicative ideal; intuitionistic fuzzy soft ideal; intuitionistic fuzzy soft positive implicative ideal.*
2010 Subject classification: *06F35; 03G25*

1 Introduction

BCK-algebras and BCI-algebras are two types of logic algebras, they were introduced by Imai and Iséki [1, 2] and have been extensively studied by many researchers. BCI-algebras are generalizations of BCK-algebras, since then, many mathematicians at home and abroad have done meaningful research. For the properties of BCK-algebras, we refer the reader to Iséki and Tanaka [3].

The real world is inherently indeterminate, imprecise and vague. The concept of fuzzy set was introduced by Zadeh [4], and the concept has now been applied to many mathematical branches, such as group, functional analysis, probability theory, topology and so on. Xi [5] applied the concept of fuzzy sets to BCK-algebras, introduced the concept of fuzzy subalgebra and fuzzy ideal and obtained some meaningful results. The soft sets theory was introduced by Molodtsov [6] in 1999 as a new mathematical tool for dealing with fuzzy and uncertain models. Since the parameters in soft sets can take arbitrary forms, the theory has been widely used in various fields. In recent years, many scholars have devoted themselves to applying soft set theory to algebraic structures, proposing concepts such as soft groups [7], soft semirings [8], soft modules [9] and soft d-algebras [10]. In 1986, Atanassov [11] introduced the concept of intuitionistic fuzzy set, which is an extension of fuzzy set theory. While fuzzy sets give the degree of membership of an element in a given set, intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership. For more details about intuitionistic fuzzy sets, we refer the reader to [12].

For the general development of BCK-algebras, the ideal theory and its intuitionistic fuzzification play an important role. The notion of positive implicative ideal and fuzzy positive implicative ideal in BCI-algebras was first introduced by Jun and Meng [13] in 1994. In [14], B. L. Meng gave some characterizations of fuzzy positive implicative ideal in BCK-algebras and investigate its some extension properties.

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In 2011, Jun et al. [15] applied soft set theory to the positive implicative ideal, as well as proposed some basic properties, then the intuitionistic fuzzy positive implicative ideal was discussed by Satyanarayana [16] in 2011.

In this paper, we introduce the notion of intuitionistic fuzzy soft positive implicative ideal in BCK-algebras and investigate related properties. We provide relations between intuitionistic fuzzy soft ideal and intuitionistic fuzzy soft positive implicative ideal, and relations between intuitionistic fuzzy soft set and intuitionistic fuzzy soft positive implicative ideal. In addition, the "extended intersection", "restricted intersection", "union" and "AND" operations of intuitionistic fuzzy soft positive implicative ideal, and homomorphism of intuitionistic fuzzy soft positive implicative ideal are considered.

We first review the definitions of the algebras we have studied, the basic definitions of soft sets and intuitionistic fuzzy soft sets and some related operations.

2 Preliminaries

2.1 Basic results on BCK-algebras

In this section, we will recall some basic notions in BCK-algebra.

Definition 2.1. [2] An algebra $(X; *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following conditions:

(1) $((x * y) * (x * z)) * (z * y) = 0$,
(2) $(x * (x * y)) * y = 0$,
(3) $x * x = 0$,
(4) $x * y = 0, y * x = 0 \Rightarrow x = y$, for all $x, y, z \in X$.
If a BCI-algebra X satisfies the following identity:
(5) $0 * x = 0$, for all $x \in X$, then X is called a BCK-algebra.

In any BCK/BCI-algebra X one can define a partial order " $\leq$ " by putting $x \leq y$ if and only if $x * y = 0$.

In any BCK-algebra X the following holds:
(1) $x * 0 = x$;
(2) $(x * y) * z = (x * z) * y$;
(3) $x \leq y \Rightarrow x * z \leq y * z, z * y \leq z * x$;
(4) $(x * z) * (y * z) \leq x * y$.

A BCK-algebras X is said to be positive implicative if $(x * y) * z = (x * z) * (y * z)$ for all $x, y, z \in X$.

A nonempty subset A of a BCK/BCI-algebra X is called a BCK/BCI-subalgebra of X if $x * y \in A$ for all $x, y \in A$.

A nonempty subset A of a BCK/BCI-algebra X is called an ideal of X if it satisfies the following axioms:
(1) $0 \in A$;
(2) $x * y \in A, y \in A \Rightarrow x \in A$, for all $x \in X$.

A nonempty subset A of a BCK-algebra X is called a positive implicative ideal of X if it satisfies the following axioms:
(1) $0 \in A$;
(2) $(x * y) * z \in A, y * z \in A \Rightarrow x * z \in A$, for all $x, y, z \in X$.

Note that, in BCK-algebras, every positive implicative ideal is an ideal, but not the converse.

Definition 2.2. [5] A fuzzy set μ in BCK/BCI-algebra X is called a fuzzy ideal of X if it satisfies the following conditions:

(1) $\mu(0) \geq \mu(x)$;

(2) $\mu(x) \geq \min\{\mu(x * y), \mu(y)\}$;
for all $x, y \in X$.

Definition 2.3. [14] A fuzzy set μ in BCK-algebra X is called a fuzzy positive implicative ideal of X if it satisfies the following conditions:
(1) $\mu(0) \geq \mu(x)$;
(2) $\mu(x * z) \geq \min\{\mu((x * y) * z), \mu(y * z)\}$;
for all $x, y, z \in X$.

Definition 2.4. [17] An intuitionistic fuzzy set in BCK/BCI-algebra X is said an intuitionistic fuzzy BCK/BCI-subalgebra of X if satisfies:
(1) $\mu(x * y) \geq \min\{\mu(x), \mu(y)\}$;
(2) $\gamma(x * y) \leq \max\{\gamma(x), \gamma(y)\}$;
for all $x, y \in X$.

Definition 2.5. [17] A mapping $f : X \to Y$ of BCK-algebras is called a homomorphism if $f(x * y) = f(x) * f(y)$ for all $x, y, z \in X$. Note that if $f : X \to Y$ is a homomorphism of BCK-algebras, then $f(0) = 0$.

Let $f : X \to Y$ is a homomorphism of BCK-algebras, for any intuitionistic fuzzy set $(\widetilde{F}, A)$ in Y, defined a new intuitionistic fuzzy set preimage $(\widetilde{F}, A)^f$ in X by $\mu_{\widetilde{F}}{}^f(x) = \mu_{\widetilde{F}}(f(x)), \gamma_{\widetilde{F}}{}^f(x) = \gamma_{\widetilde{F}}(f(x))$ for all $x \in X$.

2.2 Basic results on intuitionistic fuzzy soft sets

Molodtsov [6] defined the soft set in the following way: Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subset E$.

Definition 2.6. [6] A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\alpha \in A, F(\alpha)$ may be considered as the set of α -approximate elements of the soft set (F, A).

Definition 2.7. [11] Let U be an initial universe set and E be a set of parameters. Let $F(U)$ denote the set of all intuitionistic fuzzy sets in U. Then $(\widetilde{F}, A)$ is called an intuitionistic fuzzy soft set over U where $A \subseteq E$ and $\widetilde{F}$ is a mapping given by $\widetilde{F} : A \to F(U)$.

In general, for every $\alpha \in A, \widetilde{F}[\alpha]$ is an intuitionistic fuzzy set in U and it is called an intuitionistic fuzzy value set of parameter α. Clearly, $\widetilde{F}[\alpha]$ can be written as an intuitionistic fuzzy set such that $\widetilde{F}[\alpha] = \Big\{< x, \mu_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{F}[\alpha]}(x) >| x$ where $\mu_{\widetilde{F}[\alpha]}(x)$ and $\gamma_{\widetilde{F}[\alpha]}(x)$ denotes the degree of membership and non-membership functions respectively. If for every $\alpha \in A, \mu_{\widetilde{F}[\alpha]}(x) = 1 - \gamma_{\widetilde{F}[\alpha]}(x)$ then $\widetilde{F}[\alpha]$ will be generated to be a standard fuzzy set and then $(\widetilde{F}, A)$ will be generated to be a traditional fuzzy soft set.

