VOLUME XII
ISBN No.: 978-93-94004-01-6
Physical Science

Estd. 1957

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University, An ISO 9001:2015 Certified Institution,

Pollachi-642001

SUPPORTED BY

ABT Industries Ltd.,
Dairy Division

PROCEEDING

One day International Conference

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

NALLAMUTHU GOUNDER MAHALINGAM COLLEGE

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, Pollachi-642001.

நன்றின்பால் உய்ப்பது அறிவு

Nallamuthu Gounder Mahalingam College, Pollachi - 1

Estd. 1957

Proceeding of the

One day International Conference on

EMERGING TRENDS IN SCIENCE AND TECHNOLOGY (ETIST-2021)

27^{th} October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

ISBN No: 978-93-94004-01-6

978- 93- 94004- 01- 6

Nallamuthu Gounder Mahalingam College

An Autonomous Institution, Affiliated to Bharathiar University

An ISO 9001:2015 Certified Institution, 90 Palghat Road, Pollachi-642001.

www.ngmc.org

ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

International Conference on Emerging Trends in Science and Technology (ETIST 2021) Jointly Organized by Department of Biological Science, Physical Science and Computational Science Nallamuthu Gounder Mahalingam College, Affiliated to Bharathiar University, Tamilnadu, India.
Published by NGMC - November 2021 ISBN: 0000 - 0000

Vague Soft Fundamental Groups

M. Pavithra[1], **Saeid Jafari**[2], **V. Inthumathi**[3]

Abstract - In this paper, we initiate the study of vague soft path and vague soft path connected spaces in vague soft topological spaces. Also, we investigate the concepts of vague soft-path homotopy and vague soft fundamental groups.

Keywords *Vague soft product spaces, Vague soft-path homotopy, Vague soft-fundamental groups.*
2010 Subject classification: *03B52, 54A40, 03E72.*

1 Introduction

Soft set theory, proposed by Molodtsov [17] has been regarded as an effective Mathematical tool to deal with uncertainty. Many researchers have contributed towards the soft set theory and its applications in various fields [2, 4, 11, 12, 15, 16, 23, 25, 29]. The theory of vague sets was first proposed by Gau et al. [9]. A vague set V is defined by a truth-membership function t_V and a false-membership function f_V, where $t_V(x)$ is a lower bound on the grade of membership of x derived from the evidence for x, and $f_V(x)$ is a lower bound on the negation of x derived from the evidence against x. These true membership function and false membership function noted as $t_V(x)$ and $f_V(x)$ are associated as a real number in $[0,1]$ with each point in a basic set X, which satisfies the condition $0 \leq t_V(x) + f_V(x) \leq 1$. The vague group was first introduced by Demirci [7] in 1999. Since then the theory of vague algebraic notions has been established by [1, 3, 8, 10, 14, 20, 24].

In 2010, Xu et al. [27] combined the notions of vague sets and soft sets and introduced the notion of vague soft sets and presented its basic properties. The concept of vague soft topology was initiated by C. Wang et al. [6] which is defined over the initial universal set with a fixed set of parameter. They studied the notions vague soft interior, vague soft closure, vague soft boundary, vague soft connectedness and compacetness. The vague soft set theory also have been applied to several algebraic structures like vague soft hemirings [28], vague soft groups [26], vague soft hypergroups, vague soft hyperrings and vague soft hyperideals [21, 22] Anti vague soft R-subgroup of near ring [19]. Recently, works on the vague soft set theory are progressing rapidly. In this work, we study the algebraic structure of vague soft sets by defining the concept of vague soft-path, vague soft-path homotopy and vague soft fundamental groups.

2 Preliminaries

Definition 2.1. *[17] Let X be an initial universe set, $P(X)$ the set of all subsets of X, E a set of parameters, and $A \subseteq E$. A pair (F, A) is called a soft set over X, where F is a mapping given by $F\colon A \to$ P(X).*

[1]Research Scholar, PG and Research Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Coimbatore, Tamilnadu, India. E.mail: inthumathi65@gmail.com
[2]Professor of Mathematics, College of Vestsjaelland south Herrestraed 11, 4200 Slagelse, Denmark.
E.mail: jafaripersia@gmail.com
[3]Associate Professor and Head, PG and Research Department of Mathematics, Nallamuthu Gounder Mahalingam College, Pollachi-642001, Coimbatore, Tamilnadu, India. E.mail: inthumathi65@gmail.com

Definition 2.2. *[9] A vague set* $A = \{(x_i, [t_A(x_i), 1 - f_A(x_i)])|x_i \in X\}$ *in the universe* $X = \{x_1, x_2, ..., x_n\}$ *is characterized by a truth-membership function* $t_A : X \to [0,1]$*, and a false-membership function* $f_A : X \to [0,1]$*, where* $t_A(x_i)$ *is a lower bound on the grade of membership of* x_i *derived from the evidence of* x_i*,* $f_A(x_i)$ *is the lower bound on the negation of* x_i *derived from the evidence against* x_i *and* $0 \leq t_A(x_i) + f_A(x_i) \leq 1$ *for any* $x_i \in X$*. The grade of membership of* x_i *in the vague set is bounded to a subinterval* $[t_A(x_i), 1 - f_A(x_i)]$ *of [0,1]. The vague value* $[t_A(x_i), 1 - f_A(x_i)]$ *indicates that the exact grade of membership* $\mu_A(x_i)$ *of* x_i *may be unknown, but it is bounded by* $t_A(x_i) \leq \mu_A(x_i) \leq 1 - f_A(x_i)$*, where* $0 \leq t_A(x_i) + f_A(x_i) \leq 1$*.*

Notations: Let $I[0,1]$ denotes the family of all closed subintervals of $[0,1]$. If $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ be two elements of $I[0,1]$, we call $I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$. Similarly we understand the relations $I_1 \leq I_2$ and $I_1 = I_2$. Clearly the relation $I_1 \geq I_2$ does not necessarily imply that $I_1 \supseteq I_2$ and conversely. Also for any two unequal intervals I_1 and I_2, there is no necessity that either $I_1 \geq I_2$ and $I_1 \leq I_2$ will be true.

Definition 2.3. *[27] Let* X *be an initial universe set,* $V(X)$ *the set of all vague sets on* X*,* E *be a set of parameters, and* $A \subseteq E$*. A pair* (F, A) *is called a vague soft set over* X*, where* F *is a mapping given by* $F : A \to V(X)$*. The set of all vague soft sets on* X *is denoted by* $V\tilde{S}(X, E)$*, called vague soft classes. The interval* $[t_{F(e)}(x), 1 - f_{F(e)}(x)]$ *of* (F, A) *is called the vague soft value of* $x \in X$ *for the parameter* $e \in A$ *and is denoted by* $V_{F(e)}(x)$*.*

Definition 2.4. *[27] A vague soft set* (F, A) *over* X *is said to be a null vague soft set denoted by* $\hat{\emptyset}$*, if* $\forall e \in A$*,* $t_{F(e)}(x) = 0$*,* $1 - f_{F(e)}(x) = 0$*,* $x \in X$*.*
That is, $V_{F(e)}(x) = [0, 0]$*,* $\forall e \in A, x \in X$*.*

Definition 2.5. *[27] A vague soft set* (F, A) *over* X *is said to be an absolute vague soft set denoted by* $\hat{X}$*, if* $\forall e \in A$*,* $t_{F(e)}(x) = 1$*,* $1 - f_{F(e)}(x) = 1$*,* $x \in X$*.*
That is, $V_{F(e)}(x) = [1, 1]$*,* $\forall e \in A, x \in X$*.*

Definition 2.6. *[27] The complement of a vague soft set* (F, A) *is denoted by* $(F, A)^c$ *and is defined by* $(F, A)^c = (F^c, A)$ *and is given by* $t_{F^c(e)}(x) = f_{F(e)}(x)$*,* $1 - f_{F^c(e)}(x) = 1 - t_{F(e)}(x)$ *, for all* $e \in A$*,* $x \in X$*.*
That is, $V_{F^c(e)}(x) = [f_{F(e)}(x), 1 - t_{F(e)}(x)]$*,* $\forall e \in A, x \in X$*.*

Definition 2.7. *[6] Let* X *be an initial universe set,* E *be the nonempty* ***fixed set of parameters*** *and* τ *be the collection of vague soft sets over* X*, then* τ *is said to be a vague soft topology on* X *if*

