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27th October 2021

Jointly Organized by

Department of Biological Science, Physical Science and Computational Science

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ABOUT THE INSTITUTION

A nations's growth is in proportion to education and intelligence spread among the masses. Having this idealistic vision, two great philanthropists late. S.P. Nallamuthu Gounder and Late. Arutchelver Padmabhushan Dr.N.Mahalingam formed an organization called Pollachi Kalvi Kazhagam, which started NGM College in 1957, to impart holistic education with an objective to cater to the higher educational needs of those who wish to aspire for excellence in knowledge and values. The College has achieved greater academic distinctions with the introduction of autonomous system from the academic year 1987-88. The college has been Re-Accredited by NAAC and it is ISO 9001 : 2015 Certified Institution. The total student strength is around 6000. Having celebrated its Diamond Jubilee in 2017, the college has blossomed into a premier Post-Graduate and Research Institution, offering 26 UG, 12 PG, 13 M.Phil and 10 Ph.D Programmes, apart from Diploma and Certificate Courses. The college has been ranked within Top 100 (72nd Rank) in India by NIRF 2021.

ABOUT CONFERENCE

The International conference on “Emerging Trends in Science and Technology (ETIST-2021)” is being jointly organized by Departments of Biological Science, Physical Science and Computational Science - Nallamuthu Gounder Mahalingam College, Pollachi along with ISTE, CSI, IETE, IEE & RIYASA LABS on 27th OCT 2021. The Conference will provide common platform for faculties, research scholars, industrialists to exchange and discus the innovative ideas and will promote to work in interdisciplinary mode.

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Usance of $M^x/G(a,b)/1$ Queue Model for a Real-Life Problem

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ABSTRACT:

This study presents a formal overture to model, simulate and analyze the overall performance of a laundry service representing categorizing of clothes, soaking if necessary, washing, drying,and steam pressing and delivering the clothes. The aim of this work is to reduce the service time of the server and to maximize profit. Performance measures like expected awaiting time, expected customers in line, idle period, and busy period are ascertained.

Keywords:*Bulk Queue models, batch arrival, batch service, classical vacation.*

Mathematics Subject Classification:68M20, 90B22, 60K25.

1.INTRODUCTION:

As we all know queues are a typical every-day encounter. when resources are constrained there queues form. Having queues makes financial sense in reality.One of the majorissues within the examination of any activity system is the investigation of delay. Delay is a more unpretentious concept. Queuing theory deals with consequences which include queuing (or holding up).Typical instances might be: banks, supermarkets, computers,public transport, etc. In general, all queuing frameworks can be broken down into individual sub-systems as arrival process,service mechanism, queue characteristics, notationwhich are briefly discussedin [12].

2.LITERATURE SURVEY:

Arumuganathan and Jeyakumar[1] envisioned a queuing framework for group services with closing timesin which server goes on vacation after closedown work (such as factory lockout, computer shutdown). Malliga and Arumuganathan [2] investigated a batch system queuing model in which the server had various vacation. After serving M groups continuously, the server switches to a secondary task. In case the queue metric is greater than "a",

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at this point the server starts the job of setting up and servicing "some" of the designated clients as delayed service,after the vacation.

Jeyakumar and Arumuganathan [3] examined the bulk queue frame with vacations and optional re-service. At the completion of service, the server executes a new service onrequestionprovided the queue range is less than a. In the event that no maintenance is required and the queue length is less than a, the server takes multiple vacations until it reaches ‘a’. Haridass and Arumuganathan [4] discussed the characteristics of the stacking system with plenty of space and re-maintenance. Reservations are made in the manner required by the customer.

Arumuganathan and Ramaswami[5] looked at state-dependent arrivals, which means that while the server is busy, the approaching rate gradually increases. The arrival rate gradually declines, when the server is in vacation. Karpagam and Ayyapan [6] proposed the use of a non-Markovian and bulk service queueing model. The demonstration activates a standby server, as if the primary server were down for maintenance. Following the completion of service, break was availed by the server. After a short break or if the line length is smaller than ‘a’ server offers vacation grouping until the number of clients reaches N.