Definition 2.8. [18] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U, we say that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft subset of $(\widetilde{G}, B)$, denoted by $(\widetilde{F}, A)\widetilde{\subseteq}(\widetilde{G}, B)$, if it satisfies:
(1) $A \subseteq B$;
(2) $\widetilde{F}[e]$ and $\widetilde{G}[e]$ are identical approximations, for all $e \in A$.

Definition 2.9. [18] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U, then "extended intersection" of $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H}, C)$ satisfying the following conditions:

$$\widetilde{H}[e] = \begin{cases} \widetilde{F}[e], & \text{if } e \in A \backslash B, \\ \widetilde{G}[e], & \text{if } e \in B \backslash A, \\ \widetilde{F}[e] \cap \widetilde{G}[e], & \text{if } e \in A \cap B. \end{cases}$$

where $C = A \cup B$ and for all $e \in C$. In this case, we write $(\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, B) = (\widetilde{H}, C)$.

Definition 2.10. [18] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U such that $A \cap B \neq \emptyset$, then "restricted intersection" of $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H}, C)$ satisfying the condition: $\widetilde{H}[e] = \widetilde{F}[e] \cap \widetilde{G}[e]$,
where $C = A \cap B$ and for all $e \in C$. In this case, we write $(\widetilde{F}, A)\widetilde{\cap}_r(\widetilde{G}, B) = (\widetilde{H}, C)$.

Definition 2.11. [18] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U, then "union" of $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\widetilde{H}, C)$ satisfying the following conditions:

$$\widetilde{H}[e] = \begin{cases} \widetilde{F}[e], & \text{if } e \in A \backslash B, \\ \widetilde{G}[e], & \text{if } e \in B \backslash A, \\ \widetilde{F}[e] \cup \widetilde{G}[e], & \text{if } e \in A \cap B. \end{cases}$$

where $C = A \cup B$ and for all $e \in C$. In this case, we write $(\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B) = (\widetilde{H}, C)$.

Definition 2.12. [18] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U, then $(\widetilde{F}, A)AND(\widetilde{G}, B)$ denoted by $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$ is defined by $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$, where $\widetilde{H}[\alpha, \beta] = \widetilde{F}[\alpha] \cap \widetilde{G}[\beta]$ for all $(\alpha, \beta) \in A \times B$.

Definition 2.13. [18] Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U, then $(\widetilde{F}, A)OR(\widetilde{G}, B)$ denoted by $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B)$ is defined by $(\widetilde{F}, A)\widetilde{\vee}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$, where $\widetilde{H}[\alpha, \beta] = \widetilde{F}[\alpha] \cup \widetilde{G}[\beta]$ for all $(\alpha, \beta) \in A \times B$.

Definition 2.14. [18] Let $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft set over a common universe U, we say that the complement of $(\widetilde{F}, A)$ is denoted by $(\widetilde{F}, A)^c$ and is defined as $\overline{\mu_{\widetilde{F}}[\alpha]}(x) = 1 - \mu_{\widetilde{F}[\alpha]}(x)$ and $\overline{\gamma_{\widetilde{F}[\alpha]}}(x) = 1 - \gamma_{\widetilde{F}[\alpha]}(x)$ for all $x \in X, \alpha \in A$.

Definition 2.15. [18] Let $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft set over a common universe U, then $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F}[\alpha]}(x), \overline{\mu_{\widetilde{F}[\alpha]}}(x)\right\}$ and $\circ(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F}[\alpha]}}(x), \gamma_{\widetilde{F}[\alpha]}(x)\right\}$ for all $x \in X, \alpha \in A$.

Definition 2.16. [19] Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft set over a BCK/BCI-algebra X where A is the subset of E. We say that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft BCK/BCI-algebra over a BCK/BCI-algebra X if $\widetilde{F}[\alpha]$ is an intuitionistic fuzzy BCK/BCI-subalgebra in a BCK/BCI-algebra X for all $\alpha \in A$.

Definition 2.17. [19] Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft set, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal over a BCK/BCI-algebra X if $\widetilde{F}[\alpha] = \left\{< x, \mu_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{F}[\alpha]}(x) >| \, x \in X, \alpha \in A\right\}$ is an intuitionistic fuzzy ideal of X satisfies the following assertions:

(1) $\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$;
(2) $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;
(3) $\mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$;
(4) $\gamma_{\widetilde{F}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}$;

for all $x, y, z \in X$ and $\alpha \in A$.

3 Intuitionistic fuzzy soft positive implicative ideals

In this section, X denotes a BCK-algebra unless otherwise is specified.

Definition 3.1. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft set, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X if $\widetilde{F}[\alpha] = \left\{< x, \mu_{\widetilde{F}[\alpha]}(x), \gamma_{\widetilde{F}[\alpha]}(x) >| \ x \in X, \alpha \in A\right\}$ is an intuitionistic fuzzy positive implicative ideal of X satisfies the following assertions:

(1) $\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$;

(2) $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;

(3) $\mu_{\widetilde{F}[\alpha]}(x * z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$;

(4) $\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$;

for all $x, y, z \in X$ and $\alpha \in A$.

Let us illustrate this definition using the following example.

Example 3.1. *Let $U = \{0, a, b\}$ with Cayley table given by:*

$*$	0	a	b
0	0	0	0
a	a	0	0
b	b	b	0

Then $(U; *, 0)$ is a BCK-*algebra.*

Consider a set of parameters A = {content, sad}.

Let $\{\widetilde{F}, A\}$ is an intuitionistic fuzzy soft set over U, then $\widetilde{F}$[content] and $\widetilde{F}$[sad] are intuitionistic fuzzy sets. We define them as follows:

$\widetilde{F}$	0	a	b
content	(0.7,0.2)	(0.6,0.4)	(0.5,0.5)
sad	(0.9,0.1)	(0.9,0.1)	(0.6,0.3)

Then $\{\widetilde{F}, A\}$ is an intuitionistic fuzzy soft positive implicative ideal over U based on parameter "content" and "sad".

Theorem 3.1. *For any* BCK-*algebra X, every intuitionistic fuzzy soft positive implicative ideal is order preserving.*

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal over X based on the parameter $\alpha \in A$, Let $x, y \in X$ be such that $x \leq y$, then for all $z \in X$, putting $z = 0$ in

$\mu_{\widetilde{F}[\alpha]}(x * z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$

and

$\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$.

we have

$\mu_{\widetilde{F}[\alpha]}(x * 0) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * 0), \mu_{\widetilde{F}[\alpha]}(y * 0)\right\}$

$\Rightarrow \mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x*y), \mu_{\widetilde{F}[\alpha]}(y)\right\} = \min\left\{\mu_{\widetilde{F}[\alpha]}(0), \mu_{\widetilde{F}[\alpha]}(y)\right\} = \mu_{\widetilde{F}[\alpha]}(y)$
and
$\gamma_{\widetilde{F}[\alpha]}(x*0) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*0), \gamma_{\widetilde{F}[\alpha]}(y*0)\right\}$
$\Rightarrow \gamma_{\widetilde{F}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x*y), \gamma_{\widetilde{F}[\alpha]}(y)\right\} = \max\left\{\gamma_{\widetilde{F}[\alpha]}(0), \gamma_{\widetilde{F}[\alpha]}(y)\right\} = \gamma_{\widetilde{F}[\alpha]}(y),$
for all $x, y, z \in X, \alpha \in A$.