1. $\hat{\emptyset}_E, \hat{X}_E$ *belongs to* τ*.*
2. *the union of any number of vague soft sets in* τ *belongs to* τ*.*
3. *the intersection of any two vague soft sets in* τ *belongs to* τ*.*

The triplet (X, τ, E) *is called a vague soft topological space over* X*.*

Theorem 2.8. *[5] Let* (X, τ, E) *and* (Y, σ, K) *be two vague soft topological spaces. The vague soft function* g_{pu}*:* $V\tilde{S}(X, E) \to V\tilde{S}(Y, K)$ *is called vague soft continuous, if and only if for all* $(G, K) \in \sigma$*,* $g_{pu}^{-1}(G, K) \in \tau$*.*

Definition 2.9. *[13] Let X be an initial universal set, E be the nonempty set of parameters and $x \in X$ a fixed element in X. A vague soft set $(F, E) \in V\tilde{S}(X, E)$ is called x-**vague soft point**, if for the element $e \in E$,*

$$V_{F(e')}(x') = \begin{cases} [\alpha, 1-\beta], & \text{if } \quad e' = e \quad \text{and} \quad x' = x, \\ [0,0], & \text{Otherwise.} \end{cases}$$

for all $x' \in X$ and $e' \in E$, where $\alpha \in [0,1]$ and $\beta \in [0,1)$ are two fixed real numbers such that $0 \leq \alpha + \beta \leq 1$ with $[\alpha, 1-\beta] \neq [0,0]$. And it is denoted by $x_{e,[\alpha,1-\beta]}$ (shortly, x_e). The family of all x-vague soft points over (X, E) is denoted by $V\tilde{S}P_x(X, E)$.

Definition 2.10. *[13] Let $Y \subseteq X$. The vague characteristic set of Y on X is a vague set $\chi_{_Y} = \{(x, [t_{\chi_Y}(x), 1 - f_{\chi_Y}(x)]) \mid x \in X\}$ over X, where*

$$t_{\chi_Y}(x) = \begin{cases} 1, & \text{if } x \in Y, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and } 1 - f_{\chi_Y}(x) = \begin{cases} 1, & \text{if } x \in Y, \\ 0, & \text{otherwise.} \end{cases} \text{ for all } x \in X.$$

Definition 2.11. *[13] Let X be an initial universe set, E be the set of parameters and let $Y \subseteq X$. The vague soft characteristic set of Y over E is a vague soft set (F_Y, E) which is defined by $F_Y : E \to V(X)$ such that $F_Y(e) = \chi_{_Y}$ for all $e \in E$. Thus, for all $x \in X$,*

$$V_{\chi_{Y}(e)}(x) = \begin{cases} [1,1], & \text{if } x \in Y \\ [0,0], & \text{otherwise,} \end{cases} \text{ for all } e \in E. \text{ And it is denoted by } (\chi_{_Y}, E).$$

Notations: Throughout this paper, we use the notation $V\tilde{S}P_{x_1,x_2,x_3...}(X, E)$ is the collection of all x_i-vague soft points $V\tilde{S}P_{x_i}(X, E),\ x_i \in X$. Clearly, $V\tilde{S}P_X(X, E) = \bigcup\limits_{x \in X} V\tilde{S}P_x(X, E)$.

Definition 2.12. *[18] Euclidean space $\mathbb{R}$ is the set of all real numbers together with the topology by the Euclidean metric, $d(x, y) = |x - y|$, for all $x, y \in \mathbb{R}$.*

3 Vague Soft Fundamental Groups

Definition 3.1. *Let $(G, E) \in V\tilde{S}(X, E), (H, E') \in V\tilde{S}(X', E')$. The vague soft product of (G, E) and (H, E') is a vague soft set $(M, E \times E') = (G, E) \times (H, E')$ in $V\tilde{S}(X \times X', E \times E')$ which is defined by the mapping $M : E \times E' \to V(X \times X')$, where $M(e, e') = G(e) \times H(e')$ such that*

$$M(e, e') = \left\{ \frac{[\ min(t_{G(e)}(x), t_{H(e')}(x')),\ max(1 - f_{G(e_1)}(x), 1 - f_{H(e')}(x'))\ ]}{(x, x')};\ \forall (x, x') \in X \times X' \right\} \text{ for all}$$

$(e, e') \in E \times E'$.

Example 3.2. *Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$.*

If $(F, E) = \left\{ \begin{array}{l} \left\langle e_1, \frac{[0, 0.9]}{x_1}, \frac{[0.3, 0.6]}{x_2} \right\rangle, \\ \left\langle e_2, \frac{[0.2, 0.7]}{x_1}, \frac{[1, 1]}{x_2} \right\rangle \end{array} \right\}, (G, E) = \left\{ \begin{array}{l} \left\langle e_1, \frac{[0.2, 0.5]}{x_1}, \frac{[0.4, 0.5]}{x_2} \right\rangle, \\ \left\langle e_2, \frac{[0.2, 1]}{x_1}, \frac{[0.2, 0.8]}{x_2} \right\rangle \end{array} \right\}$ *are two vague soft sets, then their vague soft product is given by*

$$(F,E)\times(G,E)=\left\{\begin{array}{c}\left\langle (e_1,e_1),\dfrac{[0,0.9]}{(x_1,x_1)},\dfrac{[0,0.9]}{(x_1,x_2)},\dfrac{[0.2,0.6]}{(x_2,x_1)},\dfrac{[0.3,0.6]}{(x_2,x_2)}\right\rangle,\\ \left\langle (e_1,e_2),\dfrac{[0,1]}{(x_1,x_1)},\dfrac{[0,0.9]}{(x_1,x_2)},\dfrac{[0.2,1]}{(x_2,x_1)},\dfrac{[0.2,0.8]}{(x_2,x_2)}\right\rangle\\ \left\langle (e_2,e_1),\dfrac{[0.2,0.7]}{(x_1,x_1)},\dfrac{[0.2,0.7]}{(x_1,x_2)},\dfrac{[0.2,1]}{(x_2,x_1)},\dfrac{[0.4,1]}{(x_2,x_2)}\right\rangle,\\ \left\langle (e_2,e_2),\dfrac{[0.2,1]}{(x_1,x_1)},\dfrac{[0.2,0.8]}{(x_1,x_2)},\dfrac{[0.2,1]}{(x_2,x_1)},\dfrac{[0.2,1]}{(x_2,x_2)}\right\rangle\end{array}\right\}.$$

Definition 3.3. *Let (X,τ,E), (X',σ,E') be two vague soft topological spaces. The vague soft topology on $X\times X'$ having the base of the form $\mathcal{F}=\{(F,E)\times(G,E'):(F,E)\in\tau,\quad(G,E')\in\sigma\}$ is said to be the vague soft product topology (denoted by $\tau\times\sigma$) of the vague soft topologies τ and σ. The triplet $(X\times X',\tau\times\sigma,E\times E')$ is said to be the vague soft product topological space of the vague soft topological spaces (X,τ,E) and (X',σ,E').*

Proposition 3.4. *Let (X,τ,E) and (Y,σ,K) be any two vague soft topological spaces. Let U and V be the subsets of X. Let $\hat{X}_E=(\chi_U,E)\cup(\chi_V,E)$, where $(\chi_U,E),(\chi_V,E)\in\tau$. If $f:(U,\tau_U,E)\to(Y,\sigma,K)$, $h:(V,\tau_V,E)\to(Y,\sigma,K)$ are any two vague soft continuous functions such that $f(F,E)=h(F,E)$, $\forall\ (F,E)\subseteq(\chi_U,E)\cap(\chi_V,E)$, then $g:(X,\tau,E)\to(Y,\sigma,K)$ is defined by*

$$g(G,E)=\begin{cases} f(G,E), & \text{if } (G,E)\subseteq(\chi_U,E)\\ h(G,E), & \text{if } (G,E)\subseteq(\chi_V,E).\end{cases}$$

is a vague soft continuous function.