A single server bulk service queueing system has been proposed by Jayaraman, Nadarajan, and Sitrarasu[7], where servers may be repaired at any moment. The performance measurements are calculated using a matrix-geometric algorithmic approach. The condition of the server affects the customer's arrival. The bulk benefit and accessible batch queueing framework were considered by Krishnamoorthy and Ushakumari [8]. i.e. The customer entry is single, however depending on the number of clients, the service given is single or bulk. The framework has a finite capacity. This prototype is perfect for lifting, transportation, and other uses. Individual or group departures are also possible.

A server with limited buffer service queuing system has been investigated by Laxmi and Gupta [9]. They examined the framework in which the clients would arrive one by one. The server is successfully used in this type of service rule. A number of execution measures are also examined into. Gupta and Laxmi [10] examined the markovian input practice and found that the facility is provided in groups to users. If the number of clients in the line is fewer, the server waits until the number of customers in the line gains 'a,' and then launch service for all clients. Furthermore, if the number of clients is between a and b, all users are served; if not, 'b' customers are served.

Haridass and Arumuganathan[13] looked at, a bulk queueing system in which the departing set of clients may seek reservice after service, and it is up to the server to accept or reject the request. The authors [15] investigated a queueing architecture for auxiliary occupations with a variable threshold policy (vacation). After the service is completed, the server conducts auxiliary work of type one and two, again and over, until the line measures N. The server carries on the service for a group of clients. Arumuganathan, Haridass,and Senthilkumar [14] investigated a framework, where the most important consumers are served right away. If there are fewer than 'b' users, they will be served. Only 'b' customer will be served, and remaining customers wait for service and joins the orbit if the more clients arrive.

3.MODEL DESCRIPTION:

Here in this model, it is assumed that the washer men had regular customers and to identify the clothes comfortably he/she uses permanent symbol on the clothes for each customer. The clothes for laundry service arrive may be sometimes single or mostly bulk. As soon as the clothes arrive which exceeds 2 the server (in this case study a washer men) starts categorizing clothes depending upon the amount of dirt in the clothes, colours of the cloth, weight of the cloth which is referred as setup work. And the maximum clothes he can wash for one cycle is 10. The clothes with more weight are washed first as it takes more time for drying then light-coloured clothes are washed first as it takes more time for drying then light-coloured clothes are taken for washing. After completing washing the clothes are rinsed twice or thrice and then put on the string for drying. From washing to drying of clothes the process was assumed as service. During washing the washer men follows hierarchy service as weighted clothes, light colour clothes, dark colour clothes, less weight clothes, at last dirty clothes which are soaked in detergent at the beginning after completing service, in case total of clothes is fewer than minimum of batch width the washer men (server) will avail classical vacation that is being idle. As soon as the number of clothes exceeds minimum of batch size or if the clothes are dried then the vacation is interrupted. The washer man goes for washing (service) if number of clothes is greater than minimum of batch size. After service completion in case there is no cloth for washing or if the number of clothes is less than minimum, he/she performs closedown work that is again separating clothes based on its mark and steam pressing those clothes.

The above process is modelled as $M^x/G(a,b)/1$ queue model with various vacation, Setup time,Close down time. In this article first the service provider starts setup work. After completing set up work, service is rendered to a batch of clients with min magnitude ‘a’ and maxi size ‘b’. After service completion, in case the size of the queue is smaller than ‘a’ the service persons performs closedown work and avails vacation else service is continued for a next batch of clients. At the end of vacation, the laundry person moves for setup work if the width of the queue is greater than minimum batch size or it performs vacation again if the line lengthis fewer than ‘a’. The flow of system is represented diagrammatically in Fig 1.