Hence, intuitionistic fuzzy soft positive implicative ideal $(\widetilde{F}, A)$ is order preserving. □

Theorem 3.2. *For any* BCK-*algebra, every intuitionistic fuzzy soft positive implicative ideal is an intuitionistic fuzzy soft ideal.*

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal over X based on the parameter $\alpha \in A$, so for all $x, y, z \in X$, we have
$\mu_{\widetilde{F}[\alpha]}(x*z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*z), \mu_{\widetilde{F}[\alpha]}(y*z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*z), \gamma_{\widetilde{F}[\alpha]}(y*z)\right\}$.
Putting $z = 0$, we have
$\mu_{\widetilde{F}[\alpha]}(x*0) = \mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*0), \mu_{\widetilde{F}[\alpha]}(y*0)\right\} = \min\left\{\mu_{\widetilde{F}[\alpha]}(x*y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*0) = \gamma_{\widetilde{F}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*0), \gamma_{\widetilde{F}[\alpha]}(y*0)\right\} = \max\left\{\gamma_{\widetilde{F}[\alpha]}(x*y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}$,
for all $x, y, z \in X, \alpha \in A$.

Thus $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal over X based on the parameter $\alpha \in A$. □

Note that an intuitionistic fuzzy soft ideal of BCK-algebras X may not be an intuitionistic fuzzy soft positive implicative ideal of X is shown in the following example.

Example 3.2. *Let* $U = \{0, a, b, c\}$ *with Cayley table given by:*

*	0	a	b	c
0	0	0	0	0
a	a	0	0	a
b	b	a	0	b
c	c	c	c	0

Then $(U; *, 0)$ *is a* BCK-*algebra.*

Consider a set of parameters $A = \{$*mouth, hair, ear*$\}$.

Let $\{\widetilde{F}, A\}$ *be an intuitionistic fuzzy soft set over* U. *Then* $\widetilde{F}$*[mouth],* $\widetilde{F}$*[hair] and* $\widetilde{F}$*[ear] are intuitionistic fuzzy sets. We define them as follows:*

$\widetilde{F}$	0	a	b	c
mouth	(0.8,0.1)	(0.7,0.2)	(0.7,0.2)	(0.6,0.3)
hair	(0.7,0.2)	(0.5,0.4)	(0.5,0.4)	(0.3,0.5)
ear	(0.6,0.3)	(0.6,0.3)	(0.6,0.3)	(0.2,0.4)

Then $\{\widetilde{F}, A\}$ *is an intuitionistic fuzzy soft ideal over* U *based on the parameter "mouth", "hair" and "ear", but* $\{\widetilde{F}, A\}$ *is not an intuitionistic fuzzy soft positive implicative ideal over* U *based on the parameter "mouth". Since*

$\mu_{\widetilde{F}[mouth]}(b * a) = \mu_{\widetilde{F}[mouth]}(a) = 0.7$
$< \min\left\{\mu_{\widetilde{F}[mouth]}((b * a) * a), \mu_{\widetilde{F}[mouth]}(a * a)\right\}$
$= \min\left\{\mu_{\widetilde{F}[mouth]}(0), \mu_{\widetilde{F}[mouth]}(0)\right\}$
$= \min\{0.8, 0.8\} = 0.8$
and
$\gamma_{\widetilde{F}[mouth]}(b * a) = \gamma_{\widetilde{F}[mouth]}(a) = 0.2$
$> \max\left\{\gamma_{\widetilde{F}[mouth]}((b * a) * a), \gamma_{\widetilde{F}[mouth]}(a * a)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[mouth]}(0), \gamma_{\widetilde{F}[mouth]}(0)\right\}$
$= \max\{0.1, 0.1\} = 0.1.$

In the following theorem, we can see that the converse of Theorem 3.2 holds in a positive implicative BCK-algebra.

Theorem 3.3. *In a positive implicative* BCK-*algebra X, every intuitionistic fuzzy soft ideal is an intuitionistic fuzzy soft positive implicative ideal.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a positive implicative BCK-algebra X.
Let $x, y \in X$, then
$\mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}.$
By replacing x by $x * z$ and y by $y * z$, we have
$\mu_{\widetilde{F}[\alpha]}(x * z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * z) * (y * z)), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * z) * (y * z)), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}.$
Since X is a positive implicative BCK-algebra, then $(x * z) * (y * z) = (x * y) * z$, for all $x, y, z \in X$.
Hence
$\mu_{\widetilde{F}[\alpha]}(x * z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\},$
for all $x, y, z \in X, \alpha \in A$.
This shows that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X. □

Next we give the conditions under which the intuitionistic fuzzy soft ideal is the intuitionistic fuzzy soft positive implicative ideal in BCK-algebras.

Theorem 3.4. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a* BCK-*algebra X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X if and only if*
(1) $\mu_{\widetilde{F}[\alpha]}(x * y) \geq \mu_{\widetilde{F}[\alpha]}((x * y) * y)$;
(2) $\gamma_{\widetilde{F}[\alpha]}(x * y) \leq \gamma_{\widetilde{F}[\alpha]}((x * y) * y)$;
for all $x, y \in X, \alpha \in A$.

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X based on the parameter $\alpha \in A$, for all $x, y, z \in X$. If we put $z = y$ in
$\mu_{\widetilde{F}[\alpha]}(x * z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$
and

$\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$,
we have
$\mu_{\widetilde{F}[\alpha]}(x * y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * y), \mu_{\widetilde{F}[\alpha]}(y * y)\right\}$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * y), \mu_{\widetilde{F}[\alpha]}(0)\right\}$
$= \mu_{\widetilde{F}[\alpha]}((x * y) * y)$
and
$\gamma_{\widetilde{F}[\alpha]}(x * y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * y), \gamma_{\widetilde{F}[\alpha]}(y * y)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * y), \gamma_{\widetilde{F}[\alpha]}(0)\right\}$
$= \gamma_{\widetilde{F}[\alpha]}((x * y) * y)$,
for all $x, y \in X, \alpha \in A$.

Conversely, assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X satisfies the following conditions:
$\mu_{\widetilde{F}[\alpha]}(x * y) \geq \mu_{\widetilde{F}[\alpha]}((x * y) * y)$ and $\gamma_{\widetilde{F}[\alpha]}(x * y) \leq \gamma_{\widetilde{F}[\alpha]}((x * y) * y)$, for all $x, y, z \in X, \alpha \in A$.
Note that $((x * z) * z) * (y * z) \leq (x * z) * y = (x * y) * z$ for all $x, y, z \in X$, implies that
$\mu_{\widetilde{F}[\alpha]}(((x * z) * z) * (y * z)) \geq \mu_{\widetilde{F}[\alpha]}((x * y) * z)$
and
$\gamma_{\widetilde{F}[\alpha]}(((x * z) * z) * (y * z)) \leq \gamma_{\widetilde{F}[\alpha]}((x * y) * z)$.
Using
$\mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(x * y), \mu_{\widetilde{F}[\alpha]}(y)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(x * y), \gamma_{\widetilde{F}[\alpha]}(y)\right\}$,
we have
$\mu_{\widetilde{F}[\alpha]}(x * z) \geq \mu_{\widetilde{F}[\alpha]}((x * z) * z)$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}(((x * z) * z) * (y * z)), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}_{[\alpha]}}(y * z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \gamma_{\widetilde{F}[\alpha]}((x * z) * z)$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x * z) * z) * (y * z)), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$,
for all $x, y, z \in X, \alpha \in A$.

Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal over X. □

Theorem 3.5. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a* BCK-*algebra X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X if and only if*
(1) $\mu_{\widetilde{F}[\alpha]}((x * z) * (y * z)) \geq \mu_{\widetilde{F}[\alpha]}((x * y) * z)$;
(2) $\gamma_{\widetilde{F}[\alpha]}((x * z) * (y * z)) \leq \gamma_{\widetilde{F}[\alpha]}((x * y) * z)$;
for all $x, y, z \in X, \alpha \in A$.

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X by Theorem 3.2. Since
$(x * (y * z)) * (x * y) \leq y * (y * z)$,
we have
$\mu_{\widetilde{F}[\alpha]}(((x * (y * z)) * (x * y)) * z) \geq \mu_{\widetilde{F}[\alpha]}((y * (y * z)) * z) = \mu_{\widetilde{F}_{[\alpha]}}(0)$,
then
$\mu_{\widetilde{F}[\alpha]}((x * z) * (y * z))$

$= \mu_{\widetilde{F}[\alpha]}((x * (y * z)) * z)$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}(((x * (y * z)) * (x * y)) * z), \mu_{\widetilde{F}[\alpha]}((x * y) * z)\right\}$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}(0), \mu_{\widetilde{F}[\alpha]}((x * y) * z)\right\}$
$= \mu_{\widetilde{F}[\alpha]}((x * y) * z),$
then
$\mu_{\widetilde{F}[\alpha]}((x * z) * (y * z)) \geq \mu_{\widetilde{F}[\alpha]}((x * y) * z)$, for all $x, y, z \in X, \alpha \in A$.
Similarly, $\gamma_{\widetilde{F}[\alpha]}(((x * (y * z)) * (x * y)) * z) \leq \gamma_{\widetilde{F}[\alpha]}((y * (y * z)) * z) = \gamma_{\tilde{F}_{[\alpha]}}(0)$,
then
$\gamma_{\widetilde{F}[\alpha]}((x * z) * (y * z))$
$= \gamma_{\widetilde{F}[\alpha]}((x * (y * z)) * z)$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x * (y * z)) * (x * y)) * z), \gamma_{\widetilde{F}[\alpha]}((x * y) * z)\right\}$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(0), \gamma_{\widetilde{F}[\alpha]}((x * y) * z)\right\}$
$= \gamma_{\widetilde{F}[\alpha]}((x * y) * z),$
then
$\gamma_{\widetilde{F}[\alpha]}((x * z) * (y * z)) \leq \gamma_{\widetilde{F}[\alpha]}((x * y) * z)$, for all $x, y, z \in X, \alpha \in A$.
Conversely, assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X and satisfies the following conditions:
$\mu_{\widetilde{F}[\alpha]}((x * z) * (y * z)) \geq \mu_{\widetilde{F}[\alpha]}((x * y) * z)$
and
$\gamma_{\widetilde{F}[\alpha]}((x * z) * (y * z)) \leq \gamma_{\widetilde{F}[\alpha]}((x * y) * z)$, for all $x, y, z \in X, \alpha \in A$.
Then
$\mu_{\widetilde{F}[\alpha]}(x * z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * z) * (y * z)), \mu_{\widetilde{F}[\alpha]}(y * z)\right\} \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * z) * (y * z)), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\} \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$,
for all $x, y, z \in X, \alpha \in A$.
Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X. □

Theorem 3.6. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft ideal over a* BCK*-algebra X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X if and only if*
(1) $\mu_{\widetilde{F}[\alpha]}(x * y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(((x * y) * y) * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$;
(2) $\gamma_{\widetilde{F}[\alpha]}(x * y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x * y) * y) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$;
for all $x, y, z \in X, \alpha \in A$.

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X by Theorem 3.2. Using Theorem 3.5, we have
$\mu_{\widetilde{F}[\alpha]}(x * y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}((x * z) * y), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}(((x * z) * y) * 0), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}(((x * z) * y) * (y * y)), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}(((x * z) * y) * y), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
$= \min\left\{\mu_{\widetilde{F}[\alpha]}(((x * y) * y) * z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x * y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * z) * y), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$

$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x*z)*y)*0), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x*z)*y)*(y*y)), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x*z)*y)*y), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$
$= \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x*y)*y)*z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$.

Conversely, assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X and satisfies the following conditions:
$\mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(((x*y)*y)*z), \mu_{\widetilde{F}[\alpha]}(z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x*y)*y)*z), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$, for all $x, y, z \in X, \alpha \in A$.
Using $(x*y)*z = (x*z)*y$ and $(x*z)*(y*z) \leq x*y$, we have
$((x*z)*z)*(y*z) \leq (x*z)*y = (x*y)*z$.
Therefore, we obtain
$\mu_{\widetilde{F}[\alpha]}(((x*z)*z)*(y*z)) \geq \mu_{\widetilde{F}[\alpha]}((x*y)*z)$
and
$\gamma_{\widetilde{F}[\alpha]}(((x*z)*z)*(y*z)) \leq \gamma_{\widetilde{F}[\alpha]}((x*y)*z)$.
It follows from hypothesis, if we put $z = 0$, we obtain
$\mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(((x*y)*y)*0), \mu_{\widetilde{F}[\alpha]}(0)\right\} = \mu_{\widetilde{F}[\alpha]}((x*y)*y)$
and
$\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x*y)*y)*0), \gamma_{\widetilde{F}[\alpha]}(0)\right\} = \gamma_{\widetilde{F}[\alpha]}((x*y)*y)$,
then
$\mu_{\widetilde{F}[\alpha]}(x*z) \geq \mu_{\widetilde{F}[\alpha]}((x*z)*z)$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}(((x*z)*z)*(y*z)), \mu_{\widetilde{F}[\alpha]}(y*z)\right\}$
$\geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*z), \mu_{\widetilde{F}[\alpha]}(y*z)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*z) \leq \gamma_{\widetilde{F}[\alpha]}((x*z)*z)$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(((x*z)*z)*(y*z)), \gamma_{\widetilde{F}[\alpha]}(y*z)\right\}$
$\leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*z), \gamma_{\widetilde{F}[\alpha]}(y*z)\right\}$,
for all $x, y, z \in X, \alpha \in A$.
Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X. □

Combining the above results, we have characterizations of intuitionistic fuzzy soft positive implicative ideals in BCK-algebras.

Theorem 3.7. *For an intuitionistic fuzzy soft set $(\widetilde{F}, A)$ over a* BCK-*algebra X, the following are equivalent:*
(1) $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X;
(2) $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X, and satisfying the conditions in Theorem 3.4*;*
(3) $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X, and satisfying the conditions in Theorem 3.5*;*
(4) $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X, and satisfying the conditions in Theorem 3.6*.*

Next we give the conditions under which the intuitionistic fuzzy soft set is the intuitionistic fuzzy soft positive implicative ideal in BCK-algebras.

Lemma 3.1. *[19] Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft* BCK/BCI-*algebra over a* BCK/BCI-*algebra X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X if and only if it satisfies $x*y \leq z$, then*
(1) $\mu_{\widetilde{F}[\alpha]}(x) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(y), \mu_{\widetilde{F}[\alpha]}(z)\right\}$;

(2) $\gamma_{\widetilde{F}[\alpha]}(x) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(y), \gamma_{\widetilde{F}[\alpha]}(z)\right\}$*;*
for all $x, y, z \in X$*, and* $\alpha \in A$*.*

Theorem 3.8. *Let* $(\widetilde{F}, A)$ *be an intuitionistic fuzzy soft set over a* BCK*-algebra* X*, then* $(\widetilde{F}, A)$ *is an intuitionistic fuzzy soft positive implicative ideal of* X *if and only if satisfies the conditions:*

(1) $\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$ *and* $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$*;*

(2) $(((x*y)*y)*a)*b = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$

and $\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$*;*
for all $x, y, a, b \in X, \alpha \in A$*.*

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X by Theorem 3.2, and so (1) is true.