Proof. Let $(M,K)\in(Y,\sigma,K)$. Now

$$\begin{aligned} g^{-1}(M,K) &= g^{-1}(M,K)\cap\hat{X}_E\\ &= g^{-1}(M,K)\cap((\chi_U,E)\cup(\chi_U,E))\\ &= [\ g^{-1}(M,K)\cap(\chi_U,E)\ ]\ \cup\ [\ g^{-1}(M,K)\cap(\chi_V,E)\ ]\\ &= f^{-1}(M,K)\cup h^{-1}(M,K)\in\tau. \end{aligned}$$

Hence, g is vague soft continuous. □

Definition 3.5. *Let (X,T) be a topological space and Q be the set of all parameters over X. Let U be the subset of X and (χ_U,Q) be the vague soft characteristic function of U. Then the vague soft topology introduced by T is $\mathbb{V}(T)_Q=\{(\chi_U,Q):U\in T\}$ and the pair $(X,\mathbb{V}(T)_Q)$ is said to be a vague soft topological space introduced by (X,T).*

Example 3.6. *Let (X,T) be a topological space where $X=\{a,b,c\}$ and $T=\{\emptyset,\{a\},\{b,c\},X\}$. Let $E=\{e_1,e_2\}$ be the parameters over X.*

Then $\mathbb{V}(T)_E=\{(\chi_\emptyset,E),(\chi_{\{a\}},E),(\chi_{\{b,c\}},E),(\chi_X,E)\}$ forms a vague soft topology where

$$(\chi_\emptyset,E)=\hat{\emptyset}_E,\ (\chi_{\{a\}},E)=\left\{\begin{array}{c}\left\langle e_1,\frac{[1,1]}{a},\frac{[0,0]}{b},\frac{[0,0]}{c}\right\rangle,\\ \left\langle e_2,\frac{[1,1]}{a},\frac{[0,0]}{b},\frac{[0,0]}{c}\right\rangle\end{array}\right\},\ (\chi_{\{b,c\}},E)=\left\{\begin{array}{c}\left\langle e_1,\frac{[0,0]}{a},\frac{[1,1]}{b},\frac{[1,1]}{c}\right\rangle,\\ \left\langle e_2,\frac{[0,0]}{a},\frac{[1,1]}{b},\frac{[1,1]}{c}\right\rangle\end{array}\right\}\ \text{and } (\chi_X,E)=\hat{X}_E.$$

Hence $(X,\mathbb{V}(T)_E)$ is a vague soft topological space introduced by (X,T).

Notation: Let I be the unit interval and Q be the set all parameters over I. Let ξ be an Euclidean topology on I. Then $(I,\mathbb{V}(\xi)_Q)$ is a vague soft topological space introduced by the Euclidean space (I,ξ).

Definition 3.7. *Let (X, τ, E) be a vague soft topological space and $(I, \mathbb{V}(\xi)_Q)$ be a vague soft topological space introduced by the Euclidean space (I, ξ) and $x_{e,[\alpha,1-\beta]}, x'_{e',[\gamma,1-\delta]} \in V\tilde{S}P_{x,x'}(X, E)$. A **vague soft-path** ω in (X, τ, E) from $x_{e,[\alpha,1-\beta]}$ to $x'_{e',[\gamma,1-\delta]}$ is a vague soft continuous function $\omega : (I, \mathbb{V}(\xi)_Q) \to (X, \tau, E)$ such that $\omega(0) = x_{e,[\alpha,1-\beta]}$ and $\omega(1) = x'_{e',[\gamma,1-\delta]}$. Then the x-vague soft points $x_{e,[\alpha,1-\beta]}$ and $x'_{e',[\gamma,1-\delta]}$ are called the **initial** and **terminal** points of ω.*

Definition 3.8. *Let ω be the vague soft-path in (X, τ, E) from $x_{e,[\alpha,1-\beta]}$ to $x'_{e',[\gamma,1-\delta]}$, where $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$, $x'_{e',[\gamma,1-\delta]} \in V\tilde{S}P_{x'}(X, E)$. The inverse of ω is the vague soft-path in (X, τ, E) from $x'_{e',[\gamma,1-\delta]}$ to $x_{e,[\alpha,1-\beta]}$ defined by $\bar{\omega}(t) = \omega(1 - t)$ for all $t \in I$.*

Definition 3.9. *Let (X, τ, E) be a vague soft topological space and $x_{e,[\alpha_1,1-\beta_1]}, x'_{e',[\alpha_2,1-\beta_2]} \in V\tilde{S}P_{x,x'}(X, E)$. A vague soft topological space (X, τ, E) is said to be a **vague soft path connected space** if there exists a vague soft-path in (X, τ, E) for every pair $x_{e,[\alpha_1,1-\beta_1]}, x'_{e',[\alpha_2,1-\beta_2]} \in V\tilde{S}P_{x,x'}(X, E)$.*

Proposition 3.10. *Let (X, τ, E) be a vague soft topological space and $x_{e,[\alpha_1,1-\beta_1]}, x'_{e',[\alpha_2,1-\beta_2]} \in V\tilde{S}P_{x,x'}(X, E)$. If there is a vague soft-path in (X, τ, E) with initial and terminal points $x_{e,[\alpha_1,1-\beta_1]}$, $x'_{e',[\alpha_2,1-\beta_2]}$ respectively, then there exist a vague soft-path in (X, τ, E) with initial and terminal points $x'_{e',[\alpha_2,1-\beta_2]}$, $x_{e,[\alpha_1,1-\beta_1]}$ respectively.*

Proof. Let $(I, \mathbb{V}(\zeta)_R)$ be a vague soft topological spaces introduced by the Euclidean space (I, ζ) and $x_{e',[\alpha_1,1-\beta_1]}, x'_{e',[\alpha_2,1-\beta_2]} \in V\tilde{S}P_{x,x'}(X, E)$. Let ω be a vague soft-path in (X, τ, E) from $x_{e,[\alpha_1,1-\beta_1]}$ to $x'_{e',[\alpha_2,1-\beta_2]}$. Then $\omega : (I, \mathbb{V}(\zeta)_R) \to (X, \tau, E)$ is a vague soft continuous function with $\omega(0) = x_{e,[\alpha,1-\beta]}$ and $\omega(1) = x'_{e',[\gamma,1-\delta]}$. Define $\sigma : (I, \mathbb{V}(\zeta)_R) \to (X, \tau, E)$ by $\sigma(t) = \omega(1 - t)$ for every $t \in I$. Therefore σ is a vague soft continuous function with $\sigma(0) = \omega(1) = x'_{e',[\gamma,1-\delta]}$ and $\sigma(1) = \omega(0) = x_{e,[\alpha,1-\beta]}$. Therefore, σ is a vague soft -path in (X, τ, E) with initial and terminal points $x'_{e',[\alpha_2,1-\beta_2]}$, $x_{e,[\alpha_1,1-\beta_1]}$ respectively. □

Theorem 3.11. *The vague soft continuous image of a vague soft path connected space is vague soft path connected.*

Proof. Let g_{pu}: $(X, \tau, E) \to (Y, \sigma, K)$ be a vague soft continuous function from a vague soft path connected space (X, τ, E) to vague soft topological space (Y, σ, K). Let $y_{k,[\alpha,1-\beta]}, y'_{k',[\gamma,1-\delta]}$ be any two (y, y')-vague soft points in $g_{pu}(X, \tau, E)$. Then there exist $x_{e,[\alpha,1-\beta]}$, $x'_{e',[\gamma,1-\delta]} \in V\tilde{S}P_{x,x'}(X, E)$ such that $u(x) = y$, $u(x') = y'$ $p(e) = k$, $p(e') = k'$. Since (X, τ, E) is a vague soft path connected space, there exist a vague soft-path ω in (X, τ, E) from $x_{e,[\alpha,1-\beta]}$ to $x'_{e',[\gamma,1-\delta]}$. Thus, there exist a vague soft continuous function $\omega : (I, \mathbb{V}(\xi)_Q) \to (X, \tau, E)$ such that $\omega(0) = x_{e,[\alpha,1-\beta]}$ and $\omega(1) = x'_{e',[\gamma,1-\delta]}$. clearly, the vague soft mapping $g_{pu} \circ \omega : (I, \mathbb{V}(\xi)_Q) \to (Y, \sigma, K)$ is vague soft continuous with $(g_{pu} \circ \omega)(0) = g_{pu}(\omega(0)) = g_{pu}(x_{e,[\alpha,1-\beta]}) = y_{k,[\alpha,1-\beta]}$ and $(g_{pu} \circ \omega)(1) = g_{pu}(\omega(1)) = g_{pu}(x'_{e',[\gamma,1-\delta]}) = y'_{k',[\gamma,1-\delta]}$. Hence, $g_{pu}(X, \tau, E)$ is a vague soft path connected space. □