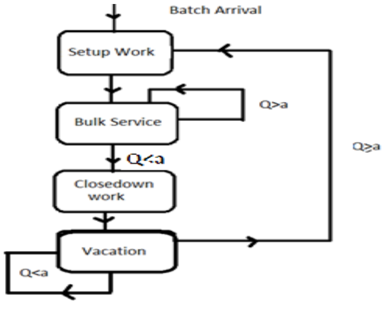


Fig 1

3.1Notations:

q-queue length, a - Minimum number of clothes for bulk service. b - Maximum number of clothes for bulk service. $N_q(t)$ denotes total customer waiting. The arrival rate is represented as λ.$N_s(t)$represents total number of customers served at time ‘t’. X be group size random variable, g_l- probability (there are l customers in the batch).X(z)- Probability generating function of X.S(),G( ),C( ),V( )stands for cdf of service time, setup time, vacation time, closedown time. Pdf of service time, setup time, closedown time, vacation time are mentioned as S(X),G(X), V(X),C(X).S_0(t) represents remaining service time.G_0(t) represents remaining setup time.V_0(t) represents remaining vacation time.C_0(t) represents remaining closedown time. $\tilde{S}(\theta)$, $\tilde{V}(\theta),\tilde{G}(\theta),\tilde{C}(\theta)$represents Laplace Stieltjes transform of S,V,G, C. $z(t) = j$ show that the server is in j^th vacation.

*cdf-Cumulative distribution function, pdf-Probability density function

$$y(t) = \begin{cases} 0 & server\ is\ busy \\ 1 & server\ \ performs\, setup\ work \\ 2 & server\ performs\, closedown\ work \\ 3 & server\, is\, in\ vacation \end{cases}$$

At various time epochs, the steady state equation for the proposed pattern is,

$$-P_{i0}^{'}(x) = -\lambda P_{i0}(x) + \sum_{k=a}^{b} P_{ki}(0)S(x) + G_i(0)S(x), \quad a \le i \le b \quad --1$$

$$-P_{ij}^{'}(x) = -\lambda P_{ij}(x) + \lambda\sum_{f=1}^{j} P_{i,j-f}(x)g_f\ , \quad j \ge 1, a \le i \le b-1 \quad --2$$

$$-P_{bj}^{'}(x) = -\lambda P_{bj}(x) + \sum_{l=a}^{b} P_{l,b+j}(0)S(x)\Delta t + G_{b+j}(0)S(x) + \lambda\sum_{l=1}^{j} P_{b,j-l}(x)g_l\ , \quad j \ge 1 \quad --3$$

$$-C_m^{'}(x) = -\lambda C_m(x) + \sum_{f=a}^{b} P_{f,m}(0)C(x) + \sum_{k=1}^{n} C_{m-k}(x)\lambda g_k\ , m \le a-1 \quad --4$$

$$-C_n^{'}(x) = -\lambda C_n(x) + \lambda\sum_{j=1}^{n-a} C_{n-j}(x)g_j\ , n \ge a \quad --5$$

$$-Q_{10}^{'}(x) = -\lambda Q_{10}(x) + C_0(0)V(x), \quad --6$$

$$-Q_{1m}^{'}(x) = -\lambda Q_{1m}(x) + \sum_{j=1}^{n} Q_{1,,m-j}(x)\lambda g_j\ + C_m(0)V(x), m \ge 1 \quad --7$$

$$-Q_{i0}^{'}(x) = -\lambda Q_{i0}(x) + Q_{i-1,0}(0)V(x), i \ge 2 \quad --8$$

$$-Q_{in}^{'}(x) = -\lambda Q_{in}(x) + Q_{i-1,0}(0)V(x) + \sum_{l=1}^{n} Q_{i,n-l}(x)\lambda g_l, \ n+1 \le a \ --9$$

$$-Q_{jn}^{'}(x) = -\lambda Q_{jn}(x) + \sum_{l=1}^{n} Q_{j,n-l}(x)\lambda g_l, \ n \ge a, 2 \le j \ --10$$

$$-G_n^{'}(x) = -\lambda G_n(x) + \sum_{k=1}^{n} G_{n-k}(x)\lambda g_k + \sum_{l=1}^{\infty} Q_{l,n}(0)g(x), \ a \le n----11$$

We get by applying the LST on both sides of the equation.