Let $x, y, a, b \in X$ be such that $(((x*y)*y)*a)*b = 0$, i.e, $((x*y)*y)*a \leq b$, it follows from Lemma 3.1 that

$$\mu_{\widetilde{F}[\alpha]}((x*y)*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$$

and

$$\gamma_{\widetilde{F}[\alpha]}((x*y)*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}, \text{ for all } \alpha \in A.$$

It follows from Theorem 3.4 that

$$\mu_{\widetilde{F}[\alpha]}(x*y) \geq \mu_{\widetilde{F}[\alpha]}((x*y)*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$$

and

$$\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \gamma_{\widetilde{F}[\alpha]}((x*y)*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}.$$

Therefore $(\widetilde{F}, A)$ satisfies

$(((x*y)*y)*a)*b = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}_{[\alpha]}}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$ and $\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$, for all $x, y, a, b \in X, \alpha \in A$, which proves (2).

Conversely, assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft set of X satisfies the following conditions:

$\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$ and $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;

$(((x*y)*y)*a)*b = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$ and $\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$, for any $x, y, a, b \in X, \alpha \in A$, and let $x, a, b \in X$ be such that $(x*a)*b = 0$, then

$$(((x*0)*0)*a)*b = 0,$$

and so

$$\mu_{\widetilde{F}[\alpha]}(x) = \mu_{\widetilde{F}[\alpha]}(x*0) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$$

and

$$\gamma_{\widetilde{F}[\alpha]}(x) = \gamma_{\widetilde{F}[\alpha]}(x*0) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}, \text{ for all } \alpha \in A.$$

By Lemma 3.1 we know that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X.
Note that $(((x*y)*y)*((x*y)*y))*0 = 0$ for all $x, y \in X$, it follows from conditions that

$$\mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x*y)*y), \mu_{\widetilde{F}[\alpha]}(0)\right\} = \mu_{\widetilde{F}[\alpha]}((x*y)*y)$$

and

$$\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x*y)*y), \gamma_{\widetilde{F}[\alpha]}(0)\right\} = \gamma_{\widetilde{F}[\alpha]}((x*y)*y), \text{ for all } \alpha \in A.$$

By Theorem 3.4, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X. □

Theorem 3.9. *Let* $(\widetilde{F}, A)$ *be an intuitionistic fuzzy soft set over a* BCK*-algebra* X*, then* $(\widetilde{F}, A)$ *is an intuitionistic fuzzy soft positive implicative ideal of* X *if and only if satisfies the conditions:*

(1) $\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$ *and* $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$*;*

(2) $(((x*y)*z)*a)*b = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}((x*z)*(y*z)) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$

and $\gamma_{\widetilde{F}[\alpha]}((x*z)*(y*z)) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$*;*
for all $x, y, z, a, b \in X, \alpha \in A$*.*

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X by Theorem 3.2, and so (1) is true.

Let $x, y, z, a, b \in X$ be such that $(((x*y)*z)*a)*b = 0$, i.e, $((x*y)*z)*a \leq b$, it follows from Lemma 3.1 that
$\mu_{\widetilde{F}[\alpha]}((x*y)*z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}((x*y)*z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$, for all $\alpha \in A$.
By Theorem 3.5 we obtain
$\mu_{\widetilde{F}[\alpha]}((x*z)*(y*z)) \geq \mu_{\widetilde{F}[\alpha]}((x*y)*z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}((x*z)*(y*z)) \leq \gamma_{\widetilde{F}[\alpha]}((x*y)*z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$,
Therefore $(\widetilde{F}, A)$ satisfies
$(((x*y)*z)*a)*b = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}((x*z)*(y*z)) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$ and $\gamma_{\widetilde{F}[\alpha]}((x*z)*(y*z)) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$, for all $x, y, a, b \in X, \alpha \in A$, which proves (2).

Conversely, assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft set of X satisfies the following conditions:
$\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$ and $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;
$(((x*y)*z)*a)*b = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}((x*z)*(y*z)) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$ and $\gamma_{\widetilde{F}[\alpha]}((x*z)*(y*z)) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$, for any $x, y, z, a, b \in X, \alpha \in A$.

We assume $(((x*y)*y)*a)*b = 0$, by the conditions, we have
$\mu_{\widetilde{F}[\alpha]}(x*y) = \mu_{\widetilde{F}[\alpha]}((x*y)*(y*y)) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a), \mu_{\widetilde{F}[\alpha]}(b)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*y) = \gamma_{\widetilde{F}[\alpha]}((x*y)*(y*y)) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a), \gamma_{\widetilde{F}[\alpha]}(b)\right\}$,
It follows from Theorem 3.8 ,$(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X. □

By the mathematical induction, the above two theorems have more general forms.

Theorem 3.10. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft set over a* BCK*-algebra X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X if and only if satisfies the conditions:*
(1) $\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$ and $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;
(2) $(\ldots(((x*y)*y)*a_1)\ldots)*a_n = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a_1), \ldots, \mu_{\widetilde{F}[\alpha]}(a_n)\right\}$
and $\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a_1), \ldots, \gamma_{\widetilde{F}[\alpha]}(a_n)\right\}$;
for all $x, y, a_1, .., a_n \in X, \alpha \in A$.

Proof. Assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X by Theorem 3.2, and so (1) is true.

Let $x, y, a_1, \ldots, a_n \in X$ be such that $(\ldots(((x*y)*y)*a_1)*\ldots)*a_n = 0$, it follows from Lemma 3.1 that
$\mu_{\widetilde{F}[\alpha]}((x*y)*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a_1), \ldots, \mu_{\widetilde{F}[\alpha]}(a_n)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}((x*y)*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a_1), \ldots, \gamma_{\widetilde{F}[\alpha]}(a_n)\right\}$, for all $\alpha \in A$.
It follows from Theorem 3.4 that
$\mu_{\widetilde{F}[\alpha]}(x*y) \geq \mu_{\widetilde{F}[\alpha]}((x*y)*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a_1), \ldots, \mu_{\widetilde{F}[\alpha]}(a_n)\right\}$
and
$\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \gamma_{\widetilde{F}[\alpha]}((x*y)*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a_1), \ldots, \gamma_{\widetilde{F}[\alpha]}(a_n)\right\}$.
Therefore $(\widetilde{F}, A)$ satisfies $(\ldots(((x*y)*y)*a_1)\ldots)*a_n = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}(x*y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a_1), \ldots, \mu_{\widetilde{F}[\alpha]}(a_n)\right\}$, and $\gamma_{\widetilde{F}[\alpha]}(x*y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a_1), \ldots, \gamma_{\widetilde{F}[\alpha]}(a_n)\right\}$, for all $x, y, a_1, \ldots, a_n \in X, \alpha \in A$, which proves (2).