Definition 3.12. *Let (X, τ, E) be a vague soft topological space and $(I, \mathbb{V}(\xi)_Q)$ be a vague soft topological space introduced by the Euclidean space (I, ξ). Let $x_{e,[\alpha_1,1-\beta_1]}$, $x'_{e',[\alpha_2,1-\beta_2]}$, $x''_{e'',[\alpha_3,1-\beta_3]} \in V\tilde{S}P_{x,x',x''}(X, E)$ and ω_1 & ω_2 be any two vague soft-path in (X, τ, E) from $x_{e,[\alpha_1,1-\beta_1]}$ to $x'_{e',[\alpha_2,1-\beta_2]}$ & from $x'_{e',[\alpha_2,1-\beta_2]}$ to $x''_{e'',[\alpha_3,1-\beta_3]}$ respectively. The product of ω_1 and ω_2 is the vague soft-path $\omega_1 * \omega_2$ in (X, τ, E) from $x_{e,[\alpha_1,1-\beta_1]}$ to $x''_{e'',[\alpha_3,1-\beta_3]}$ which is defined by*

$$(\omega_1 * \omega_2)(t_{q,[\mu,1-\nu]}) = \begin{cases} \omega_1((2t)_{q,[\mu,1-\nu]}), & \text{if } 0 \le t \le \frac{1}{2}, \\ \omega_2((2t-1)_{q,[\mu,1-\nu]}), & \text{if } \frac{1}{2} \le t \le 1, \end{cases}$$

for each $t_{q,[\mu,1-\nu]} \in (I, \mathbb{V}(\xi)_Q)$.

Definition 3.13. *Let* (X, τ, E) *be a* $V\tilde{S}TS$. *Let* $(I, \mathbb{V}(\zeta)_R)$ *and* $(I, \mathbb{V}(\xi)_Q)$ *be any two vague soft topological spaces introduced by the Euclidean spaces* (I, ζ) *and* (I, ξ) *respectively. Two vague soft-paths* ω_1 *&* ω_2 *in* (X, τ, E) *from* $x_{e,[\alpha_1,1-\beta_1]}$ *to* $x'_{e',[\alpha_2,1-\beta_2]}$ *are said to be a* ***vague soft-path homotopic*** *if there exists a vague soft continuous function* $\mathcal{H} : (I, \mathbb{V}(\zeta)_R) \times (I, \mathbb{V}(\xi)_Q) \to (X, \tau, E)$ *such that*

$$\mathcal{H}(t_{r,[\gamma,1-\delta]}, 0) = \omega_1(t_{r,[\gamma,1-\delta]}) \quad \text{and} \quad \mathcal{H}(t_{r,[\gamma,1-\delta]}, 1) = \omega_2(t_{r,[\gamma,1-\delta]})$$

for each $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I, R)$ *in* $(I, \mathbb{V}(\zeta)_R)$ *where* $t \in I$.

$$\mathcal{H}(0, t'_{q,[\mu,1-\nu]}) = x_{e,[\alpha_1,1-\beta_1]} \quad \text{and} \quad \mathcal{H}(1, t'_{q,[\mu,1-\nu]}) = x'_{e',[\alpha_2,1-\beta_2]}$$

for each $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I, Q)$ *in* $(I, \mathbb{V}(\xi)_Q)$ *where* $t' \in I$.

Moreover, the function $\mathcal{H}$ *is said to be a vague soft-path homotopy between* ω_1 *and* ω_2, *denoted as* $\omega_1 \cong \omega_2$.

Definition 3.14. *Let* (X, τ, E) *be a vague soft topological space and* $(I, \mathbb{V}(\xi)_Q)$ *be a vague soft topological space introduced by the Euclidean space* (I, ξ). *Let* $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$. *Let* $\omega : (I, \mathbb{V}(\xi)_Q) \to (X, \tau, E)$ *be a vague soft-path. If the initial point of* ω *equals its terminal point, that is,* $\omega(0) = \omega(1) = x_{e,[\alpha,1-\beta]}$, *then the vague soft-path* ω *is called as* ***vague soft-loop*** *based at the vague soft base point* $x_{e,[\alpha,1-\beta]}$. *The collection of all vague soft-loops based at* $x_{e,[\alpha,1-\beta]}$ *in* (X, τ, E) *is denoted by* $\Omega(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$.

Proposition 3.15. *Let* (X, τ, E) *be a vague soft topological space and* $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$. *Then the relation* $\cong$ *is an equivalence relation on* $\Omega(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$.

Proof. The proof is obvious. □

Definition 3.16. *Let* (X, τ, E) *be a vague soft topological space and* $x_{e,[\alpha_1,1-\beta_1]} \in V\tilde{S}P_x(X, E)$. *If* $\omega \in \Omega(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$, *then* $[\omega]$ *denotes the vague soft-path homotopy equivalence classes of vague soft loops based at* $x_{e,[\alpha,1-\beta]}$ *that contains* ω *and* $\Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ *denotes the set of all vague soft-path homotopy equivalence classes on* $\Omega(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$. *Define an operation* $\circ$ *on* $\Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ *by* $[\omega_1] \circ [\omega_2] = [\omega_1 * \omega_2]$.

Proposition 3.17. *Let* (X, τ, E) *be a vague soft topological space and* $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$. *Let* $\omega_1,\ \omega_2,\ \lambda_1,\ \lambda_2 \in \Omega(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ *be vague soft-loops in* (X, τ, E). *If* $\omega_1 \cong \omega_2$ *and* $\lambda_1 \cong \lambda_2$, *then* $\omega_1 * \lambda_1 \cong \omega_2 * \lambda_2$.

Proof. Let $(I, \mathbb{V}(\zeta)_R)$ and $(I, \mathbb{V}(\xi)_Q)$ be any two vague soft topological spaces introduced by the Euclidean space (I, ζ) and (I, ξ) respectively. Since $\omega_1 \cong \omega_2$ and $\lambda_1 \cong \lambda_2$, there exist vague soft continuous functions $\mathcal{H},\ \mathcal{K} : (I, \mathbb{V}(\zeta)_R) \times (I, \mathbb{V}(\xi)_Q) \to (X, \tau, E)$ such that

$$\mathcal{H}(t_{r,[\gamma,1-\delta]}, 0) = \omega_1(t_{r,[\gamma,1-\delta]}), \qquad \mathcal{H}(t_{r,[\gamma,1-\delta]}, 1) = \omega_2(t_{r,[\gamma,1-\delta]}),$$
$$\mathcal{K}(t_{r,[\gamma,1-\delta]}, 0) = \lambda_1(t_{r,[\gamma,1-\delta]}) \quad \text{and} \quad \mathcal{K}(t_{r,[\gamma,1-\delta]}, 1) = \lambda_2(t_{r,[\gamma,1-\delta]})$$

for each $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I, R)$ in $(I, \mathbb{V}(\zeta)_R)$,

$$\mathcal{H}(0, t'_{q,[\mu,1-\nu]}) = \mathcal{H}(1, t'_{q,[\mu,1-\nu]}) = \mathcal{K}(0, t'_{q,[\mu,1-\nu]}) = \mathcal{K}(1, t'_{q,[\mu,1-\nu]}) = x_{e,[\alpha,1-\beta]}$$

for each $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I, Q)$ in $(I, \mathbb{V}(\xi)_Q)$ and $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$.

Let $\mathcal{G} : (I, \mathbb{V}(\zeta)_R) \times (I, \mathbb{V}(\xi)_Q) \to (X, \tau, E)$ be such that

$$\mathcal{G}(t_{r,[\gamma,1-\delta]}, t'_{q,[\mu,1-\nu]}) = \begin{cases} \mathcal{H}((2t)_{r,[\gamma,1-\delta]}, t'_{q,[\mu,1-\nu]}), & \text{if } 0 \le t \le \frac{1}{2} \text{ and } 0 \le t' \le 1, \\ \mathcal{K}((2t-1)_{r,[\gamma,1-\delta]}, t'_{q,[\mu,1-\nu]}), & \text{if } \frac{1}{2} \le t \le 1 \text{ and } 0 \le t' \le 1, \end{cases}$$ for all

$t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I, R)$ in $(I, \mathbb{V}(\zeta)_R)$, $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I, Q)$ in $(I, \mathbb{V}(\xi)_Q)$. Thus $\mathcal{G}$ is vague soft continuous function.