$$\theta \tilde{P}_{i0}(\theta)-P_{i0}(0)=\lambda \tilde{P}_{i0}(\theta)-\sum_{k=a}^{b}P_{ki}(0)\tilde{S}(\theta)-G_i(0)\tilde{S}(\theta),\ \ a\le i\le b\text{-----}12$$

$$\theta \tilde{P}_{ij}(\theta)-P_{ij}(0)=\lambda \tilde{P}_{ij}(\theta)-\lambda\sum_{f=1}^{j}P_{i,j-f}(\theta)g_f,\ \ a\le i\le b-1,\ j\ge 1\text{-----}13$$

$$\theta \tilde{P}_{bj}(\theta)-P_{bj}(0)=\lambda \tilde{P}_{bj}(\theta)-\sum_{k=a}^{b}P_{k,b+j}(0)\tilde{S}(\theta)-\lambda\sum_{k=1}^{j}\tilde{P}_{b,j-k}(\theta)g_k-G_{b+j}(0)\tilde{S}(\theta),\ \ j\ge 1\text{--}14$$

*LST-Laplace Stieltjes Transform

$$\theta \tilde{C}_n(\theta)-C_n(0)=\lambda \tilde{C}_n(\theta)+\sum_{f=a}^{b}P_{fn}(0)\tilde{C}(\theta)+\sum_{k=1}^{n}\tilde{C}_{n-k}(\theta)\lambda g_k,\ \ n\le a-1\text{-----}15$$

$$\theta \tilde{C}_m(\theta)-C_m(0)=\lambda \tilde{C}_m(\theta)-\lambda\sum_{k=1}^{n-a}\tilde{C}_{m-k}(\theta)g_k,\ \ n\ge a\text{------}16$$

$$\theta \tilde{Q}_{10}(\theta)-Q_{10}(0)=\lambda \tilde{Q}_{10}(\theta)-C_0(0)\tilde{V}(\theta)\text{-----------}17$$

$$\theta \tilde{Q}_{1n}(\theta)-Q_{1n}(0)=\lambda \tilde{Q}_{1n}(\theta)-\lambda\sum_{j=1}^{n}\tilde{Q}_{1,n-j}(\theta)g_j-C_n(0)\tilde{V}(\theta),n\ge 1\text{--------------}18$$

$$\theta \tilde{Q}_{i0}(\theta)-Q_{i0}(0)=\lambda \tilde{Q}_{i0}(\theta)-Q_{i-1,0}(0)\tilde{V}(\theta),i\ge 2\text{-----------}19$$

$$\theta \tilde{Q}_{in}(\theta)-Q_{in}(0)=\lambda \tilde{Q}_{in}(\theta)-Q_{i-1,0}(0)\tilde{V}(\theta)-\lambda\sum_{k=1}^{n}\tilde{Q}_{i,n-k}(\theta)g_k,n\le a-1\text{--------}20$$

$$\theta \tilde{Q}_{jn}(\theta)-Q_{jn}(0)=\lambda \tilde{Q}_{jn}(\theta)-\lambda\sum_{l=1}^{n}\tilde{Q}_{j,n-l}(\theta)g_l,n\ge a,\ j\ge 2\text{---------}21$$

$$\theta \tilde{G}_n(\theta)-G_n(0)=\lambda \tilde{G}_n(\theta)-\lambda\sum_{k=1}^{n}\tilde{G}_{n-k}(\theta)g_k-\sum_{l=1}^{\infty}\tilde{Q}_{l,n}(0)\tilde{G}(\theta),n\ge a\text{---------}22$$

The PGF's are as follows

$$\tilde{P}_i(z,\theta)=\sum_{k=0}^{\infty}\tilde{P}_{ik}(\theta)z^k\ \ \tilde{P}_i(z,0)=\sum_{l=0}^{\infty}P_{il}(0)z^l\ \ a\le i\le b$$

$$\tilde{Q}_j(z,\theta)=\sum_{s=1}^{\infty}\tilde{Q}_{sj}(\theta)z^j\ \ \tilde{Q}_j(z,0)=\sum_{f=1}^{\infty}Q_{fj}(0)z^j\ \ j\ge 1$$

$$\tilde{C}(z,\theta)=\sum_{y=0}^{\infty}\tilde{C}_y(\theta)z^y\ \ \tilde{C}(z,0)=\sum_{y=0}^{\infty}C_y(0)z^y$$

$$\tilde{G}(z,\theta)=\sum_{a=0}^{\infty}\tilde{G}_a(\theta)z^a\ \ \tilde{G}(z,0)=\sum_{b=0}^{\infty}G_b(0)z^b\ \text{----------}23$$