Conversely, assume that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft set of X satisfies the following conditions:

$\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$ and $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;

$(\ldots(((x * y) * y) * a_1) \ldots) * a_n = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}(x * y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a_1), \ldots, \mu_{\widetilde{F}[\alpha]}(a_n)\right\}$ and $\gamma_{\widetilde{F}[\alpha]}(x * y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a_1), \ldots\right.$

for all $x, y, a_1, \ldots, a_n \in X, \alpha \in A$, and let $x, a_1, \ldots, a_n \in X$ be such that $((x * a_1) * \ldots) * a_n = 0$, then

$((((x * 0) * 0) * a_1) * \ldots) * a_n = 0$

and so

$\mu_{\widetilde{F}[\alpha]}(x) = \mu_{\widetilde{F}[\alpha]}(x * 0) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a_1), \ldots, \mu_{\widetilde{F}[\alpha]}(a_n)\right\}$

and

$\gamma_{\widetilde{F}[\alpha]}(x) = \gamma_{\widetilde{F}[\alpha]}(x * 0) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a_1), \ldots, \gamma_{\widetilde{F}[\alpha]}(a_n)\right\}$, for all $\alpha \in A$.

By Lemma 3.1 we know that $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft ideal of X.

Note that $((((x * y) * y) * ((x * y) * y)) * 0) * \ldots) * 0 = 0$ for all $x, y \in X$, it follows from conditions that

$\mu_{\widetilde{F}[\alpha]}(x * y) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * y), \mu_{\widetilde{F}[\alpha]}(0), \ldots, \mu_{\widetilde{F}[\alpha]}(0)\right\} = \mu_{\widetilde{F}[\alpha]}((x * y) * y)$

and

$\gamma_{\widetilde{F}[\alpha]}(x * y) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * y), \gamma_{\widetilde{F}[\alpha]}(0), \ldots, \gamma_{\widetilde{F}[\alpha]}(0)\right\} = \gamma_{\widetilde{F}[\alpha]}((x * y) * y)$, for all $\alpha \in A$.

By Theorem 3.4, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X. □

Theorem 3.11. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft set over a* BCK-*algebra X, then $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X if and only if satisfies the conditions:*

(1) $\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$ and $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$;

(2) $(\ldots(((x * y) * z) * a_1) * \ldots) * a_n = 0 \Rightarrow \mu_{\widetilde{F}[\alpha]}((x * z) * (y * z)) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}(a_1), \ldots, \mu_{\widetilde{F}[\alpha]}(a_n)\right\}$

and $\gamma_{\widetilde{F}[\alpha]}((x * z) * (y * z)) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}(a_1), \ldots, \gamma_{\widetilde{F}[\alpha]}(a_n)\right\}$;

for all $x, y, z, a_1, \ldots, a_n \in X, \alpha \in A$.

Proof. It is similar to Theorem 3.9 and is omitted. □

4 Some properties of intuitionistic fuzzy soft positive implicative ideals

In this section, X denotes a BCK-algebra unless otherwise is specified, we will consider Some properties of intuitionistic fuzzy soft positive implicative ideals in BCK-algebras.

Theorem 4.1. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft positive implicative ideal over a* BCK-*algebra X, then $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F}[\alpha]}(x), \overline{\mu_{\widetilde{F}[\alpha]}}(x)\right\}$ is also an intuitionistic fuzzy soft positive implicative ideal of X, for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X, we have

$\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$

$\Rightarrow 1 - \mu_{\widetilde{F}[\alpha]}(0) \leq 1 - \mu_{\widetilde{F}[\alpha]}(x)$

$\Rightarrow \overline{\mu_{\widetilde{F}[\alpha]}}(0) \leq \overline{\mu_{\widetilde{F}[\alpha]}}(x)$,

for all $x \in X, \alpha \in A$.

Consider for any $x, y, z \in X, \alpha \in A$,

$\mu_{\widetilde{F}[\alpha]}(x * z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$

$\Rightarrow 1 - \overline{\mu_{\widetilde{F}[\alpha]}}(x * z) \geq \min\left\{1 - \overline{\mu_{\widetilde{F}[\alpha]}}((x * y) * z), 1 - \overline{\mu_{\widetilde{F}[\alpha]}}(y * z)\right\}$

$\Rightarrow \overline{\mu_{\widetilde{F}[\alpha]}}(x * z) \leq 1 - \min\left\{1 - \overline{\mu_{\widetilde{F}[\alpha]}}((x * y) * z), 1 - \overline{\mu_{\widetilde{F}[\alpha]}}(y * z)\right\}$

$\Rightarrow \overline{\mu_{\widetilde{F}[\alpha]}}(x * z) \leq \max\left\{\overline{\mu_{\widetilde{F}[\alpha]}}((x * y) * z), \overline{\mu_{\widetilde{F}[\alpha]}}(y * z)\right\}$

Hence, $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F}[\alpha]}(x), \overline{\mu_{\widetilde{F}[\alpha]}}(x)\right\}$ is an intuitionistic fuzzy soft positive implicative ideal of X, for all $x \in X, \alpha \in A$. □

Theorem 4.2. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft positive implicative ideal over a* BCK*-algebra X, then* $^{\circ}(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F}[\alpha]}}(x), \gamma_{\widetilde{F}[\alpha]}(x)\right\}$ *is also an intuitionistic fuzzy soft positive implicative ideal of X, for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X, we have

$\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$
$\Rightarrow 1 - \gamma_{\widetilde{F}[\alpha]}(0) \geq 1 - \gamma_{\widetilde{F}[\alpha]}(x)$
$\Rightarrow \overline{\gamma_{\widetilde{F}[\alpha]}}(0) \geq \overline{\gamma_{\widetilde{F}[\alpha]}}(x)$

for all $x \in X, \alpha \in A$.

Consider for any $x, y, z \in X, \alpha \in A$,

$\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$
$\Rightarrow 1 - \overline{\gamma_{\widetilde{F}[\alpha]}}(x * z) \leq \max\left\{1 - \overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), 1 - \overline{\gamma_{\widetilde{F}[\alpha]}}(y * z)\right\}$
$\Rightarrow \overline{\gamma_{\widetilde{F}[\alpha]}}(x * z) \geq 1 - \max\left\{1 - \overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), 1 - \overline{\gamma_{\widetilde{F}[\alpha]}}(y * z)\right\}$
$\Rightarrow \overline{\gamma_{\widetilde{F}[\alpha]}}(x * z) \geq \min\left\{\overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), \overline{\gamma_{\widetilde{F}[\alpha]}}(y * z)\right\}$

Hence, $^{\circ}(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F}[\alpha]}}(x), \gamma_{\widetilde{F}[\alpha]}(x)\right\}$ is an intuitionistic fuzzy soft positive implicative ideal of X, for all $x \in X, \alpha \in A$. □

Theorem 4.3. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft positive implicative ideal over a* BCK*-algebra X if and only if $\mu_{\widetilde{F}[\alpha]}(x)$ and $\overline{\gamma_{\widetilde{F}[\alpha]}}(x)$ are fuzzy soft positive implicative ideals of X for all $x \in X, \alpha \in A$.*

Proof. Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X, clearly $\mu_{\widetilde{F}[\alpha]}(x)$ is a fuzzy soft positive implicative ideal of X.

Let $x, y, z \in X, \alpha \in A$, then

$\overline{\gamma_{\widetilde{F}[\alpha]}}(0) = 1 - \gamma_{\widetilde{F}[\alpha]}(0) \geq 1 - \gamma_{\widetilde{F}[\alpha]}(x) = \overline{\gamma_{\widetilde{F}[\alpha]}}(x)$

and

$\overline{\gamma_{\widetilde{F}[\alpha]}}(x * z) = 1 - \gamma_{\widetilde{F}[\alpha]}(x * z)$
$\geq 1 - \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$
$= \min\left\{1 - \gamma_{\widetilde{F}[\alpha]}((x * y) * z), 1 - \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$
$= \min\left\{\overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), \overline{\gamma_{\widetilde{F}[\alpha]}}(y * z)\right\}.$

Hence, $\overline{\gamma_{\widetilde{F}[\alpha]}}(x)$ is a fuzzy soft positive implicative ideal of X.