Also, $$\mathcal{G}(t_{r,[\gamma,1-\delta]}, 0) = \begin{cases} \mathcal{H}((2t)_{r,[\gamma,1-\delta]}, 0), & \text{if } 0 \le t \le \frac{1}{2}, \\ \mathcal{K}((2t-1)_{r,[\gamma,1-\delta]}, 0), & \text{if } \frac{1}{2} \le t \le 1, \end{cases}$$

$$= \begin{cases} \omega_1((2t)_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t \le \frac{1}{2}, \\ \lambda_1((2t-1)_{r,[\gamma,1-\delta]}), & \text{if } \frac{1}{2} \le t \le 1, \end{cases}$$

$\mathcal{G}(t_{r,[\gamma,1-\delta]}, 0) = (\omega_1 * \lambda_1)(t_{r,[\gamma,1-\delta]})$ for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I, R)$ in $(I, \mathbb{V}(\zeta)_R)$,

Similarly, $$\mathcal{G}(t_{r,[\gamma,1-\delta]}, 1) = \begin{cases} \mathcal{H}((2t)_{r,[\gamma,1-\delta]}, 1), & \text{if } 0 \le t \le \frac{1}{2}, \\ \mathcal{K}((2t-1)_{r,[\gamma,1-\delta]}, 1), & \text{if } \frac{1}{2} \le t \le 1, \end{cases}$$

$$= \begin{cases} \omega_2((2t)_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t \le \frac{1}{2}, \\ \lambda_2((2t-1)_{r,[\gamma,1-\delta]}), & \text{if } \frac{1}{2} \le t \le 1, \end{cases}$$

$\mathcal{G}(t_{r,[\gamma,1-\delta]}, 1) = (\omega_2 * \lambda_2)(t_{r,[\gamma,1-\delta]})$ for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I, R)$ in $(I, \mathbb{V}(\zeta)_R)$ and

$\mathcal{G}(0, t'_{q,[\mu,1-\nu]}) = \mathcal{H}(0, t'_{q,[\mu,1-\nu]}) = x_{e,[\alpha,1-\beta]}$, $\mathcal{G}(1, t'_{q,[\mu,1-\nu]}) = \mathcal{K}(1, t'_{q,[\mu,1-\nu]}) = x_{e,[\alpha,1-\beta]}$

for each $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I, Q)$ in $(I, \mathbb{V}(\xi)_Q)$ and $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$.
Hence, $\omega_1 * \lambda_1 \cong \omega_2 * \lambda_2$. □

Proposition 3.18. *Let (X, τ, E) be a vague soft topological space. Let $[\omega_1]$, $[\omega_2] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ where $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$ be a x-vague soft point. Then $[\omega_1] \circ\ [\omega_2]\ \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$.*

Proof. Assume that, $[\omega_1]$, $[\omega_2] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ where $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$. We know that, $[\omega_1] \circ [\omega_2] = [\omega_1 * \omega_2]$.

Now by the Definition 3.12, the product $\omega_1 * \omega_2$ is also a vague soft-loop based at $x_{e,[\alpha,1-\beta]}$. Clearly, $[\omega_1] \circ\ [\omega_2] = [\omega_1 * \omega_2]\ \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$. □

Proposition 3.19. *Let (X, τ, E) be a vague soft topological space. Let $[\omega_1]$, $[\omega_2]$, $[\omega_3] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ where $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$ be a x-vague soft point. Then $(\ [\omega_1] \circ\ [\omega_2]\ ) \circ\ [\omega_3] =\ [\omega_1] \circ\ (\ [\omega_2] \circ\ [\omega_3]\ )$.*

Proof. It is sufficient to prove that $(\ \omega_1 *\ \omega_2\ ) *\ \omega_3 \cong\ \omega_1 *\ (\ \omega_2 *\ \omega_3\ )$. Now for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I, R)$ in $(I, \mathbb{V}(\zeta)_R)$,

$$((\omega_1 * \omega_2) * \omega_3)(t_{r,[\gamma,1-\delta]}) = \begin{cases} (\omega_1 * \omega_2)((2t)_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t \le \frac{1}{2}, \\ \omega_3((2t-1)_{r,[\gamma,1-\delta]}), & \text{if } \frac{1}{2} \le t \le 1. \end{cases}$$

$$((\omega_1 * \omega_2) * \omega_3)(t_{r,[\gamma,1-\delta]}) = \begin{cases} \omega_1((4t)_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t \le \frac{1}{4}, \\ \omega_2((4t-1)_{r,[\gamma,1-\delta]}), & \text{if } \frac{1}{4} \le t \le \frac{1}{2}, \\ \omega_3((2t-1)_{r,[\gamma,1-\delta]}), & \text{if } \frac{1}{2} \le t \le 1. \end{cases}$$

and $$(\omega_1 * (\omega_2 * \omega_3))(t_{r,[\gamma,1-\delta]}) = \begin{cases} \omega_1((2t)_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t \le \frac{1}{2}, \\ \omega_2((4t-2)_{r,[\gamma,1-\delta]}), & \text{if } \frac{1}{2} \le t \le \frac{3}{4}, \\ \omega_3((4t-3)_{r,[\gamma,1-\delta]}), & \text{if } \frac{3}{4} \le t \le 1. \end{cases}$$

Now define $\mathcal{G} : (I, \mathbb{V}(\zeta)_R) \times (I, \mathbb{V}(\xi)_Q) \to (X, \tau, E)$ by

$$\mathcal{G}(t_{r,[\gamma,1-\delta]}, t'_{q,[\mu,1-\nu]}) = \begin{cases} \omega_1((\frac{4t}{1+t'})_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t' \le 1 \text{ and } 0 \le t \le \frac{1+t'}{4}, \\ \omega_2((4t-1-t')_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t' \le 1 \text{ and } \frac{1+t'}{4} \le t \le \frac{2+t'}{4}, \\ \omega_3((\frac{(4t-2-t')}{2-t'})_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t' \le 1 \text{ and } \frac{2+t'}{4} \le t \le 1. \end{cases}$$

for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I,R)$ in $(I, \mathbb{V}(\zeta)_R)$, $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I,Q)$ in $(I, \mathbb{V}(\xi)_Q)$. Hence $\mathcal{G}$ is a vague soft continuous function, by Proposition 3.4.
Also, $\mathcal{G}(t_{r,[\gamma,1-\delta]}, 0) = ((\omega_1 * \omega_2) * \omega_3)(t_{r,[\gamma,1-\delta]})$ and $\mathcal{G}(t_{r,[\gamma,1-\delta]}, 1) = (\omega_1 * (\omega_2 * \omega_3))(t_{r,[\gamma,1-\delta]})$ for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I,R)$ in $(I, \mathbb{V}(\zeta)_R)$ and $\mathcal{G}(0, t'_{q,[\mu,1-\nu]}) = \mathcal{G}(1, t'_{q,[\mu,1-\nu]}) = x_{e,[\alpha,1-\beta]}$ for each $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I,Q)$ in $(I, \mathbb{V}(\xi)_Q)$ and $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X,E)$.
Hence $([\omega_1] \circ [\omega_2]) \circ [\omega_3] = [\omega_1] \circ ([\omega_2] \circ [\omega_3])$. □

Proposition 3.20. *Let (X, τ, E) be a vague soft topological space and $(I, \mathbb{V}(\zeta)_R)$ be a vague soft topological space introduced by the Euclidean space (I, ζ). Also let $\varrho : (I, \mathbb{V}(\zeta)_R) \to (X, \tau, E)$ be the vague soft-loop defined by $\varrho(t_{r,[\gamma,1-\delta]}) = x_{e,[\alpha,1-\beta]}$ for each $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I,R)$ in $(I, \mathbb{V}(\zeta)_R)$ and $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X,E)$. Then $[\omega] \circ [\varrho] = [\varrho] \circ [\omega] = [\omega]$ for each $[\omega] \in \Pi((X, \tau, E), x_{e,[\alpha,1-\beta]})$.*