On multiplying the equations with z^n and totalling, we get

$$(\theta-\lambda+\lambda X(z))\tilde{P}_j(z,\theta)=\tilde{P}_j(z,0)-\tilde{S}(\theta)\sum_{l=a}^{b}P_{li}(0)-\tilde{S}(\theta)\tilde{G}_j(0),\ \ a\le j\le b-1\text{---------}24$$

$$\left(\theta-\lambda+\lambda X(z)\right)\tilde{P}_b\left(z,\theta\right)=\tilde{P}_b\left(z,0\right)-\frac{\tilde{S}(\theta)}{z^b}\left[\begin{array}{l}\sum_{m=a}^{b}\left[P_m(z,0)-\sum_{j=0}^{b-1}P_{mj}(0)z^n\right]\\+G\left(z,0\right)+\sum_{n=0}^{b-1}G_n(0)z^n\end{array}\right]\text{-----------}25$$

$$\left(\theta-\lambda+\lambda X(z)\right)\tilde{C}\left(z,\theta\right)-C\left(z,0\right)=\tilde{C}\left(z,\theta\right)\left(\sum_{n=0}^{a-1}\sum_{m=a}^{b}P_{m,n}\left(0\right)z^n\right)\text{----------}26$$

$$\left(\theta-\lambda+\lambda X(z)\right)\tilde{Q}_1\left(z,\theta\right)-Q_1\left(z,0\right)=C\left(z,0\right)\tilde{V}\left(\theta\right)\text{-------}27$$

$$\left(\theta-\lambda+\lambda X(z)\right)\tilde{Q}_j\left(z,\theta\right)=-\tilde{V}\left(\theta\right)\sum_{n=0}^{a-1}Q_{j-1,n}\left(0\right)z^n+Q_j\left(z,0\right),\ j\geq 2\text{------------------}28$$

$$\left(\theta-\lambda+\lambda X(z)\right)\tilde{G}\left(z,\theta\right)=G\left(z,0\right)-\tilde{G}\left(\theta\right)\sum_{l=1}^{\infty}\left[Q_1(z,0)-\sum_{j=0}^{a-1}Q_{l,j}\left(0\right)z^j\right]\text{--------}29$$

Substituting $\theta=\lambda-\lambda X\left(z\right)$ in equation, and considering $P_i=\sum_{k=a}^{b}P_{ki}(0)$, $q_i=\sum_{j=1}^{\infty}Q_{ji}(0)$ and $G_j=G_j(0)$

$$\tilde{C}\left(z,\theta\right)=\frac{\sum_{g=0}^{a-1}\sum_{f=a}^{b}P_{fg}(0)z^g\left[\tilde{C}\left(\lambda-\lambda X(z)\right)-C(\theta)\right]}{\left(\theta-\lambda+\lambda X(z)\right)}\text{---}30$$

$$\tilde{G}\left(z,\theta\right)=\frac{\sum_{k=1}^{\infty}\left[Q_k(z,0)-\sum_{j=0}^{a-1}Q_{kj}(0)z^j\right]\left[\tilde{G}\left(\lambda-\lambda X(z)\right)-G(\theta)\right]}{\left(\theta-\lambda+\lambda X(z)\right)}\text{-------}31$$

$$Q_1\left(z,\theta\right)=\frac{C(z,\theta)\left[\tilde{V}\left(\lambda-\lambda X(z)\right)-V(\theta)\right]}{\left(\theta-\lambda+\lambda X(z)\right)}\text{----------}32$$

$$Q_j\left(z,\theta\right)=\frac{\left[\sum_{f=0}^{a-1}Q_{j-1,f}(0)z^f\right]\left[\tilde{V}\left(\lambda-\lambda X(z)\right)-V(\theta)\right]}{\left(\theta-\lambda+\lambda X(z)\right)},\quad j\geq 2\text{------}33$$