Conversely, assume that $\mu_{\widetilde{F}[\alpha]}(x)$ and $\overline{\gamma_{\widetilde{F}[\alpha]}}(x)$ are fuzzy soft positive implicative ideals of X for all $x \in X, \alpha \in A$. For all $x \in X$, we have

$\mu_{\widetilde{F}[\alpha]}(0) \geq \mu_{\widetilde{F}[\alpha]}(x)$

and

$1 - \gamma_{\widetilde{F}[\alpha]}(0) = \overline{\gamma_{\widetilde{F}[\alpha]}}(0) \geq \overline{\gamma_{\widetilde{F}[\alpha]}}(x) = 1 - \gamma_{\widetilde{F}[\alpha]}(x).$

Which show that $\gamma_{\widetilde{F}[\alpha]}(0) \leq \gamma_{\widetilde{F}[\alpha]}(x)$.

Now let $x, y, z \in X, \alpha \in A$, then

$\mu_{\widetilde{F}[\alpha]}(x * z) \geq \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}$

and

$1 - \gamma_{\widetilde{F}[\alpha]}(x * z) = \overline{\gamma_{\widetilde{F}[\alpha]}}(x * z)$
$\geq \min\left\{\overline{\gamma_{\widetilde{F}[\alpha]}}((x * y) * z), \overline{\gamma_{\widetilde{F}[\alpha]}}(y * z)\right\}$
$= \min\left\{1 - \gamma_{\widetilde{F}[\alpha]}((x * y) * z), 1 - \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$
$= 1 - \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$

and so

$\gamma_{\widetilde{F}[\alpha]}(x * z) \leq \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}$.
Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of X. □

Theorem 4.4. *Let $(\widetilde{F}, A)$ be an intuitionistic fuzzy soft positive implicative ideal over a* BCK*-algebra X if and only if $\neg(\widetilde{F}, A) = \left\{\mu_{\widetilde{F}[\alpha]}(x), \overline{\mu_{\widetilde{F}[\alpha]}}(x)\right\}$ and $^\circ(\widetilde{F}, A) = \left\{\overline{\gamma_{\widetilde{F}[\alpha]}}(x), \gamma_{\widetilde{F}[\alpha]}(x)\right\}$ are intuitionistic fuzzy soft positive implicative ideals of X, for all $x \in X, \alpha \in A$.*

Proof. It is straightforward by Theorem 4.3. □

In the following we discuss other properties of intuitionistic fuzzy soft positive implicative ideals in BCK-algebras.

Theorem 4.5. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft positive implicative ideals over a* BCK*-algebra X, then the "extended intersection" $(\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, B)$ is an intuitionistic fuzzy soft positive implicative ideal over* X.

Proof. Let $(\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, B) = (\widetilde{H}, C)$ be the "extended intersection" of intuitionistic fuzzy soft positive implicative ideal $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over X, where $C = A \cup B$. For any $e \in C$,

if $e \in A\backslash B$, then $\widetilde{H}[e] = \widetilde{F}[e]$ is an intuitionistic fuzzy positive implicative ideal in X because $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X;

if $e \in B\backslash A$, then $\widetilde{H}[e] = \widetilde{G}[e]$ is an intuitionistic fuzzy positive implicative ideal in X because $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X;

if $A \cap B \neq \emptyset$, then $\widetilde{H}[e] = \widetilde{F}[e] \cap \widetilde{G}[e]$ is an intuitionistic fuzzy positive implicative ideal for all $e \in A \cap B$, since the intersection of two intuitionistic fuzzy positive implicative ideals is an intuitionistic fuzzy positive implicative ideal.

Therefore $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, B)$ is an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X. □

The following two corollaries are straightforward results of Theorem 4.5.

Corollary 4.1. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, A)$ be two intuitionistic fuzzy soft positive implicative ideals over a* BCK*-algebra X, then the "extended intersection" $(\widetilde{F}, A)\widetilde{\cap}_e(\widetilde{G}, A)$ is an intuitionistic fuzzy soft positive implicative ideal over X.*

Corollary 4.2. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft positive implicative ideals over a* BCK*-algebra X, then the "restricted intersection" $(\widetilde{F}, A)\widetilde{\cap}_r(\widetilde{G}, B)$ is an intuitionistic fuzzy soft positive implicative ideal over X.*

Theorem 4.6. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft positive implicative ideals over a* BCK*-algebra X, if A and B are disjoint, then the "union" $(\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft positive implicative ideal over X.*

Proof. Let $(\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B) = (\widetilde{H}, C)$ be the "union" of intuitionistic fuzzy soft positive implicative ideal $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ over X. Since A and B are disjoint, then for all $e \in C$, either $e \in A\backslash B$ or $e \in B\backslash A$, by means of Definition 2.11,

if $e \in A\backslash B$, then $\widetilde{H}[e] = \widetilde{F}[e]$ is an intuitionistic fuzzy positive implicative ideal in X because $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X;

if $e \in B\backslash A$, then $\widetilde{H}[e] = \widetilde{G}[e]$ is an intuitionistic fuzzy positive implicative ideal in X because $(\widetilde{G}, B)$ is an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X.

Hence $(\widetilde{H}, C) = (\widetilde{F}, A)\widetilde{\cup}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft positive implicative ideal over a BCK-algebra X. □

Theorem 4.7. *Let $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft positive implicative ideals over a* BCK*-algebra X, then the "AND " $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B)$ is an intuitionistic fuzzy soft positive implicative ideal over X.*

Proof. By means of Definition 2.12, we know that $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$, where $\widetilde{H}[\alpha,\beta] = \widetilde{F}[\alpha] \cap \widetilde{G}[\beta]$ for all $(\alpha,\beta) \in A \times B$.

Since $(\widetilde{F}, A)$ and $(\widetilde{G}, B)$ be two intuitionistic fuzzy soft positive implicative ideals over a BCK-algebra X, then $\widetilde{F}[\alpha]$ and $\widetilde{G}[\alpha]$ are intuitionistic fuzzy positive implicative ideals of X, then the intersection is $\widetilde{F}[\alpha] \cap \widetilde{G}[\beta]$ also an intuitionistic fuzzy positive implicative ideal of X.

Hence, $\widetilde{H}[\alpha,\beta]$ is an intuitionistic fuzzy positive implicative ideal of X for all $(\alpha,\beta) \in A \times B$.

Therefore, $(\widetilde{F}, A)\widetilde{\wedge}(\widetilde{G}, B) = (\widetilde{H}, A \times B)$ is an intuitionistic fuzzy soft positive implicative ideal over X based on the parameter (α,β). □

At the end of the paper, we discuss the homomorphism between intuitionistic fuzzy soft positive implicative ideals in BCK-algebras.

Theorem 4.8. *Let $f : X \to Y$ is an onto homomorphism of* BCK*-algebras. If an intuitionistic fuzzy soft set $(\widetilde{F}, A)$ of Y is an intuitionistic fuzzy soft positive implicative ideal, then preimage $(\widetilde{F}, A)^f$ is also an intuitionistic fuzzy soft positive implicative ideal of X.*

Proof. Since $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of Y, and $(\widetilde{F}, A)^f$ is the preimage of $(\widetilde{F}, A)$ under f of X, then $\mu_{\widetilde{F}[\alpha]}(f(x)) = \mu_{\widetilde{F}[\alpha]}{}^f(x), \gamma_{\widetilde{F}[\alpha]}(f(x)) = \gamma_{\widetilde{F}[\alpha]}{}^f(x)$ for all $x \in X, \alpha \in A$.