Proof. Let $\omega : (I, \mathbb{V}(\xi)_Q) \to (X, \tau, E)$ be a vague soft-loop such that $\omega(0) = \omega(1) = x_{e,[\alpha,1-\beta]}$ where $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X,E)$ is a x-vague soft point.
Define $\mathcal{G} : (I, \mathbb{V}(\zeta)_R) \times (I, \omega S(\xi)_Q) \to (X, \tau, E)$ by

$$\mathcal{G}(t_{r,[\gamma,1-\delta]}, t'_{q,[\mu,1-\nu]}) = \begin{cases} \omega((\frac{2t}{1+t'})_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t' \le 1 \text{ and } 0 \le t \le \frac{1+t'}{2}, \\ \varrho((2t-1)_{r,[\gamma,1-\delta]}) = x_{e,[\alpha,1-\beta]}, & \text{if } 0 \le t' \le 1 \text{ and } \frac{1+t'}{2} \le t \le 1. \end{cases}$$

for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I,R)$ in $(I,\mathbb{V}(\zeta)_R)$, $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I,Q)$ in $(I,\mathbb{V}(\xi)_Q)$. Hence

$$\mathcal{G}\ (t_{r,[\gamma,1-\delta]},0) = \begin{cases} \omega((2t)_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t \le \dfrac{1}{2}, \\ \varrho((2t-1)_{r,[\gamma,1-\delta]}) = x_{e,[\alpha,1-\beta]}, & \text{if } \dfrac{1}{2} \le t \le 1. \end{cases}$$

Thus, $\mathcal{G}\ (t_{r,[\gamma,1-\delta]},0) = (\omega * \varrho)(t_{r,[\gamma,1-\delta]})$, $\mathcal{G}\ (t_{r,[\gamma,1-\delta]},1) = \omega(t_{r,[\gamma,1-\delta]})$, for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I,R)$ in $(I,\mathbb{V}(\zeta)_R)$, $\mathcal{G}\ (0,t'_{q,[\mu,1-\nu]}) = \omega(0) = x_{e,[\alpha,1-\beta]}$ and $\mathcal{G}\ (1,t'_{q,[\mu,1-\nu]}) = \varrho(1) = x_{e,[\alpha,1-\beta]}$ for each $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I,Q)$ in $(I,\mathbb{V}(\xi)_Q)$ and $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X,E)$. By Proposition 3.4, $\mathcal{G}$ is vague soft-continuous function and $\omega * \varrho \cong \omega$. Thus $[\omega] \circ [\varrho] = [\omega]$.

Similarly the proof of $[\varrho] \circ [\omega] = [\omega]$ can be obtained by defining the vague soft-path homotopy $\mathcal{H} : (I,\mathbb{V}(\zeta)_R) \times (I,\mathbb{V}(\xi)_Q) \to (X,\tau,E)$ by

$$\mathcal{H}\ (t_{r,[\gamma,1-\delta]},t'_{q,[\mu,1-\nu]}) = \begin{cases} \varrho((2t)_{r,[\gamma,1-\delta]}) = x_{e,[\alpha,1-\beta]}, & \text{if } 0 \le t' \le 1 \text{ and } 0 \le t \le \dfrac{1-t'}{2}, \\ \omega((\dfrac{2t+t'-1}{1+t'})_{r,[\gamma,1-\delta]}), & \text{if } 0 \le t' \le 1 \text{ and } \dfrac{1-t'}{2} \le t \le 1. \end{cases}$$

for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I,R)$ in $(I,\mathbb{V}(\zeta)_R)$, $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I,Q)$ in $(I,\mathbb{V}(\xi)_Q)$.

Hence $$\mathcal{H}\ (t_{r,[\gamma,1-\delta]},0) = \begin{cases} \varrho((2t)_{r,[\gamma,1-\delta]}) = x_{e,[\alpha,1-\beta]}, & \text{if } 0 \le t \le \dfrac{1}{2}, \\ \omega((2t-1)_{r,[\gamma,1-\delta]}), & \text{if } \dfrac{1}{2} \le t \le 1. \end{cases}$$

Thus, $\mathcal{H}\ (t_{r,[\gamma,1-\delta]},0) = (\varrho * \omega)(t_{r,[\gamma,1-\delta]})$, $\mathcal{H}\ (t_{r,[\gamma,1-\delta]},1) = \omega(t_{r,[\gamma,1-\delta]})$, for all $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I,R)$ in $(I,\mathbb{V}(\zeta)_R)$, $\mathcal{H}\ (0,t'_{q,[\mu,1-\nu]}) = \varrho(0) = x_{e,[\alpha,1-\beta]}$ and $\mathcal{H}\ (1,t'_{q,[\mu,1-\nu]}) = \omega(1) = x_{e,[\alpha,1-\beta]}$ for each $t'_{q,[\mu,1-\nu]} \in V\tilde{S}P_{t'}(I,Q)$ in $(I,\mathbb{V}(\xi)_Q)$ and $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X,E)$. By Proposition 3.4, $\mathcal{H}$ is vague soft-continuous function and $\varrho * \omega \cong \omega$. Thus $[\varrho] \circ [\omega] = [\omega]$. Hence $[\omega] \circ [\varrho] = [\varrho] \circ [\omega] = [\omega]$. □

Proposition 3.21. *Let (X,τ,E) be a vague soft topological space and let $[\omega] \in \Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )$ where $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X,E)$. Then $[\bar{\omega}] \in \Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]})$.*

Proof. Assume that, (X,τ,E) is a vague soft topological space and $[\omega] \in \Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )$ where $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X,E)$. Then $\omega \in \Omega(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )$ is vague soft-loop based at $x_{e,[\alpha,1-\beta]}$. Clearly, the inverse $\bar{\omega}$ is also a vague soft-loop based at $x_{e,[\alpha,1-\beta]}$ in (X,τ,E). Therefore, $\bar{\omega} \in \Omega(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )$ and hence $[\bar{\omega}] \in \Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]})$.

□

Proposition 3.22. *Let (X,τ,E) be a vague soft topological space and $x_{e,[\alpha_1,1-\beta_1]} \in V\tilde{S}P_x(X,E)$. Let $[\omega] \in \Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )$. Then there exists $[\bar{\omega}] \in \Pi(\ (X,\tau,E),\ x_{e,[\alpha_1,1-\beta_1]}\ )$ such that $[\omega]\circ[\bar{\omega}] = [\varrho] = [\bar{\omega}]\circ[\omega]$.*

Proof. The existence of $[\bar{\omega}]$ follows from the above Proposition 3.21.

Now consider $\varrho : (I,\mathbb{V}(\zeta)_R) \to (X,\tau,E)$ be the vague soft-loop defined by $\varrho(t_{r,[\gamma,1-\delta]}) = x_{e,[\alpha,1-\beta]}$ for each $t_{r,[\gamma,1-\delta]} \in V\tilde{S}P_t(I,R)$ in $(I,\mathbb{V}(\zeta)_R)$.
Next, to prove $[\omega] \circ [\bar{\omega}] = [\varrho] = [\bar{\omega}] \circ [\omega]$, it is sufficient to prove that $\omega * \bar{\omega} \cong \varrho \cong \bar{\omega} * \omega$.

Let $\mathcal{H} : (I,\mathbb{V}(\zeta)_R) \times (I,\mathbb{V}(\xi)_Q) \to (X,\tau,E)$ be defined by

$$\mathcal{H}\ (t_{r,[\gamma,1-\delta]},t'_{q,[\mu,1-\nu]})=\begin{cases} x_{e,[\alpha,1-\beta]}, & \text{if } 0\le t'\le 1 \text{ and } 0\le t\le \dfrac{t'}{2},\\ \omega((2t-t')_{r,[\gamma,1-\delta]}), & \text{if } 0\le t'\le 1 \text{ and } \dfrac{t'}{2}\le t\le \dfrac{1}{2},\\ \bar{\omega}((2t-1-t')_{r,[\gamma,1-\delta]}), & \text{if } 0\le t'\le 1 \text{ and } \dfrac{1}{2}\le t\le 1-\dfrac{t'}{2},\\ x_{e,[\alpha,1-\beta]}, & \text{if } 0\le t'\le 1 \text{ and } 1-\dfrac{t'}{2}\le t\le 1,\end{cases}$$

for all $t_{r,[\gamma,1-\delta]}\in V\tilde{S}P_t(I,R)$ in $(I,\mathbb{V}(\zeta)_R)$, $t'_{q,[\mu,1-\nu]}\in V\tilde{S}P_{t'}(I,Q)$ in $(I,\mathbb{V}(\xi)_Q)$. Thus

$$\mathcal{H}\ (t_{r,[\gamma,1-\delta]},0)=\begin{cases} \omega(2t)_{r,[\gamma,1-\delta]}, & \text{if } 0\le t\le \dfrac{1}{2},\\ \bar{\omega}(2t-1)_{r,[\gamma,1-\delta]}, & \text{if } \dfrac{1}{2}\le t\le 1.\end{cases}$$

$\Rightarrow$ $\mathcal{H}\ (t_{r,[\gamma,1-\delta]},0)=(\omega*\bar{\omega})(t_{r,[\gamma,1-\delta]})$, and $\mathcal{H}\ (t_{r,[\gamma,1-\delta]},1)=x_{e,[\alpha,1-\beta]}=\varrho(t_{r,[\gamma,1-\delta]})$ for all $t_{r,[\gamma,1-\delta]}\in V\tilde{S}P_t(I,R)$ in $(I,\mathbb{V}(\zeta)_R)$, $\mathcal{H}\ (0,t'_{q,[\mu,1-\nu]})=\mathcal{H}\ (1,t'_{q,[\mu,1-\nu]})=x_{e,[\alpha,1-\beta]}$ for each $t'_{q,[\mu,1-\nu]}\in V\tilde{S}P_{t'}(I,Q)$ in $(I,\mathbb{V}(\xi)_Q)$ and $x_{e,[\alpha,1-\beta]}\in V\tilde{S}P_x(X,E)$. Thus, $\omega*\bar{\omega}\cong\varrho$.