$$P_i\left(z,\theta\right)=\frac{\left[\tilde{S}\left(\lambda-\lambda X(z)\right)-S(\theta)\right]}{\left(\theta-\lambda+\lambda X(z)\right)}\left[\sum_{n=a}^{b}P_{ni}(0)+G_i(0)\right],\quad a\leq i\leq b-1\text{---------------}34$$

$$P_b\left(z,\theta\right)=\frac{\left[\tilde{S}\left(\lambda-\lambda X(z)\right)-S(\theta)\right]k(z)}{\left(\theta-\lambda+\lambda X(z)\right)\left(z^b-S\left(\lambda-\lambda X(z)\right)\right)}\text{--------------}35$$

Where

$$k(z) = G(\lambda - \lambda X(z))\sum_{l=1}^{\infty}\left\{\sum_{n=0}^{a-1} Q_{j-1,n}(0) z^n \tilde{V}(\lambda - \lambda X(z))\right\} - \sum_{j=0}^{b-1} P_j z^j -$$

$$\sum_{n=0}^{b-1} G_n z^n + \sum_{m=a}^{b} \tilde{S}(\lambda - \lambda X(z))[P_i + G_i] \text{-------} 36$$

The PGF P(z) of the line length at any time is obtained, and this can be used to assess performance metrices.

The PGF of the line size is

$$P(z) = \frac{\left[\begin{array}{l}(z^b - 1)\left(\tilde{G}\right)\left(\tilde{V} - 1\right)\sum\limits_{n=0}^{a-1} q_n z^n + \sum\limits_{j=0}^{b-1} P_j z^j \left(z^b - \tilde{S}\right)\left(\tilde{C}\tilde{G}\tilde{V} - 1\right) \\ \qquad + \sum\limits_{i=a}^{b-1} [P_i]\left(\tilde{S} - 1\right)(z^b - z^i) + \left(1 - \tilde{S}\right)\sum\limits_{i=a}^{b-1} [G_i] z^b \end{array}\right]}{(-\lambda + \lambda X(z))(z^b - S(\lambda - \lambda X(z)))} \text{------(37)}$$

The above equation represents the PGF of total of clients in the line at a random time epoch. By implementing suitable numerical approach, we can find solution to these equations.

The PGF has to full fill P(1)=1,andwe get $\rho = \frac{\lambda E(X) E(S)}{b}$. $\rho < 1$ is the requirement for maintenance of steady state.

Theorem 1: Let G$_i$be expressed in relation to p$_i$ as $G_i = \left[\sum_{n=a}^{i} \gamma_{i-n} \sum_{u=0}^{a-1}\left[h_{n-u} + \sum_{j=0}^{a-1-u} k_j \alpha_{n-j-u}\right]\right] du,$ $i = a, a+1, \ldots b-1$ where γ_i is the possibility that 'i' customers come during the exceptional last vacation time.

Theorem 2: Let q$_i$be expelled in terms of p$_i$ as $\mathrm{q}_\mathrm{k} = \sum_{\mathrm{i}=0}^{\mathrm{n}} \mathrm{S}_\mathrm{i} \mathrm{P}_{\mathrm{k-i}},\quad k = 0,1,2,3 \ldots a-1$ where

$S_k = \frac{h_k + \sum_{i=1}^{n} \alpha_i S_{k-i}}{1 - \alpha_0},\ k = 1,2,3,\ldots a-1$ with $S_0 = \frac{\alpha_0 \beta_0}{1 - \alpha_0},\ h_n = \sum_{i=0}^{n} \alpha_0 \beta_{n-i}$ also chance of the 'i' customers appear during close-down time and vacation time are taken as β_i's andα_i's respectively.