Since $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of Y, then for any $x \in X, \alpha \in A$, we have

$$\mu_{\widetilde{F\alpha]}}{}^f(x) = \mu_{\widetilde{F}[\alpha]}(f(x)) \leq \mu_{\widetilde{F}[\alpha]}(0) = \mu_{\widetilde{F}[\alpha]}(f(0)) = \mu_{\widetilde{F}[\alpha]}{}^f(0)$$

and

$$\gamma_{\widetilde{F}[\alpha]}{}^f(x) = \gamma_{\widetilde{F}[\alpha]}(f(x)) \geq \gamma_{\widetilde{F}[\alpha]}(0) = \gamma_{\widetilde{F}[\alpha]}(f(0)) = \gamma_{\widetilde{F}[\alpha]}{}^f(0).$$

Moreover,

$$\begin{aligned}
&\min\left\{\mu_{\widetilde{F}[\alpha]}{}^f((x * y) * z), \mu_{\widetilde{F}[\alpha]}{}^f(y * z)\right\}\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}(f((x * y) * z)), \mu_{\widetilde{F}[\alpha]}(f(y * z))\right\}\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}(f(x * y) * f(z)), \mu_{\widetilde{F}[\alpha]}(f(y) * f(z))\right\}\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}((f(x) * f(y)) * f(z)), \mu_{\widetilde{F\alpha]}}(f(y) * f(z))\right\}\\
&\leq \mu_{\widetilde{F}[\alpha]}(f(x) * f(z))\\
&= \mu_{\widetilde{F}[\alpha]}(f(x * z))\\
&= \mu_{\widetilde{F}[\alpha]}{}^f(x * z)
\end{aligned}$$

and

$$\begin{aligned}
&\max\left\{\gamma_{\widetilde{F}[\alpha]}{}^f((x * y) * z), \gamma_{\widetilde{F}[\alpha]}{}^f(y * z)\right\}\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}(f((x * y) * z)), \gamma_{\widetilde{F}[\alpha]}(f(y * z))\right\}\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}(f(x * y) * f(z)), \gamma_{\widetilde{F}[\alpha]}(f(y) * f(z))\right\}\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}((f(x) * f(y)) * f(z)), \gamma_{\widetilde{F}_{\alpha]}}(f(y) * f(z))\right\}\\
&\geq \gamma_{\widetilde{F}[\alpha]}(f(x) * f(z))\\
&= \gamma_{\widetilde{F}[\alpha]}(f(x * z))\\
&= \gamma_{\widetilde{F}[\alpha]}{}^f(x * z).
\end{aligned}$$

Hence, $(\widetilde{F}, A)^f$ is also an intuitionistic fuzzy soft positive implicative ideal of X, for any $x, y, z \in X, \alpha \in A$. □

If we strengthen the condition of f, then we can construct the converse of the above theorem as follows.

Theorem 4.9. *Let $f : X \to Y$ is an epimorphism of* BCK-*algebras. If an intuitionistic fuzzy soft set $(\widetilde{F}, A)^f$ is an intuitionistic fuzzy soft positive implicative ideal of X, then $(\widetilde{F}, A)$ is also an intuitionistic fuzzy soft positive implicative ideal of Y.*

Proof. Since $(\widetilde{F}, A)^f$ is an intuitionistic fuzzy soft positive implicative ideal of X, and $(\widetilde{F}, A)^f$ is the preimage of $(\widetilde{F}, A)$ under f of X, then $\mu^f_{\widetilde{F}_{[\alpha]}}(x) = \mu_{\widetilde{F}[\alpha]}(f(x)), {\gamma_{\widetilde{F}[\alpha]}}^f(x) = \gamma_{\widetilde{F}[\alpha]}(f(x))$ for all $x \in X, \alpha \in A$.

Let $x, y, z \in Y, \alpha \in A$, there exist $a, b, c \in X$ such that $f(a) = x, f(b) = y$ and $f(c) = z$.
Now,

$\mu_{\widetilde{F}[\alpha]}(x) = \mu_{\widetilde{F}[\alpha]}(f(a)) = {\mu_{\widetilde{F}[\alpha]}}^f(a) \leq {\mu_{\widetilde{F}[\alpha]}}^f(0) = \mu_{\widetilde{F}[\alpha]}(f(0)) = \mu_{\widetilde{F}[\alpha]}(0)$
and

$\gamma_{\widetilde{F}\alpha]}(x) = \gamma_{\widetilde{F}[\alpha]}(f(a)) = {\gamma_{\widetilde{F}[\alpha]}}^f(a) \geq {\gamma_{\widetilde{F}[\alpha]}}^f(0) = \gamma_{\widetilde{F}[\alpha]}(f(0)) = \gamma_{\widetilde{F}[\alpha]}(0)$.
Moreover,

$$
\begin{aligned}
&\mu_{\widetilde{F}[\alpha]}(x * z)\\
&= \mu_{\widetilde{F}[\alpha]}(f(a) * f(c))\\
&= \mu_{\widetilde{F}[\alpha]}(f(a * c))\\
&= {\mu_{\widetilde{F}[\alpha]}}^f(a * c)\\
&\geq \min\left\{{\mu_{\widetilde{F}[\alpha]}}^f((a * b) * c), {\mu_{\widetilde{F}[\alpha]}}^f(b * c)\right\}\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}(f((a * b) * c)), \mu_{\widetilde{F}[\alpha]}(f(b * c))\right\}\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}(f(a * b) * f(c)), \mu_{\widetilde{F}[\alpha]}(f(b) * f(c))\right\}\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}((f(a) * f(b)) * f(c)), \mu_{\widetilde{F}[\alpha]}(f(b) * f(c))\right\}\\
&= \min\left\{\mu_{\widetilde{F}[\alpha]}((x * y) * z), \mu_{\widetilde{F}[\alpha]}(y * z)\right\}
\end{aligned}
$$
and

$$
\begin{aligned}
&\gamma_{\widetilde{F}[\alpha]}(x * z)\\
&= \gamma_{\widetilde{F}[\alpha]}(f(a) * f(c))\\
&= \gamma_{\widetilde{F}[\alpha]}(f(a * c))\\
&= {\gamma_{\widetilde{F}[\alpha]}}^f(a * c)\\
&\leq \max\left\{{\gamma_{\widetilde{F}[\alpha]}}^f((a * b) * c), {\gamma_{\widetilde{F}[\alpha]}}^f(b * c)\right\}\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}(f((a * b) * c)), \gamma_{\widetilde{F}[\alpha]}(f(b * c))\right\}\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}(f(a * b) * f(c)), \gamma_{\widetilde{F}[\alpha]}(f(b) * f(c))\right\}\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}((f(a) * f(b)) * f(c)), \gamma_{\widetilde{F}[\alpha]}(f(b) * f(c))\right\}\\
&= \max\left\{\gamma_{\widetilde{F}[\alpha]}((x * y) * z), \gamma_{\widetilde{F}[\alpha]}(y * z)\right\}.
\end{aligned}
$$
Hence, $(\widetilde{F}, A)$ is an intuitionistic fuzzy soft positive implicative ideal of Y. □

5 Conclusion

In order to study the structure of algebraic systems, ideals with special properties obviously play an important role. We introduced the notion of intuitionistic fuzzy soft positive implicative ideal, and investigated related properties. Meanwhile, we discussed relations between intuitionistic fuzzy soft ideal and intuitionistic fuzzy soft positive implicative ideal over a BCK-algebras, and relations between intuitionistic fuzzy soft set and intuitionistic fuzzy soft positive implicative ideal over a BCK-algebras. We believe that such discussion could clarify some misunderstandings and clean the ground for further development of that interesting theory.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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