Similarly, the proof for $\bar{\omega}*\omega\cong\varrho$ can be obtained by defining the vague soft-path homotopy $\mathcal{G}:(I,\mathbb{V}(\zeta)_R)\times(I,\mathbb{V}(\xi)_Q)\to(X,\tau,E)$ by

$$\mathcal{G}\ (t_{r,[\gamma,1-\delta]},t'_{q,[\mu,1-\nu]})=\begin{cases} x_{e,[\alpha,1-\beta]}, & \text{if } 0\le t'\le 1 \text{ and } 0\le t\le \dfrac{t'}{2},\\ \bar{\omega}((2t-t')_{r,[\gamma,1-\delta]}), & \text{if } 0\le t'\le 1 \text{ and } \dfrac{t'}{2}\le t\le \dfrac{1}{2},\\ \omega((2t-1-t')_{r,[\gamma,1-\delta]}), & \text{if } 0\le t'\le 1 \text{ and } \dfrac{1}{2}\le t\le 1-\dfrac{t'}{2},\\ x_{e,[\alpha,1-\beta]}, & \text{if } 0\le t'\le 1 \text{ and } 1-\dfrac{t'}{2}\le t\le 1,\end{cases}$$

for all $t_{r,[\gamma,1-\delta]}\in V\tilde{S}P_t(I,R)$ in $(I,\mathbb{V}(\zeta)_R)$, $t'_{q,[\mu,1-\nu]}\in V\tilde{S}P_{t'}(I,Q)$ in $(I,\mathbb{V}(\xi)_Q)$. Thus, $\bar{\omega}*\omega\cong\varrho$. Hence $[\omega]\circ[\bar{\omega}]=[\bar{\omega}]\circ[\omega]=[\varrho]$. ☐

Theorem 3.23. *The set* $\mathbf{\Pi}(\ (\mathbf{X},\tau,\mathbf{E}),\ \mathbf{x}_{\mathbf{e},[\alpha,\mathbf{1}-\beta]}\ )$ *of vague soft-path homotopy equivalence classes of vague soft-loops at* $x_{e,[\alpha,1-\beta]}$ *forms a group under an operation* $\circ$, *is called the* ***vague soft fundamental group*** *of* (X,τ,E) *relative to the vague soft base point* $x_{e,[\alpha,1-\beta]}$.

Proof. It follows from the Propositions 3.18, 3.19, 3.20, 3.22. ☐

Definition 3.24. *Let* $\Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )$ *and* $\Pi(\ (Y,\sigma,K),\ y_{k,[\gamma,1-\delta]})$ *be any two vague soft fundamental groups. A function* $\mathfrak{f}:\Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )\to\Pi(\ (Y,\sigma,K),\ y_{k,[\gamma,1-\delta]})$ *is said to be a vague soft homomorphism if* $f([\omega_1]\circ[\omega_2])=f([\omega_1])\circ f([\omega_2])$ *for all* $[\omega_1],\ [\omega_2]\in\Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )$. *Moreover the vague soft homomorphism is said to be vague soft isomorphism if it is bijective.*

Proposition 3.25. *Let* (X,τ,E) *be a vague soft-path connected space and* $x_{e,[\alpha,1-\beta]},x'_{e',[\alpha',1-\beta']}\in V\tilde{S}P_{x,x'}(X,E)$. *Then there exists a vague soft isomorphism of* $\Pi(\ (X,\tau,E),\ x_{e,[\alpha,1-\beta]}\ )$ *onto* $\Pi(\ (X,\tau,E),\ x'_{e',[\alpha',1-\beta']}\ )$.

Proof. Proof is obvious. $\square$

Note 3.26. *If $f : (X, \tau, E) \to (Y, \sigma, K)$ is a vague soft continuous function and if ω_1, ω_2 are the vague soft-paths with $\omega_1(1) = \omega_2(0)$, then $f(\omega_1 * \omega_2) = f(\omega_1) * f(\omega_2)$.*

Definition 3.27. *Let (X, τ, E) and (Y, σ, K) be any two vague soft path connected spaces, $[\omega] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha}$ and $f : (X, \tau, E) \to (Y, \sigma, K)$ be a vague soft continuous function with $f(x_{e,[\alpha,1-\beta]}) = y_{k,[\gamma,1-\delta]}$. Then the function $\mathfrak{f}_* : \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ ) \to \Pi(\ (Y, \sigma, K),\ y_{k,[\gamma,1-\delta]})$, defined by $\mathfrak{f}_*(\ [\omega]) = [f\omega]$ for all $[\omega] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ is called the vague soft function induced by f.*

Proposition 3.28. *Let (X, τ, E) and (Y, σ, K) be any two vague soft-path connected spaces. Let $f : (X, \tau, E) \to (Y, \sigma, K)$ be a vague soft continuous function and $x_{e,[\alpha,1-\beta]} \in V\tilde{S}P_x(X, E)$. Then f induces a vague soft homomorphism $\mathfrak{f}_* : \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ ) \to \Pi(\ (Y, \sigma, K),\ f(x_{e,[\alpha,1-\beta]})\ )$.*

Proof. For each $[\omega_1], [\omega_2] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$,

$$
\begin{aligned}
\mathfrak{f}_*([\omega_1] \circ [\omega_2]) &= \mathfrak{f}_*(\ [\omega_1 * \omega_2]) && \text{(by Definition 3.16)}\\
&= [f(\omega_1 * \omega_2)] && \text{(by Definition 3.27)}\\
&= [f\omega_1 * f\omega_2] && \text{(by note 3.26)}\\
&= [f\omega_1] \circ [f\omega_2] && \text{(by Definition 3.16)}\\
&= \mathfrak{f}_*([\omega_1]) \circ \mathfrak{f}_*([\omega_2]) && \text{(by Definition 3.27)}
\end{aligned}
$$

Thus, $\mathfrak{f}_*([\omega_1] \circ [\omega_2]) = \mathfrak{f}_*([\omega_1]) \circ \mathfrak{f}_*([\omega_2]) \quad \forall\ [\omega_1], [\omega_2] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$.
Hence $\mathfrak{f}_*$ is a vague soft homomorphism. $\square$

Proposition 3.29. *Let (X, τ, E), (X', τ', E') and (X'', τ'', E'') be any three vague soft-path connected spaces. If $g : (X, \tau, E) \to (X', \tau', E')$ and $f : (X', \tau', E') \to (X'', \tau'', E'')$ are two vague soft continuous functions and $x_{e,[\alpha,1-\beta]}$ is a x-vague soft point of (X, E), then $(\mathfrak{f} \circ \mathfrak{g})_* = \mathfrak{f}_* \circ \mathfrak{g}_*$.*

Proof. For each $[\omega] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$,

$$
\begin{aligned}
(\mathfrak{f} \circ \mathfrak{g})_*(\ [\omega]\ ) &= [(f \circ g)\omega]\\
&= [f(g(\omega))]\\
&= \mathfrak{f}_*([g(\omega)])\\
&= \mathfrak{f}_*(\mathfrak{g}_*([\omega]))\\
&= (\mathfrak{f}_* \circ \mathfrak{g}_*)([\omega])
\end{aligned}
$$

Thus,$(\mathfrak{f} \circ \mathfrak{g})_*(\ [\omega]\ ) = (\mathfrak{f}_* \circ \mathfrak{g}_*)([\omega]), \ \ \forall\ [\omega] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$. Hence, $(\mathfrak{f} \circ \mathfrak{g})_* = \mathfrak{f}_* \circ \mathfrak{g}_*$. $\square$

Theorem 3.30. *Let f be a vague soft isomorphism between the vague soft-path connected spaces (X, τ, E) and (X', τ', E'). Then $\mathfrak{f}_* : \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ ) \to \Pi(\ (X', \tau', E'),\ f(x_{e,[\alpha,1-\beta]})\ )$ is a vague soft isomorphism.*

Proof. Let (X, τ, E) and (X', τ', E') be any two vague soft topological spaces. Let $I_d(X, E) : (X, \tau, E) \to (X, \tau, E)$ and $\mathfrak{I}_{\mathfrak{d}(\Pi(X,E),\ x_{e,[\alpha,1-\beta]})} : \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ ) \to \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ be any two vague soft identity functions on (X, τ, E) and $\Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$ respectively.