3.2Particular Case:

*Case:1*By taking setup time alone as zero the equation (37)reduces to

$$P(z)=\frac{\left\{\begin{array}{l}\sum_{i=a}^{b-1}\left[z^{b}-z^{i}\right]P_{i}\left[\tilde{S}\left(\lambda-\lambda X(z)\right)-1\right]\\ +(z^{b}-1)\sum_{i=0}^{a-1}P_{i}z^{i}\left[\tilde{V}\left(\lambda-\lambda X(z)\right)\tilde{C}\left(\lambda-\lambda X(z)\right)-1)\right]\\ +(z^{b}-1)\sum_{i=0}^{a-1}q_{i}z^{i}\left[\tilde{V}\left(\lambda-\lambda X(z)\right)-1)\right]\end{array}\right\}}{\left(-\lambda+\lambda X(z)\right)\left(z^{b}-S\left(\lambda-\lambda X(z)\right)\right)} \text{------38}$$

Which coexist with the results considered in [13].

Case:2 Assuming the setup time and lockdown time to zero we get,

$$P(z)=\frac{\left\{\begin{array}{l}\left[\tilde{S}\left(\lambda-\lambda X(z)\right)-1\right]\sum_{i=a}^{b-1}\left[z^{b}-z^{i}\right]P_{i}\\ +(z^{b}-1)\left[\tilde{V}\left(\lambda-\lambda X(z)\right)-1)\right]\left[\sum_{i=0}^{a-1}P_{i}z^{i}+\sum_{i=0}^{a-1}q_{i}z^{i}\right]\end{array}\right\}}{\left(-\lambda+\lambda X(z)\right)\left(z^{b}-S\left(\lambda-\lambda X(z)\right)\right)} \text{------------39}$$

Which is nothing but the works of Krishnareddy and etal[10] if there in no setup work and N=a.

4.PERFORMANCE MEASURES:

4.1Expected length of busy period:

L is a random variable and defined as

$$L=\begin{cases}0 & if\ the\ server\ has\ less\ than\ 'a'\ customers\ after\ the\ residence\ time\\ 1 & if\ the\ server\ has\ atleast\ 'a'\ customers\ after\ the\ residence\ time\end{cases}$$

The duration of the anticipated engaged period is expressed as $E(B)=E(T)/P(L=0)$

$$E(B)=E(T)/\sum_{i=0}^{a-1}p_{i} \text{-------------29}$$

*4.2Expected length of idle period*E(I)*:*

The predicted idle period is now calculated as $E(I)=E(C)+E(G)+E(I_{1})$ where

$$\mathrm{E(I_1)}=\frac{\mathrm{E(V)}}{1-\sum_{n=0}^{a-1}Q_{1n}(0)}=\frac{E(V)}{1-\sum_{n=0}^{a-1}\sum_{i=0}^{n}\left\{\sum_{j=0}^{n-1}\alpha_{j}\beta_{n-i-j}\right\}p_{i}}\text{---}(30)$$

where E(C) means anticipated lockdown time and E(G) denotes awaited frame-up time, and I1 denotes the Idle Time owing to multiple vacation processes.

4.3Expected Queue Length:

On differentiating P(z) at z= We calculated the average line length E(Q) at any time epoch as

$$E(Q)=\frac{\begin{array}{l}\sum_{n=0}^{a-1} q_n\big[\big(3(b^2+2nb-b)\big)k_3-3bk_2+3b\lambda k_4+6bk_3k_5\big]\big(2\lambda E(x)(b+k_1)\big)\\ +\sum_{m=0}^{b} P_m(w_2)\big(2\lambda E(x)(b+k_1)\big)+\sum_{f=0}^{b-1} P_f(w_3)\big(2\lambda E(x)(b+k_1)\big)-w_5 2k_1(b-f)\\ +\sum_{i=a}^{b-1} G_i\big[(w_4)\big(2\lambda E(x)(b+k_1)\big)-w_5(k_9-\lambda k_9+bk_1+bk_1E(s))\big]\end{array}}{6\big[(\lambda E(x))(b+k_1)\big]^2} ---(31)$$

4.4 *Expected Waiting Time:* The expected waiting time E(w)is obtained as $E(w)=\dfrac{E(Q)}{\lambda E(x)}$

5.COST MODEL:

We drive a formulation to estimate the total average cost with the following undertaking. Let us denote the establishment cost be denoted as C_s. The several cost per unit time is as follows. Carrying cost is mentioned as C_h. Operating expenses is indicated as C_o, Renumeration is referred as C_r and frame-up cost is shown as C_g. Now, the expected cycle length $E(T_c)$, is obtained as $E(T_C)=E(I)+E(B)$

The total mean cost per unit is obtained by,

Total average cost=establishment cost/cycle + frame-up time cost + carrying cost of customer in the queue–renumeration on account of vacation + Closedown time cost+ operating expenses *ρ.

$$Total\ average\ \text{cost}=\left[C_s-\frac{C_rE(v)}{P(u=0)}+C_uE(c)+C_gE(G)\right]\frac{1}{E(T_c)}+C_hE(Q)+C_o.\rho$$

where $\rho=\dfrac{\lambda[E(S)]E(X)}{b}$

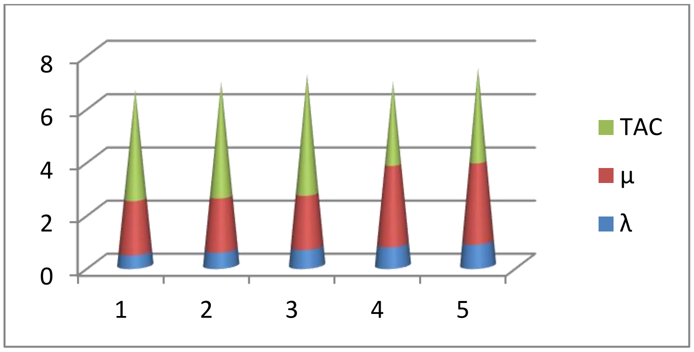
6.NUMERICAL SAMPLE:

A numerical prototype was examined to sketch in what way laundry men can utilize the above result to reduce the total average cost (TAC). The cloths arrive in bulk from consumers in the laundry service and are processed. The washerman continues his vacations after closing down work till he finds the bare minimum of clothes. He merely takes advantage of this opportunity to rest. When the washer man returns after a break and the quantity of clothing available exceeds the minimal value, he requests a setup time to categorise the garments by colour, amount of impurity, fabric weight, and other factors. In Laundry Service, the clothes arrive in group from customers and obey Poisson process (λ). After closedown work the washer man continues vacations until he finds minimum number of clothes. He utilizes this time only for taking rest. When the washer man returns from rest and in case the total of clothes in stock is more than min value, he demands for a setup time to categorize the clothes based on colour amount of dirt, heaviness of cloth etc. In order to utilize manpower and time consumption the washer man wishes to optimize that reduces the total cost. The above process has formed as $M^x/G(a,b)/1$ bulk service queuing structure.

The subsequent claims are made. With k=2, the service time distribution follows the K-Erlang distribution, μ=6.Batcharrival follows geometric distribution. Vacation time is distributed exponentially with a parameter $V = 10$. And setup time and closedown time are exponential with parameter $S = 7, C = 6$ .The various cost per unit time are as follows Establishment cost: Rs.4.00. frameup amount: Rs 0.50. Carrying charge for each customer: Rs 0.50.Operating expenses: Rs 5.00. Closedown time price: Rs 0.25. Remuneration figure on account of vacations: Rs 1.00. With b=4, the outcome for mixed threshold values and performance indicators are listed below.

λ	μ	ρ	EQL	EBP	EIP	EWT	TAC
0.5	2	0.25	5.1662	16.605	0.2494	5.1662	4.1149
0.6	2	0.3	4.9369	14.3668	0.2494	4.1141	4.2934
0.7	**2**	**0.35**	**4.7355**	**12.702**	**0.2493**	**3.3825**	**4.4845**
0.8	3	0.4	1.0904	8.0947	0.2529	0.6815	3.1142
0.9	3	0.45	1.387	7.716	0.2561	0.7706	3.5394

The overall average cost for their respective arrival and service rates is shown in the graph below.



7.CONCLUSION:

The proposed model's queue size Probability Generating Function is calculated. Different performance matrices are also conferred. The work helps laundry system management to take opinion in fixing the threshold value to initiate service. Some special cases also discussed.

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