For each $[\omega] \in \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ )$,

$$(I_d(X, E))_*([\omega]) = [I_d(X, E)\ (\omega)] = [\omega] = \mathfrak{I}_{\mathfrak{d}(\Pi(X,E),\ x_{e,[\alpha,1-\beta]})}([\omega]).$$

Thus, $(I_d(X, E))_* = \mathfrak{I}_{\mathfrak{d}(\Pi(X,E),\ x_{e,[\alpha,1-\beta]})}$.

By Proposition 3.28, f induces a vague soft homomorphism $\mathfrak{f}_* : \Pi(\ (X, \tau, E),\ x_{e,[\alpha,1-\beta]}\ ) \to \Pi(\ (X', \tau', E'),\ f(x_{e,[\alpha,1-\beta]})\ )$.

Now $(\mathfrak{f}^{-1})_* \circ \mathfrak{f}_* = (\mathfrak{f}^{-1} \circ \mathfrak{f})_* = (I_d(X, E))_* = \mathfrak{I}_{\mathfrak{d}(\Pi(X,E),\ x_{e,[\alpha,1-\beta]})}$ and similarly, $\mathfrak{f}_* \circ (\mathfrak{f}^{-1})_* = \mathfrak{I}_{\mathfrak{d}(\Pi(X,E),\ x_{e,[\alpha,1-\beta]})}$.
Since $(\mathfrak{f}_*)^{-1} = (\mathfrak{f}^{-1})_*$, we have $\mathfrak{f}_*$ is bijective. Hence $\mathfrak{f}_*$ is a vague soft isomorphism. $\square$

Acknowledgement

This work is catalyzed and financially supported by Tamilnadu State Council for Science and Technology, Department of Higher Education, Government of Tamilnadu under Ref. No. TNSCST/RFRS/VR/07/2019-20/3024.

References

[1] Muhammad Akram and Kar-Ping Shum. Vague lie subalgebras over a vague field. *Quasigroups and Related Systems*, 17(2):119–134, 2009.

[2] Hacı Aktaş. Some algebraic applications of soft sets. *Applied Soft Computing*, 28:327–331, 2015.

[3] Dionís Boixader, Gaspar Mayor, and Jordi Recasens. Aggregating fuzzy subgroups and t-vague groups. In *International Summer School on Aggregation Operators*, pages 40–52, 2017.

[4] Naim Çağman and Serdar Enginoğlu. Soft set theory and uni–int decision making. *European journal of operational research*, 207(2):848–855, 2010.

[5] Chang Wang, V. Inthumathi, and M. Pavithra. Vague soft separation axioms and vague soft continuous functions in vague soft topological spaces. *iItalian Journal of Pure and Applied Mathematics,*, N.46:1047–1067, 2021.

[6] Chang Wang and Y. Li. Topological structure of vague soft sets. *Abstract and Applied Analysis*, Volume 2014(Article Id 504021):1–8, 2014.

[7] Mustafa Demirci. Vague groups. *Journal of mathematical analysis and applications*, 230(1):142–156, 1999.

[8] S Cicily Flora, I Arockiarani, and Ganeshsree Selvachandran. An approach to bipolar vague group and its properties. *International Journal of Innovative Science, Engineering and Technology*, 5(1):70–76, 2018.

[9] W.L. Gau and D.J. Buehrer. Vague sets. *IEEE Transactions on Systems Man and Cybernetics*, 23(2):610–614, 1993.

[10] AR Hadipour and Arsham Borumand Saeid. Vague bf-algebras. *Buletinul Academiei de Ştiinţe a Republicii Moldova. Matematica*, (3):11–22, 2010.

[11] S. Hussain and B. Ahmad. Soft separation axioms in soft topological spaces. *Hacettepe Journal of Mathematics and Statistics*, 44(3):559–568, 2015.

[12] V Inthumathi, V Chitra, and S Jayasree. The role of operators on soft sets in decision making problems. *Int J Comput Appl Math*, 12(3):899–910, 2017.

[13] V. Inthumathi and M. Pavithra. Vague soft separation axioms via vague soft quasi-coincident. *Poincare Journal of Analysis and Applications*, 8(1(I) Special Issue):147–164, 2021.

[14] Hakimuddin Khan, Musheer Ahmad, and Ranjit Biswas. On vague groups. *International Journal of computational cognition*, 5(1):27–30, 2007.

[15] Pabitra Kumar Maji, Ranjit Biswas, and AkhilRanjan Roy. Soft set theory. *Computers and Mathematics with Applications*, 45(4-5):555–562, 2003.

[16] Won Keun Min. A note on soft topological spaces. *Computers and Mathematics with Applications*, 62(9):3524–3528, 2011.

[17] D. Molodtsov. Soft set theory - first results. *Computers and Mathematics with Applications*, 37(4-5):19–31, 1999.

[18] James R Munkres. Topology. 2000. *Prenctice Hall, US.*

[19] Sushama Patil and JD Yadav. Anti vague soft r-subgroup of near-ring. *Electronic Journal of Mathematical Analysis and Applications*, 8(2):297–304, 2020.

[20] K V Rama Rao and V Amarendra Babu. Vague vector space and vague modules. *International Journal of Pure and Applied Mathematics*, 111(2):179–188, 2016.

[21] Ganeshsree Selvachandran and Abdul Razak Salleh. Vague soft hypergroups and vague soft hypergroup homomorphism. *Advances in Fuzzy Systems*, 2014(Article ID 758637):1–10, 2014.

[22] Ganeshsree Selvachandran and Abdul Razak Salleh. Algebraic hyperstructures of vague soft sets associated with hyperrings and hyperideals. *The Scientific World Journal*, 2015(Article ID 780121):1–12, 2015.

[23] Güzide Senel and Naim Cagman. Soft topological subspaces. *Annals of Fuzzy Mathematics and Informatics*, 10(4):525–535, 2015.

[24] Sevda Sezer. Vague rings and vague ideals. *Iranian Journal of Fuzzy Systems*, 8(1):145–157, 2011.

[25] Muhammad Shabir and Munazza Naz. On soft topological spaces. *Computers & Mathematics with Applications*, 61(7):1786–1799, 2011.

[26] Banu Pazar Varol, A Aygunoglu, and H Aygun. On vague soft groups. *Annals of Fuzzy Mathematics and Informatics*, 7(2):205–218, 2014.

[27] Wei Xu, Jian Ma, Shouyang Wang, and Gang Hao. Vague soft sets and their properties. *Computers & Mathematics with Applications*, 59(2):787–794, 2010.

[28] Yunqiang Yin, Young Bae Jun, and Jianming Zhan. Vague soft hemirings. *Computers & Mathematics with Applications*, 62(1):199–213, 2011.

[29] I. Zorlutuna, M. Akdag, W K Min, and S. Atmaca. Remarks on soft topological spaces. *Annals of Fuzzy Mathematics and Informatics*, 3(2):171–185, 2012.

BIOGRAPHY

M. Pavithra is a Ph. D. Scholar of Mathematics under the guidance of Dr. V. Inthumathi, Associate professor and Head, PG and Research Department of Mathematics, Nallamuthu Gounder Mahalingam college, Pollachi. She is doing research in the area of Vague Soft Topological Spaces. She has published 7 research articles in reputed Journals.

V. Inthumathi received Doctoral degree in the field of Ideal topological spaces from Bharathiar University in 2012. She is working as an Associate Professor in Department of Mathematics in Nallamuthu Gounder Mahalingam college, Pollachi, India. She has 24 years of teaching experience. she has published about 45 research articles in reputed journals. She guided 23 M.Phil scholars and guiding 4 Ph.D. Scholars. Currently she is doing research in the area of soft topological spaces and Nano Ideal Topological Spaces.